Accelerator Physics
Bernhard Holzer
CERN
A Short Introduction
What we will do …

... introduce some “funny” keywords that you always wanted to understand and never really asked for.

trajectory / closed orbit / tune / resonances / chromaticity & dispersion
Higgs / structure of matter / beam emittance / adiabatic shrinking
beam size / beta function, focusing matrix / lattice cell
mini-beta insertion / “beta-star” / dynamic aperture

... and why do the particles not follow gravity and just drop down to the bottom of the vacuum chamber (... or do they do so ?)
Luminosity Run of a typical storage ring:

- guide the particles on a well defined orbit („design orbit“)
- focus the particles to keep each single particle trajectory within the vacuum chamber of the storage ring, i.e. close to the design orbit.
1.) Introduction and Basic Ideas

"... in the end and after all it should be a kind of circular machine"

→ need transverse deflecting force

Lorentz force

\[ \vec{F} = q^* (\vec{E} + \vec{v} \times \vec{B}) \]

typical velocity in high energy machines:

\[ v \approx c \approx 3 \times 10^8 \text{ m/s} \]

Example:

\[ B = 1 \text{T} \quad \rightarrow \quad F = q * 3 \times 10^8 \frac{m}{s} \cdot \frac{V}{s} \]

\[ F = q * 300 \frac{MV}{m} \]

Technical limit for electrical fields:

\[ E \leq 1 \frac{MV}{m} \]

equivalent electrical field:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit

\[ \rightarrow \text{Magnetic Guide Field} \]

condition for circular orbit:

- **Lorentz force**
  \[ F_L = e \nu B \]

- **centrifugal force**
  \[ F_{\text{centr}} = \frac{\gamma m_0 \nu^2}{\rho} \]

\[ \frac{\gamma m_0 \nu^3}{\rho} = e \nu B \]

\[ \frac{p}{e} = B \rho \]

"beam rigidity"

**Dipole Magnets:** define the ideal orbit

*homogeneous field* created by two flat pole shoes
The Magnetic Guide Field

The dipole magnets of a storage ring (or synchrotron) create a circle (... better polygon) of circumference \(2\pi \rho\) and define the maximum momentum of the particle beam.

Example LHC: \[2\pi \rho = 17.6 \text{ km}\]
\[\approx 66\%\]

About 1/3 of the ring size is still needed for straight sections, rf cavities, diagnostics, injection, extraction, high energy physics detectors etc etc
The Problem:

LHC Design Magnet current: \( I = 11850 \, A \)

and the machine is 27 km long !!!

Ohm’s law:  \( U = R \times I, \quad P = R \times I^2 \)

Task:
with \( I = 12000 \, A \) we have to reduce ohmic losses to the absolute minimum
The Solution: Super Conductivity ... and why we run at 1.9 K

discovery of sc. by H. Kammerling Onnes, Leiden 1911

thermal conductivity of fl. Helium in supra fluid state
LHC: The -1232- Main Dipole Magnets

required field quality: \( \Delta B/B = 10^{-4} \)

6 µm Ni-Ti filament

\[ \text{NbTi:} \quad B = 8.3 \, T \]

\[ \text{Nb}_3\text{Sn} \quad B = 16 \, T \]
3.) Focusing Properties – Transverse Beam Optics

... keeping the flocs together:
In addition to the pure bending of the beam we have to keep $10^{11}$ particles close together

classical mechanics: pendulum

there is a restoring force, proportional to the elongation $x$:

$$m \frac{d^2 x}{dt^2} = -c \cdot x$$

general solution: free harmonic oszillation

$$x(t) = A \cdot \cos(\omega t + \varphi)$$

this is how grandma‘s Kuckuck‘s clock is working!!!
**Quadrupole Magnets:**

**Storage Rings:** linear increasing Lorentz force to keep trajectories in vicinity of the ideal orbit

linear increasing magnetic field \( B_y = g \times \) \( x \) \quad \( B_x = g \times y \)

\[ F(x) = q \times v \times B(x) \]

**LHC main quadrupole magnet** \( g \approx 25 \ldots 220 \text{T/m} \)

---

**Table 7.13: Parameter list for main quadrupole magnets (MQ) at 7.0 TeV**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrated Gradient</td>
<td>690</td>
<td>T</td>
</tr>
<tr>
<td>Nominal Temperature</td>
<td>1.9</td>
<td>K</td>
</tr>
<tr>
<td>Nominal Gradient</td>
<td>223</td>
<td>T/m</td>
</tr>
<tr>
<td>Peak Field in Conductor</td>
<td>6.85</td>
<td>T</td>
</tr>
<tr>
<td>Temperature Margin</td>
<td>2.19</td>
<td>K</td>
</tr>
<tr>
<td>Working Point on Load Line</td>
<td>80.3</td>
<td>%</td>
</tr>
<tr>
<td>Nominal Current</td>
<td>11870</td>
<td>A</td>
</tr>
<tr>
<td>Magnetic Length</td>
<td>3.10</td>
<td>M</td>
</tr>
<tr>
<td>Beam Separation distance (cold)</td>
<td>194.0</td>
<td>mm</td>
</tr>
</tbody>
</table>
Focusing forces and particle trajectories:

normalise magnet fields to momentum
(remember: $B^*\rho = p / q$)

\[
\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}
\]

Dipole Magnet

\[
k := \frac{g}{p/q}
\]

Quadrupole Magnet
4.) A Bit of Theory

*The large Storage Rings and „Synchrotrons“*
The Equation of Motion:

\[ \frac{B(x)}{p/e} = \frac{1}{\rho} + k \ x + \frac{1}{2!} m \ x^2 + \frac{1}{3!} n \ x^3 + \ldots \]

only terms linear in \( x, y \) taken into account dipole fields
quadrupole fields

Separate Function Machines:

Split the magnets and optimise them according to their job:

bending, focusing etc

Example:

heavy ion storage ring TSR

\[ \text{man sieht nur dipole und quads } \quad \text{linear} \]
The Equation of Motion:

* Equation for the horizontal motion:

\[ x'' + x \left( \frac{1}{\rho^2} + k \right) = 0 \]

\[ x = \text{particle amplitude} \]
\[ x' = \text{angle of particle trajectory (wrt ideal path line)} \]

* Equation for the vertical motion:

\[ \frac{1}{\rho^2} = 0 \quad \text{no dipoles ... in general ...} \]
\[ k \leftrightarrow -k \quad \text{quadrupole field changes sign} \]
\[ \Rightarrow \text{UPSSSSSS} \]

\[ y'' - k y = 0 \]
\[ x'' + \left( \frac{1}{\rho^2} - k \right) x = 0 \]

**Remarks:**

... there seems to be a focusing even without a quadrupole gradient

"weak focusing of dipole magnets"

\[ k = 0 \quad \Rightarrow \quad x'' = -\frac{1}{\rho^2} x \]

even without quadrupoles there is a retrieving force (i.e. focusing) in the bending plane of the dipole magnets

... in large machines it is weak. (!)

Mass spectrometer: particles are separated according to their energy and focused due to the $1/\rho^2$ effect of the dipole
* Hard Edge Model:

\[ x'' + \left( \frac{1}{\rho^2} - k \right) x = 0 \]

\[ x''(s) + \left( \frac{1}{\rho^2(s)} - k(s) \right) x(s) = 0 \]

---

... this equation is not correct !!!

bending and focusing fields ... are functions of the independent variable „s“

Inside a magnet we assume constant focusing properties!

\[ \frac{1}{\rho} = \text{const} \quad k = \text{const} \]

---

\[ B l_{eff} = \int_{0}^{l_{mag}} B \, ds \]
5.) Solution of Trajectory Equations

Define ... hor. plane: \( K = \frac{1}{\rho^2} + k \)
... vert. Plane: \( K = -k \)

Differential Equation of harmonic oscillator ... with spring constant \( K \)

Ansatz: Hor. Focusing Quadrupole \( K > 0 \):

\[
x(s) = x_0 \cdot \cos(\sqrt{|K|} s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|} s)
\]

\[
x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|} s) + x'_0 \cdot \cos(\sqrt{|K|} s)
\]

For convenience expressed in matrix formalism:

\[
\begin{pmatrix}
x \\
x'
\end{pmatrix}_{s_1} = M_{foc} \begin{pmatrix}
x \\
x'
\end{pmatrix}_{s_0}
\]

\[
M_{foc} = \begin{pmatrix}
\cos(\sqrt{|K|} l) & \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|} l) \\
-\sqrt{|K|} \sin(\sqrt{|K|} l) & \cos(\sqrt{|K|} l)
\end{pmatrix}
\]
**hor. defocusing quadrupole:**

\[ x'' - K x = 0 \]

**Remember from school:**

\[ f(s) = \cosh(s), \quad f'(s) = \sinh(s) \]

**Ansatz:**

\[ x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s) \]

**drift space:** \( K = 0 \)

\[
M_{\text{defoc}} = \begin{pmatrix}
\cosh \sqrt{|K|l} & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|l} \\
\sqrt{|K|} \sinh \sqrt{|K|l} & \cosh \sqrt{|K|l}
\end{pmatrix}
\]

\[
M_{\text{drift}} = \begin{pmatrix}
1 & l \\
0 & 1
\end{pmatrix}
\]
Combining the two planes:

Clear enough (hopefully ... ?) : a quadrupole magnet that is focussing in one plane acts as defocusing lens in the other plane ... et vice versa.

**hor foc. quadrupole lens**

\[ M_{\text{foc}} = \begin{pmatrix} \cos(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s) \\ -\sqrt{|K|} \sin(\sqrt{|K|}s) & \cos(\sqrt{|K|}s) \end{pmatrix} \]

**matrix of the same magnet in the vert. plane:**

\[ M_{\text{defoc}} = \begin{pmatrix} \cosh \sqrt{|K|}l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|}l \\ \sqrt{|K|} \sinh \sqrt{|K|}l & \cosh \sqrt{|K|}l \end{pmatrix} \]

\[
\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{f} = \begin{pmatrix} \cos(\sqrt{|k|}s) & 0 & 0 & 0 \\ -\sqrt{|k|} \sin(\sqrt{|k|}s) & \cos(\sqrt{|k|}s) & 0 & 0 \\ 0 & 0 & \cosh(\sqrt{|k|}s) & \frac{1}{\sqrt{|k|}} \sinh(\sqrt{|k|}s) \\ 0 & 0 & \sqrt{|k|} \sinh(\sqrt{|k|}s) & \cosh(\sqrt{|k|}s) \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{i}
\]
"veni vidi vici ...“ .... or in english .... „we got it !“

* we can calculate the trajectory of a single particle, inside a storage ring magnet (lattice element)
* for arbitrary initial conditions $x_0, x'_0$
* we can combine these trajectory parts (also mathematically) and so get the complete transverse trajectory around the storage ring

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_D^*$$

Beispiel:
Speichering für Fußgänger (Wille)
Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

\[
M_{\text{total}} = M_{QF} \ast M_{D} \ast M_{QD} \ast M_{\text{Bend}} \ast M_{D^*}.....
\]

\[
\begin{pmatrix}
    x \\
    x'
\end{pmatrix}_{s_2} = M(s_2,s_1) \ast 
\begin{pmatrix}
    x \\
    x'
\end{pmatrix}_{s_1}
\]

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator !!!

**typical values in a strong foc. machine:**
\[x \approx \text{mm}, \ x' \leq \text{mrad}\]
First turn steering "by sector:"

- One beam at the time
- Beam through 1 sector (1/8 ring), correct trajectory, open collimator and move on.
6. Orbit & Tune:

Tune: number of oscillations per turn

64.31
59.32

Relevant for beam stability:
non integer part

LHC revolution frequency: 11.3 kHz

0.31*11.3 = 3.5 kHz

... and the tunes in x and y are different.

i.e. we can apply different focusing forces in the two planes

i.e. we can create different beam sizes in the two planes
**Dipole Magnets** ... 
... bend the particle trajectories onto a „polygon“ (... welll kind of ring),
... define the geometry of the machine
... define the maximum momentum (... or energy) that the particle beam will have
... have a small contribution to the focusing of the beam

**Quadrupole Magnets** ... 
... focus every single particle trajectory towards the centre of the vacuum chamber
... define the beam size
... „produce“ the tune
... increase the luminosity

**Trajectory** ... 
... under the influence of the focusing fields the particles follow a certain path along the machine. They are oscillating transversely, while moving around the „ring“.
Closed Orbit …
… There is one (!) trajectory that closes upon itself. It is given by the foc. fields and it is what we „see“ when we observe the BPM readings of the stored beam.

… The single particle will perform transverse oscillations and so the single particle trajectories will oscillate (= betatron oscillations) around this closed orbit.

The Tune …
… is the number of these transverse oscillations per turn and corresponds to the „Eigenfrequency“ or sound of the particle oscillations.
There is a tune for the horizontal, the vertical and the longitudinal oscillation.
And we could even hear it … if there were no vacuum.