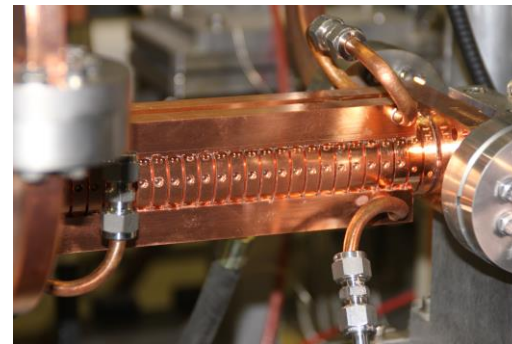
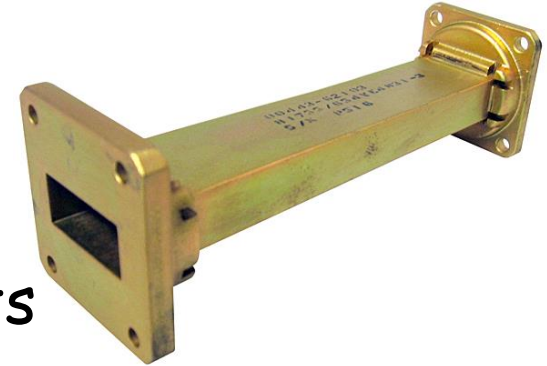


# RF Systems

Frank Tecker, CERN, BE-OP



- Waveguides and components
- Cavities
- RF sources
- RF systems



**Basics of Accelerator Physics and Technology**  
**ESI, Archamps, 25-29 June 2018**

# RF Systems

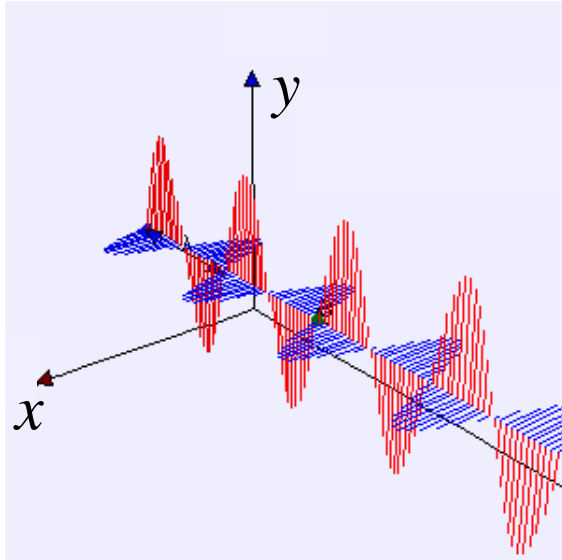
Frank Tecker  
CERN, BE-OP

Many thanks to Erk Jensen  
from whom I inherited the course  
for using much of his material

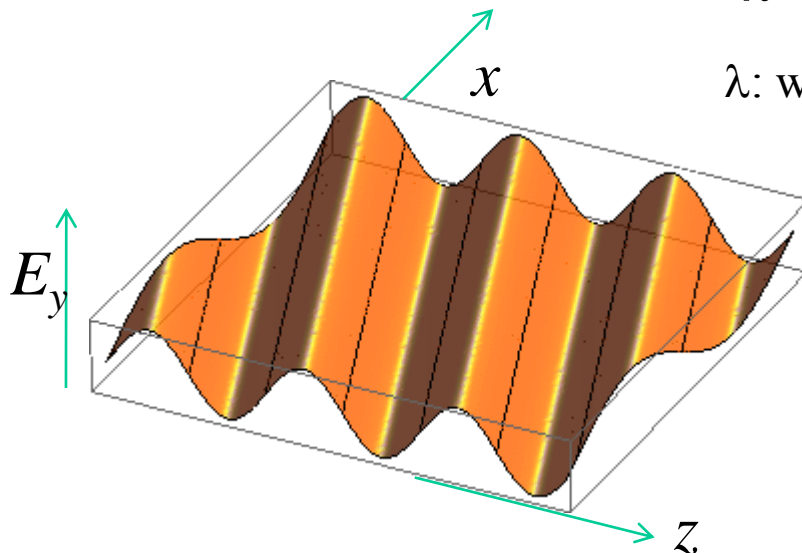
- Waves in waveguides and modes in cavities
- Types of cavities
  - Standing wave and travelling wave structures
- Cavity parameters:
  - Shunt impedance, transit time factor, quality factor, filling time
- Higher Order Modes and Wakefields
- Power and coupling to cavities
- RF systems and feedback loops

**Basics of Accelerator Physics and Technology**  
**ESI, Archamps, 25-29 June 2018**

# Electromagnetic Homogeneous Plane Wave



Courtesy: <http://weelookang.blogspot.ch/2011/10/ejs-open-source-propagation-of.html>



In free space:

**Electric and magnetic fields** are **perpendicular** to each other and to the direction of the wave

$$\vec{E} \propto \vec{u}_y \cos(\omega t - \vec{k} \cdot \vec{r})$$

$$\vec{B} \propto \vec{u}_x \cos(\omega t - \vec{k} \cdot \vec{r})$$

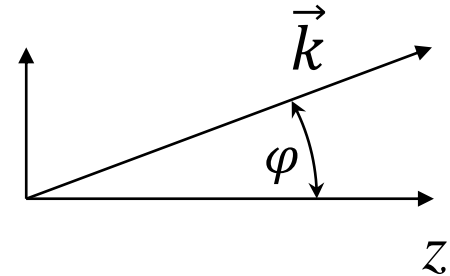
$$\omega = 2\pi f$$

t: time

**Wave number  $k$ :**

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = |\vec{k}|$$

$\lambda$ : wavelength,  $c$ : light speed



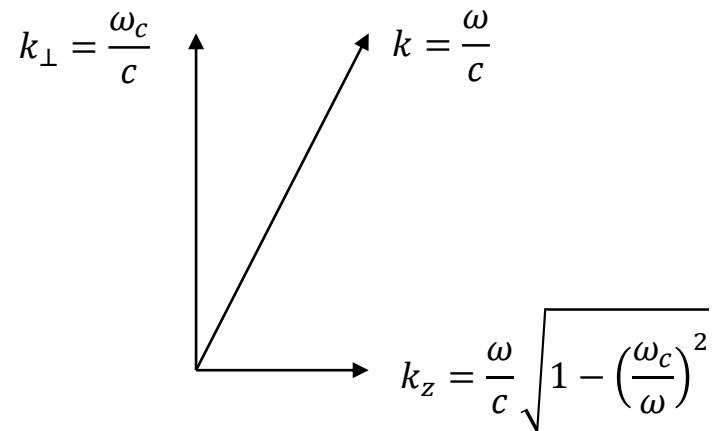
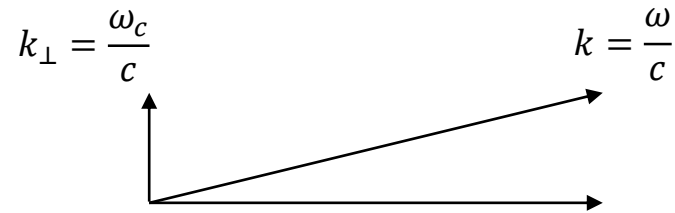
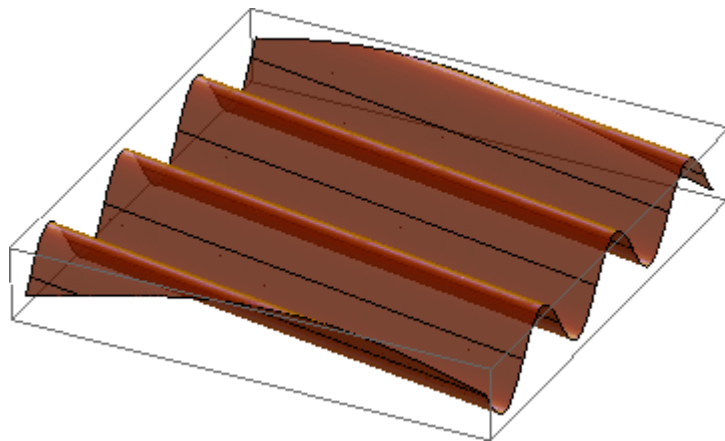
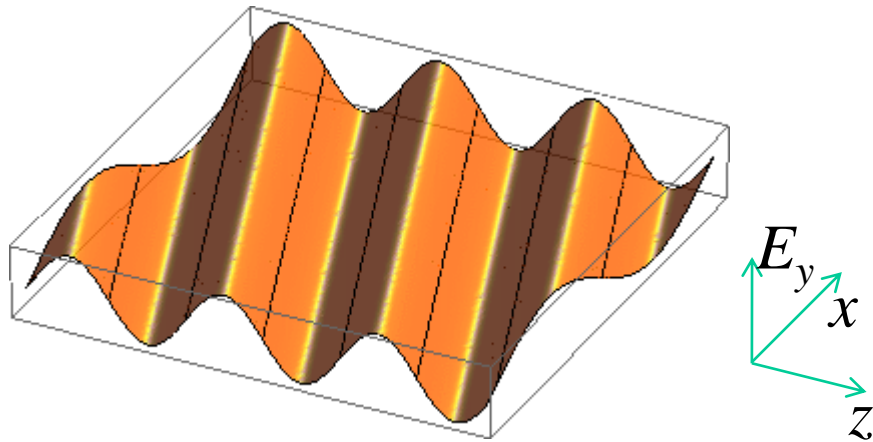
**Wave vector  $\vec{k}$ :**

- orthogonal to phase front
- the direction of  $\vec{k}$  is (usually) the direction of propagation
- the length of  $\vec{k}$  is the phase shift per unit length

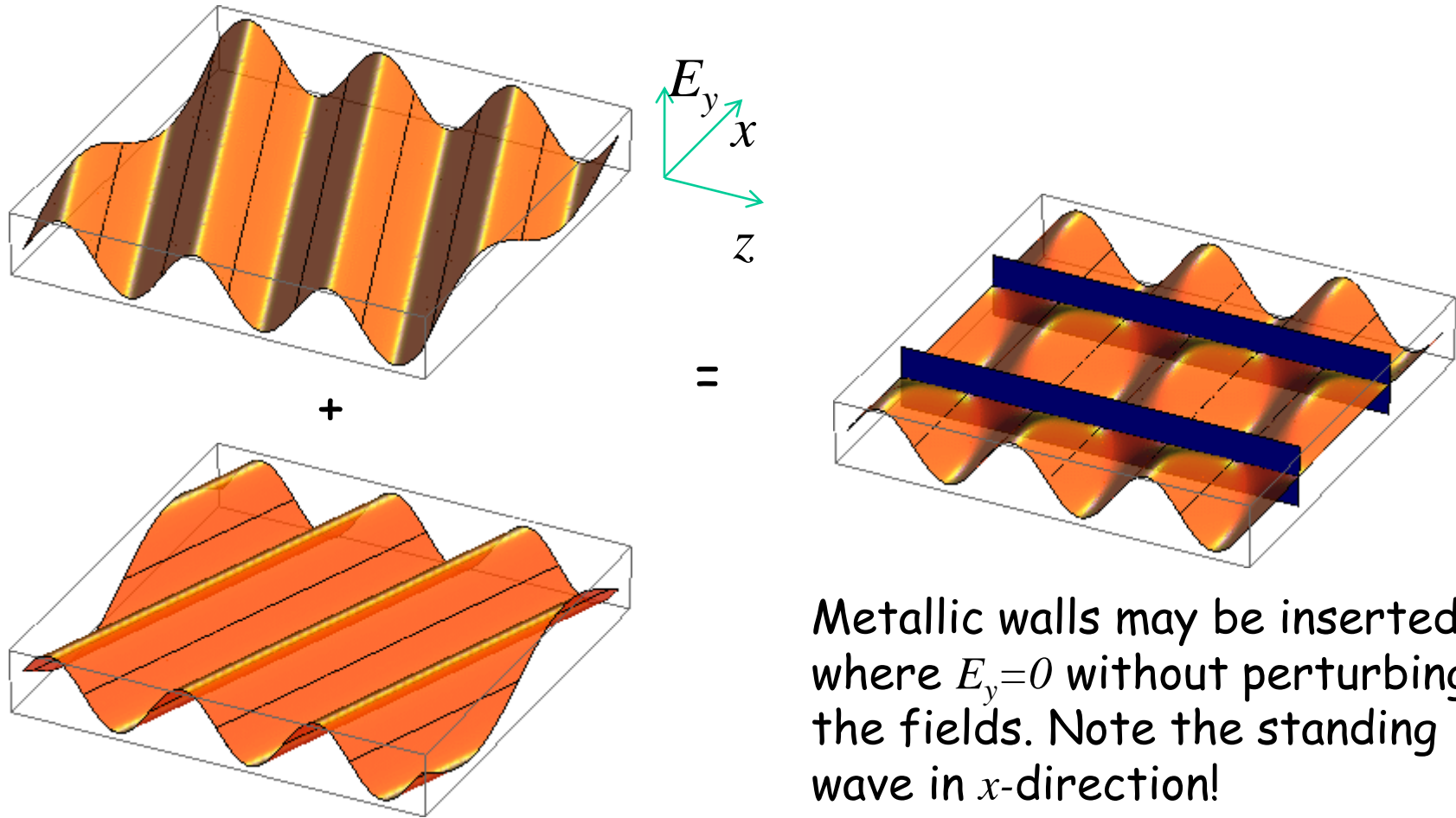
# Wave length, phase velocity

The components of  $\vec{k}$  are related to

- the wavelength in the direction of that component as  $\lambda_z = \frac{2\pi}{k_z}$  etc.
- to the phase velocity as  $v_{\phi,z} = \frac{\omega}{k_z} = f\lambda_z$ .



# Superposition of 2 homogeneous plane waves



Metallic walls may be inserted where  $E_y=0$  without perturbing the fields. Note the standing wave in  $x$ -direction!

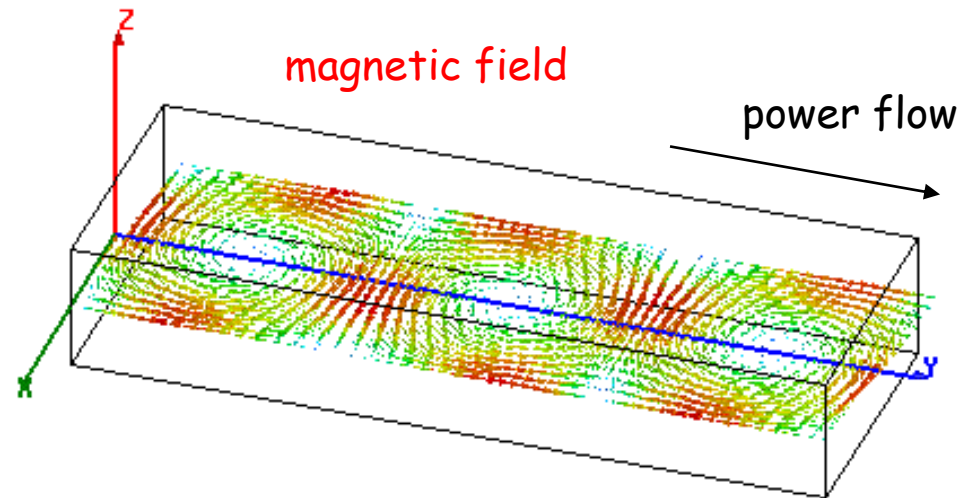
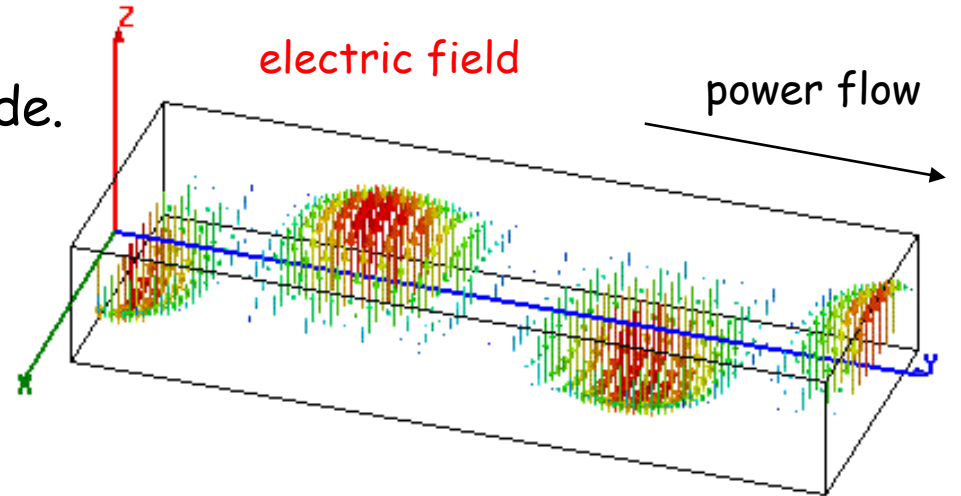
This way one gets a hollow rectangular waveguide!

# Rectangular waveguide

Fundamental ( $TE_{10}$  or  $H_{10}$ ) mode  
in a standard rectangular waveguide.  
E.g. forward wave

Electric and magnetic field  
travel in phase in the waveguide

$$\text{power flow: } \frac{1}{2} \text{Re}\left\{\iint \vec{E} \times \vec{H}^* dA\right\}$$

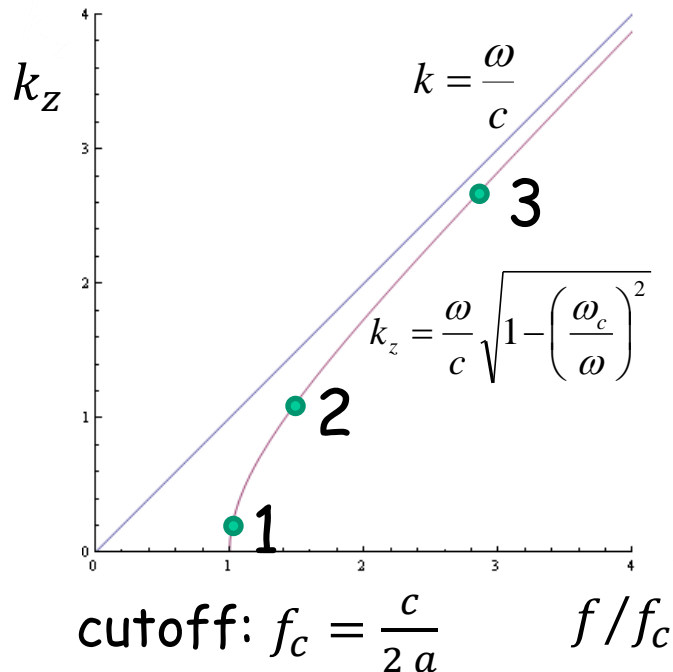


# Waveguide dispersion

Different waveguide width  $a$ :

Waves with  $\lambda > 2a$  don't propagate. Only frequencies higher **Cutoff**  $f_c = \frac{c}{2a}$  enter.

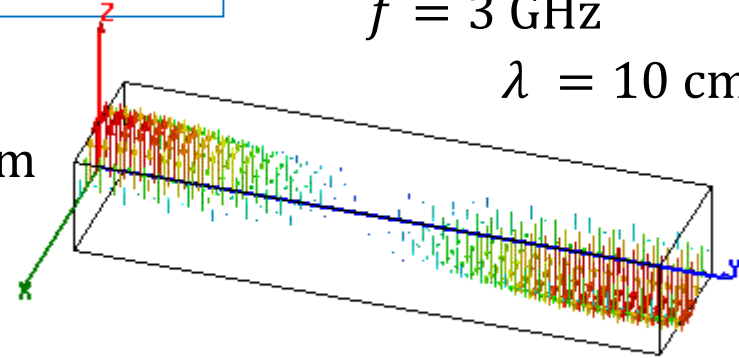
The "guided wavelength"  $\lambda_g$  varies from  $\infty$  at  $f_c$  to  $\lambda$  at very high frequencies.



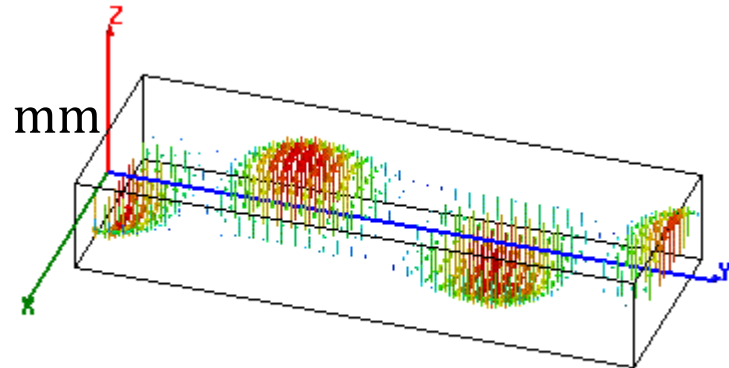
$$f = 3 \text{ GHz}$$

$$\lambda = 10 \text{ cm}$$

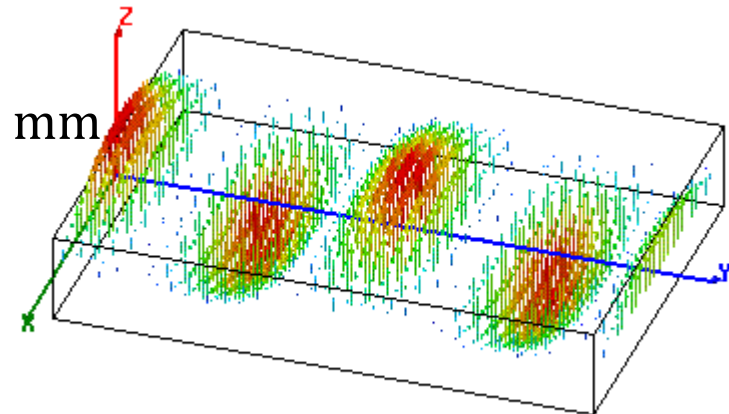
1:  
 $a = 52 \text{ mm}$   
 $\frac{f}{f_c} = 1.04$



2:  
 $a = 72.14 \text{ mm}$   
 $\frac{f}{f_c} = 1.44$



3:  
 $a = 144.3 \text{ mm}$   
 $\frac{f}{f_c} = 2.88$





# Phase velocity $v_\phi$

The phase velocity is the speed with which the crest or a zero-crossing travels.

Note in the animations that, at constant  $f$ , it is  $v_\phi \propto \lambda_g$ .

Note that at  $f = f_c$ ,  $v_\phi = \infty$ !

With  $f \rightarrow \infty$ ,  $v_\phi \rightarrow c$ !

Energy travels with group velocity

In a hollow waveguide:

- phase velocity  $v_\phi > c$

- group velocity  $v_{gr} < c$

$$v_{gr} \cdot v_\phi = c^2$$

In fixed dimension waveguide

⇒ Different frequencies travel with different speed.

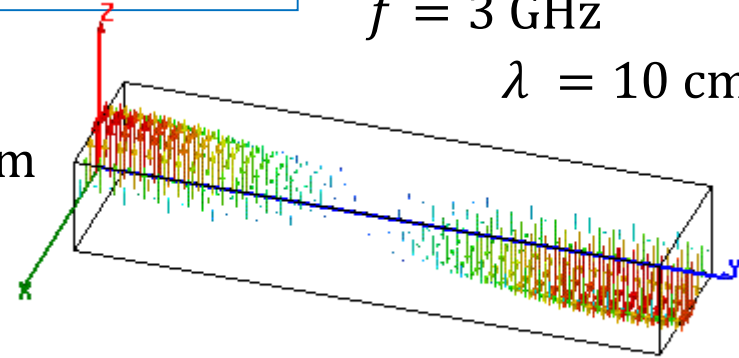
$$f = 3 \text{ GHz}$$

$$\lambda = 10 \text{ cm}$$

1:

$$a = 52 \text{ mm}$$

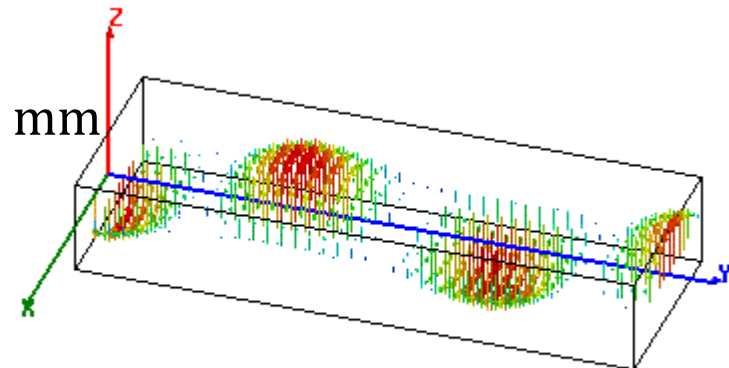
$$\frac{f}{f_c} = 1.04$$



2:

$$a = 72.14 \text{ mm}$$

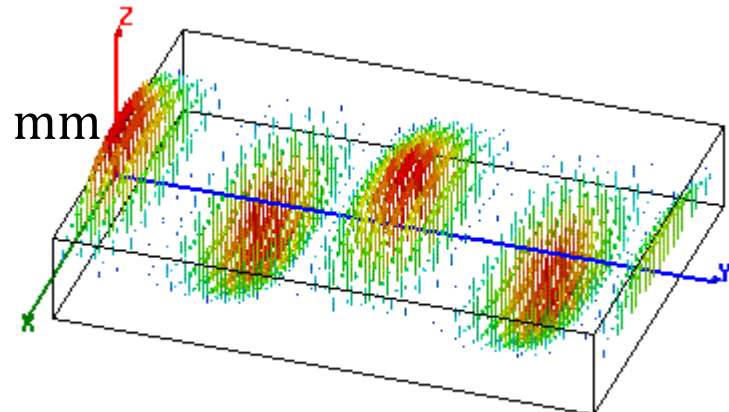
$$\frac{f}{f_c} = 1.44$$



3:

$$a = 144.3 \text{ mm}$$

$$\frac{f}{f_c} = 2.88$$

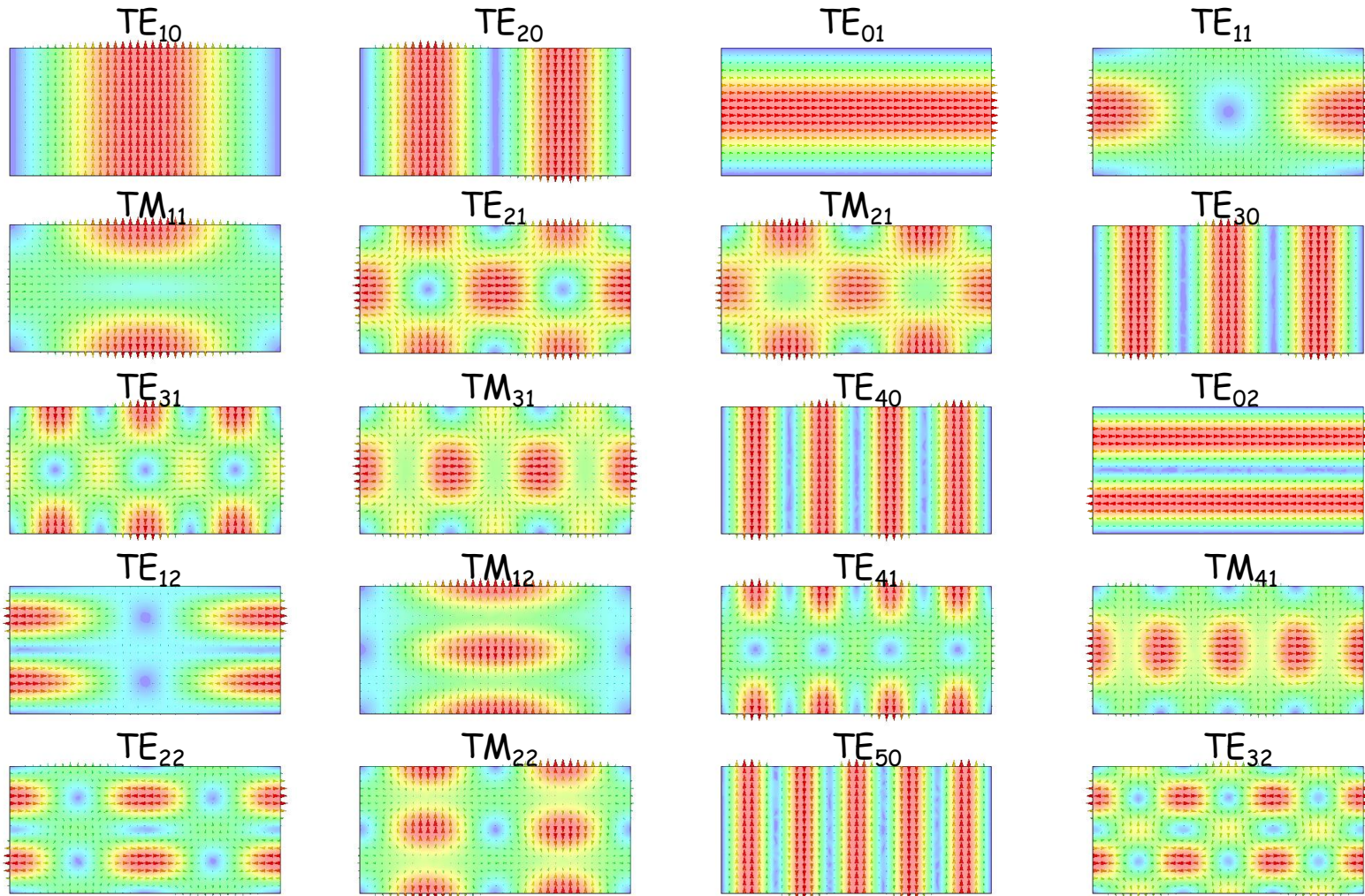




# Rectangular waveguide modes

Types:  $TM_{xy}$  (transverse magnetic) or  $TE_{xy}$  (transverse electric)

**Indices** indicate **number of half-waves** in transverse directions.

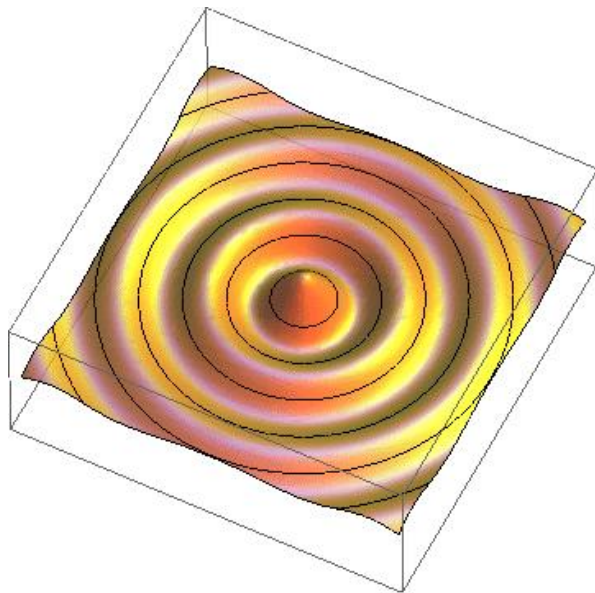


plotted: E-field

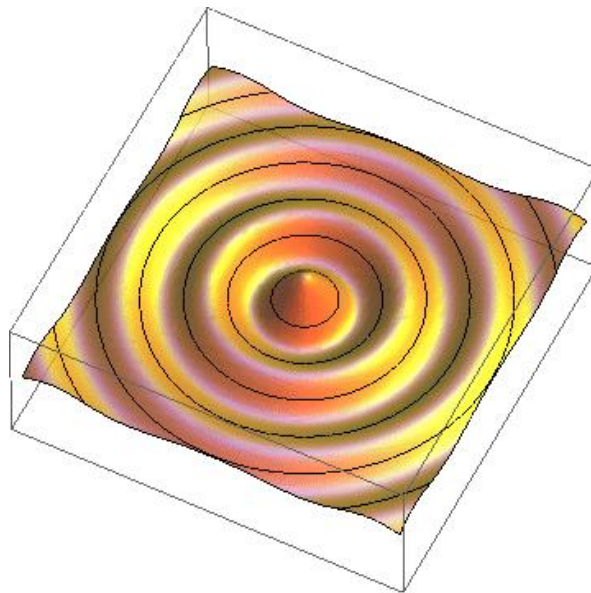
# Radial waves

Also radial waves may be interpreted as superposition of plane waves.

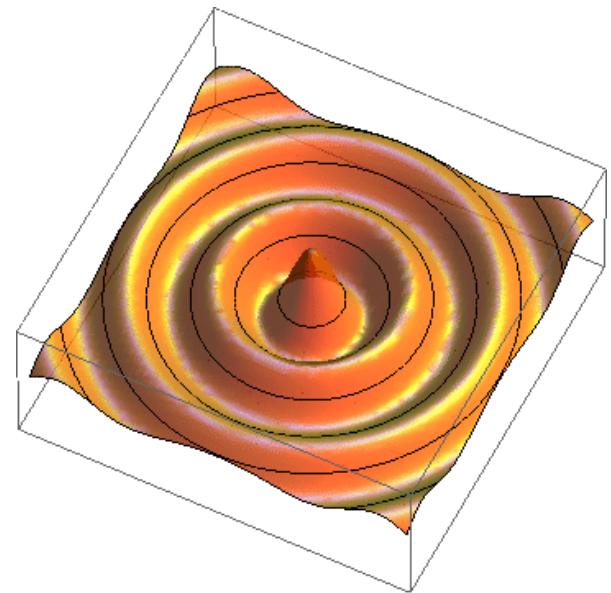
The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.



$$E_z \propto H_n^{(2)}(k_\rho \rho) \cos(n\varphi)$$



$$E_z \propto H_n^{(1)}(k_\rho \rho) \cos(n\varphi)$$

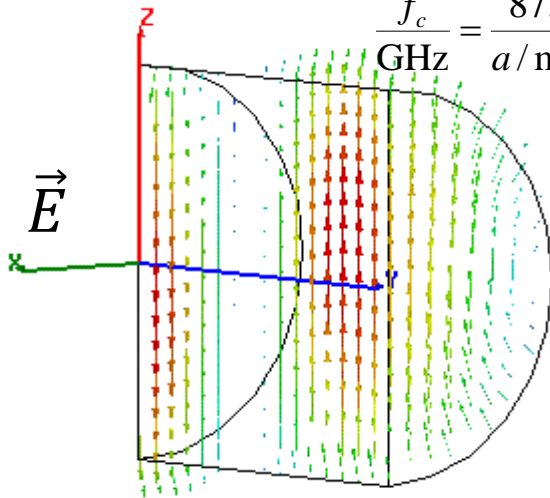


$$E_z \propto J_n(k_\rho \rho) \cos(n\varphi)$$

# Round waveguide modes

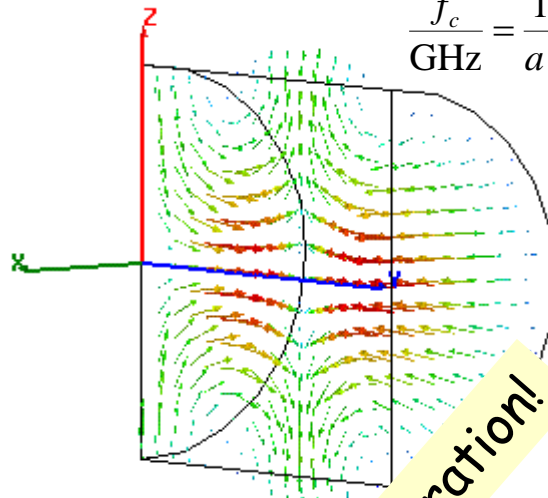
TE<sub>11</sub> - fundamental

$$\frac{f_c}{\text{GHz}} = \frac{87.9}{a/\text{mm}}$$



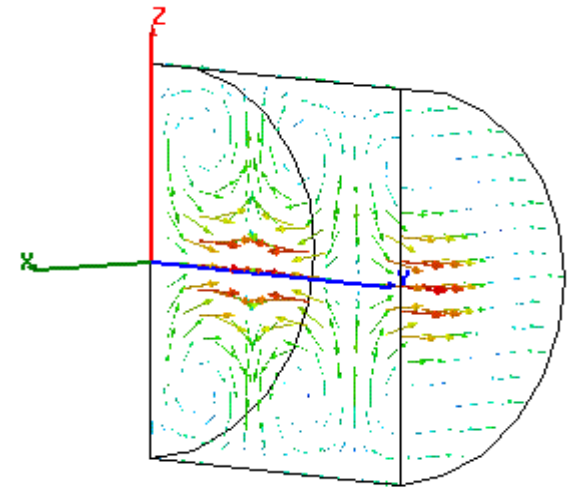
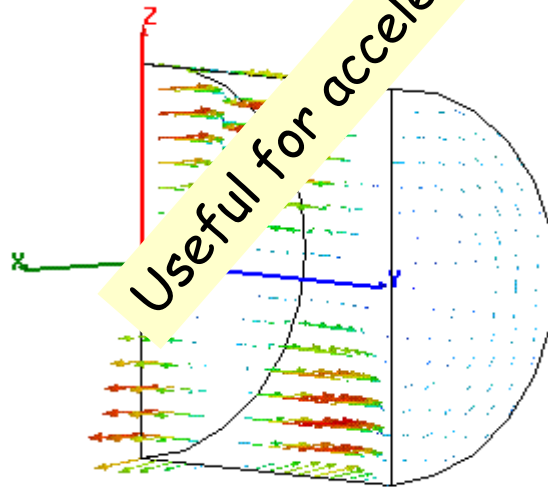
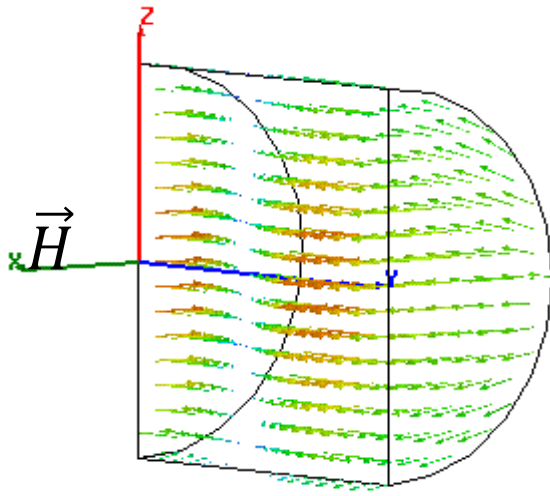
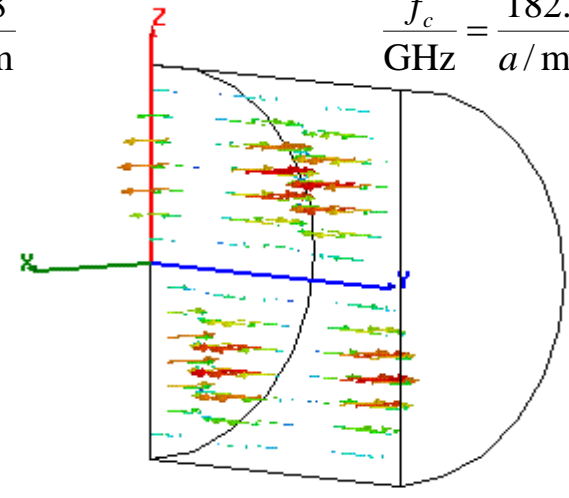
TM<sub>01</sub> - axial field

$$\frac{f_c}{\text{GHz}} = \frac{114.8}{a/\text{mm}}$$



TE<sub>01</sub> - low loss

$$\frac{f_c}{\text{GHz}} = \frac{182.9}{a/\text{mm}}$$

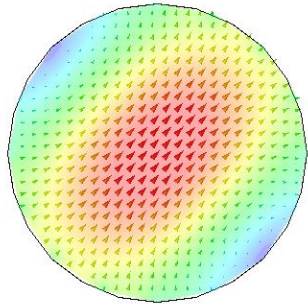


Useful for acceleration!

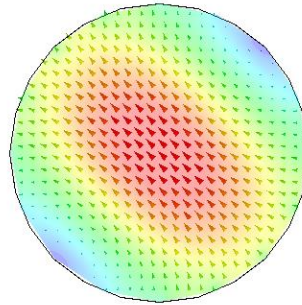


# Circular waveguide modes

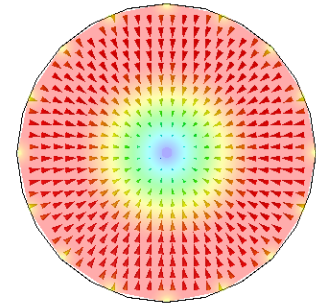
Indices linked to the number of field knots in polar co-ordinates  $\varphi, r$



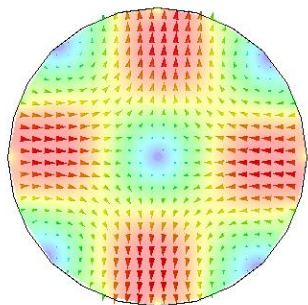
TE<sub>11</sub>



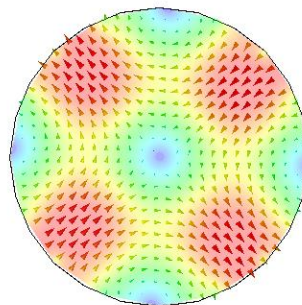
TE<sub>11</sub>



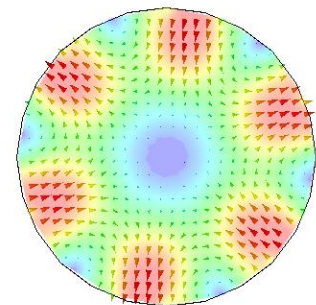
TM<sub>01</sub>



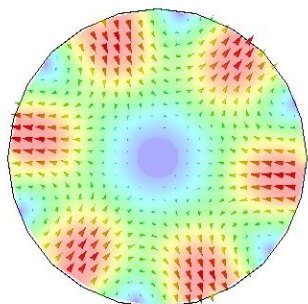
TE<sub>21</sub>



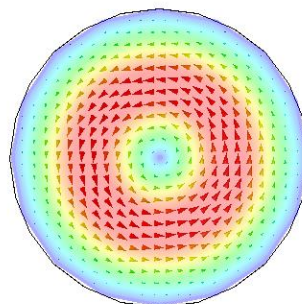
TE<sub>21</sub>



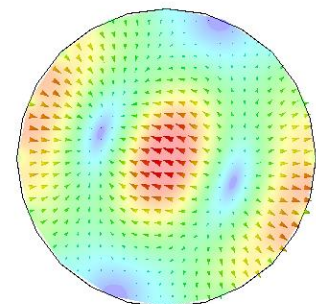
TE<sub>31</sub>



TE<sub>31</sub>



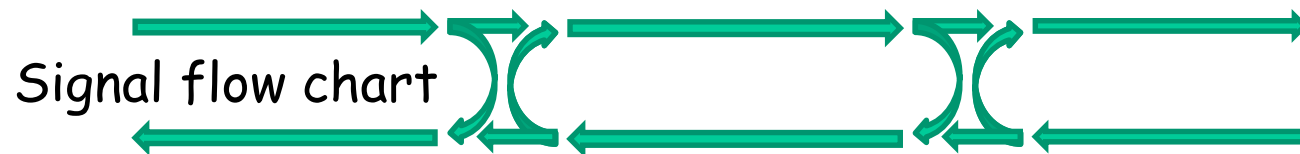
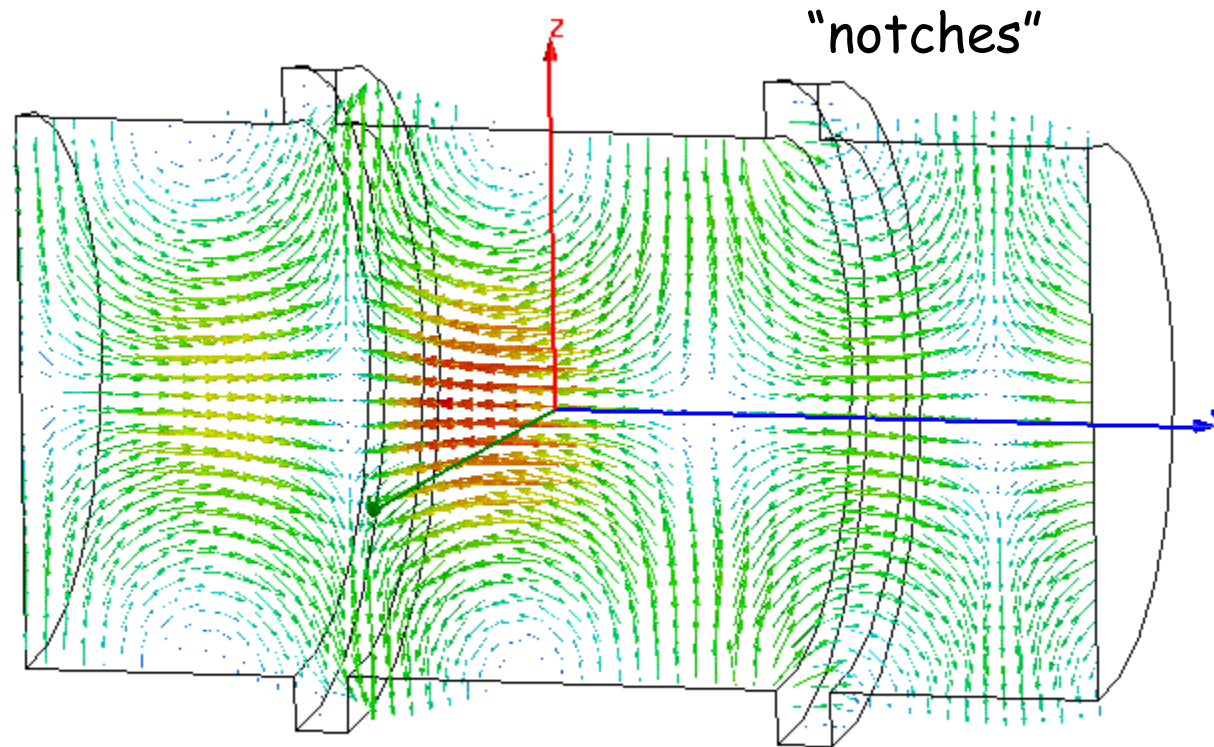
TE<sub>01</sub>



TM<sub>11</sub>

plotted:  $E$ -field

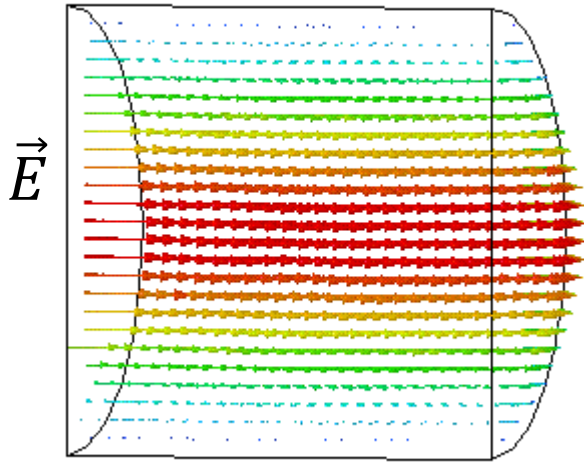
# Waveguide perturbed by discontinuities (notches)



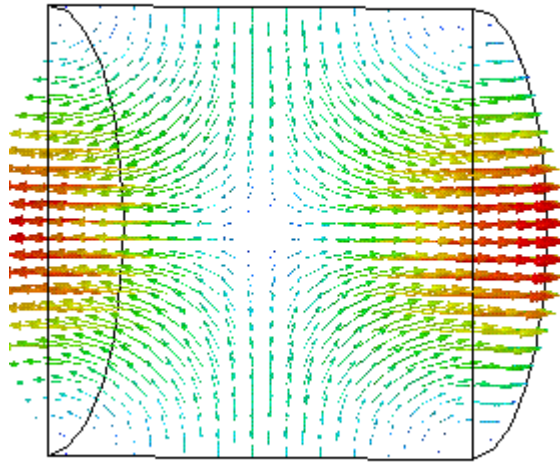
Reflections from notches lead to a superimposed standing wave pattern.  
"Trapped mode"

# Short-circuited waveguide -> Cavity

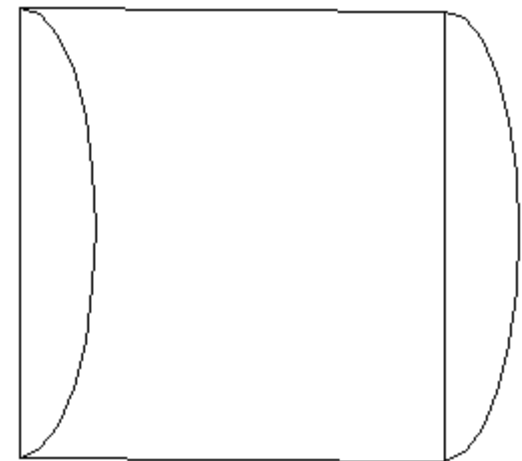
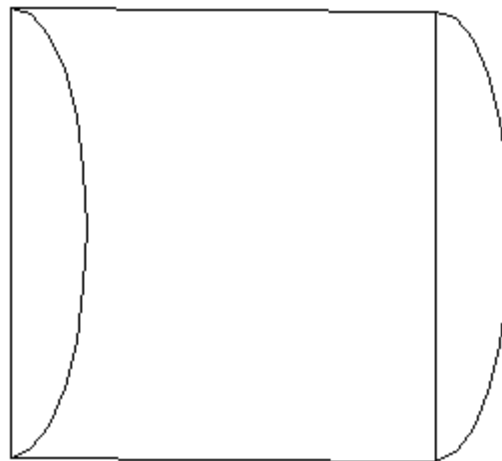
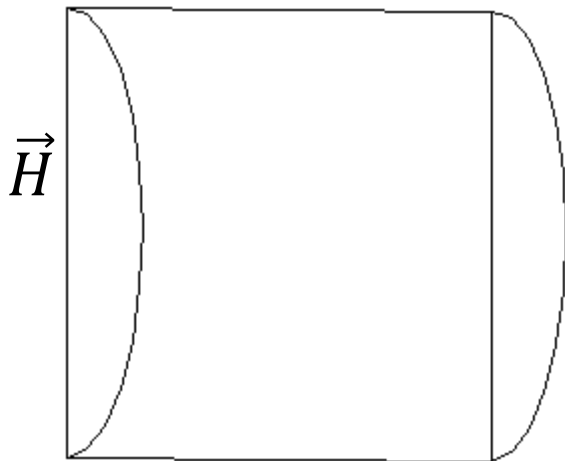
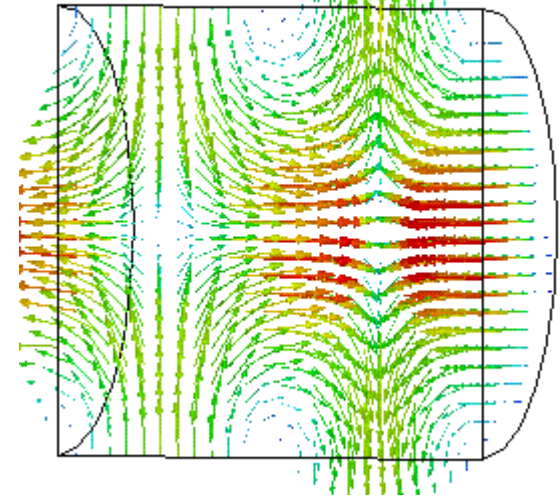
$TM_{010}$  (no axial dependence)



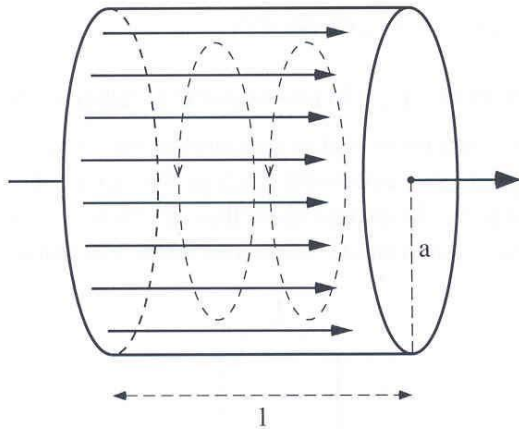
$TM_{011}$



$TM_{012}$



# The 'Pill Box' Cavity



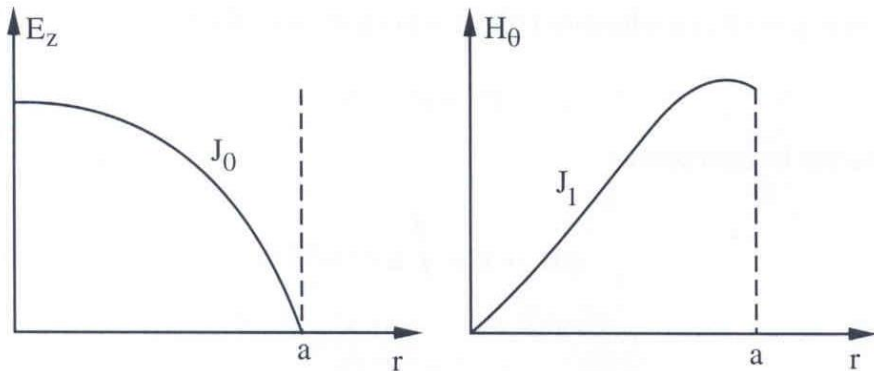
—→  $E_z$     - - - - -→  $H_\theta$

The **wave solutions** for E and H are **oscillating modes**, at **discrete frequencies**.

Modes can be type  $TM_{xyz}$  (transverse magnetic) or  $TE_{xyz}$  (transverse electric).

**Indices** linked to the **number of field knots** in polar co-ordinates  $\phi$ ,  $r$  and  $z$ .

For  $l < 2a$  the most simple mode,  $TM_{010}$ , has the lowest frequency, and has only two field components:



$$E_z = J_0(kr) e^{i\omega t}$$

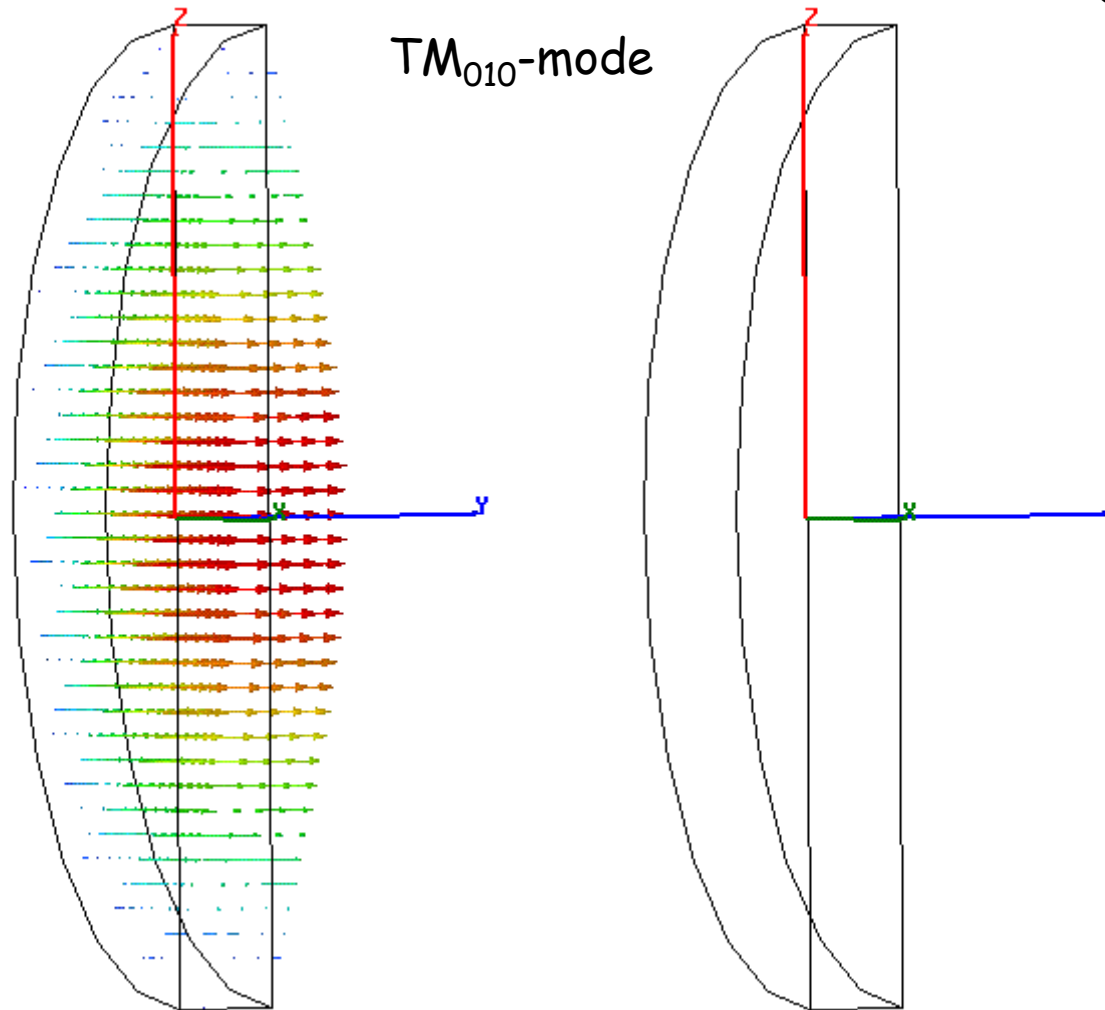
$$H_\theta = -\frac{i}{Z_0} J_1(kr) e^{i\omega t}$$

$$k = \frac{2\rho}{l} = \frac{\omega}{c} \quad l = 2.62a \quad Z_0 = 377\Omega$$



# Simple pillbox cavity

(only 1/2 shown)

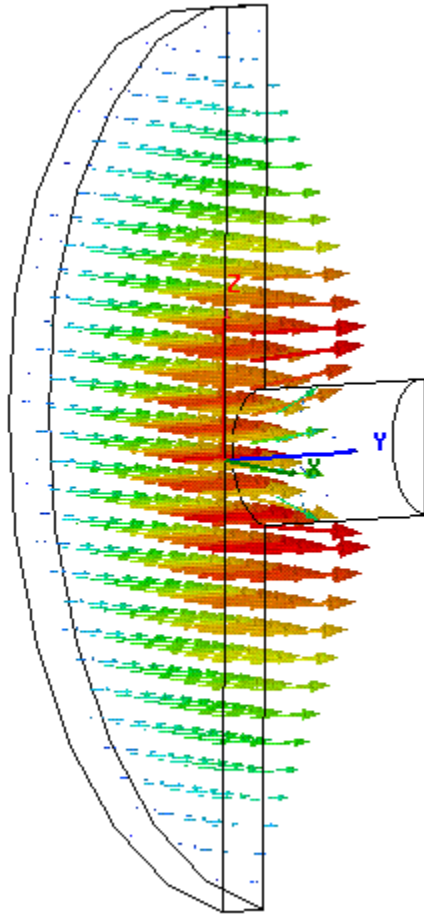


electric field (purely axial)      magnetic field (purely azimuthal)

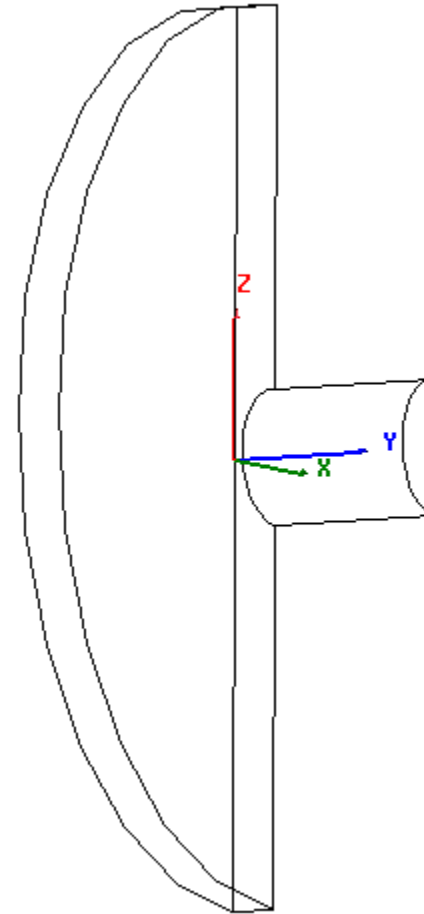
# Pillbox with beam pipe

$TM_{010}$ -mode (only 1/4 shown)

One needs a hole for the beam pipe - circular waveguide below cutoff



electric field

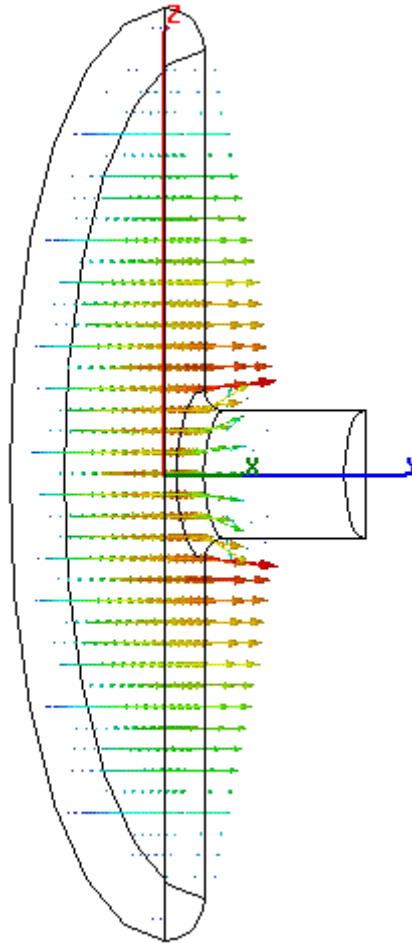


magnetic field

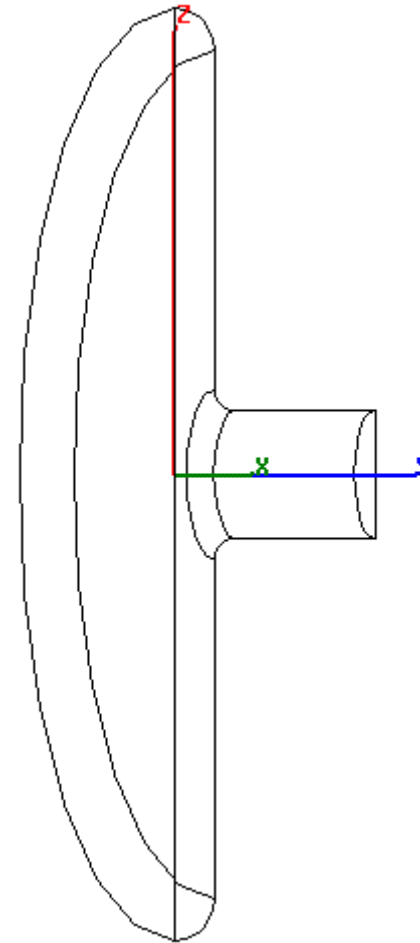
# A more practical pillbox cavity

Round off sharp  
edges (field  
enhancement!)

$TM_{010}$ -mode (only 1/4 shown)



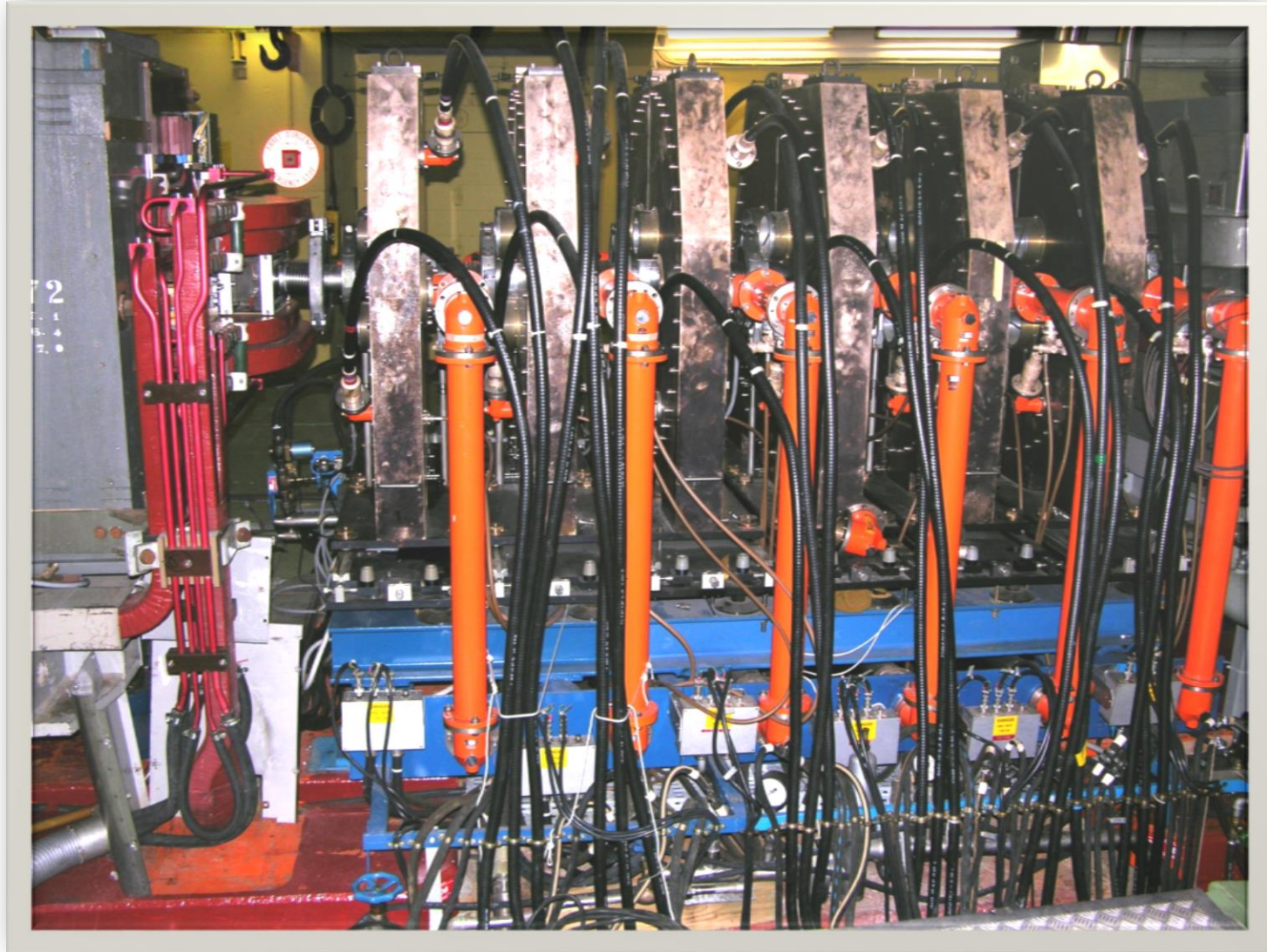
electric field



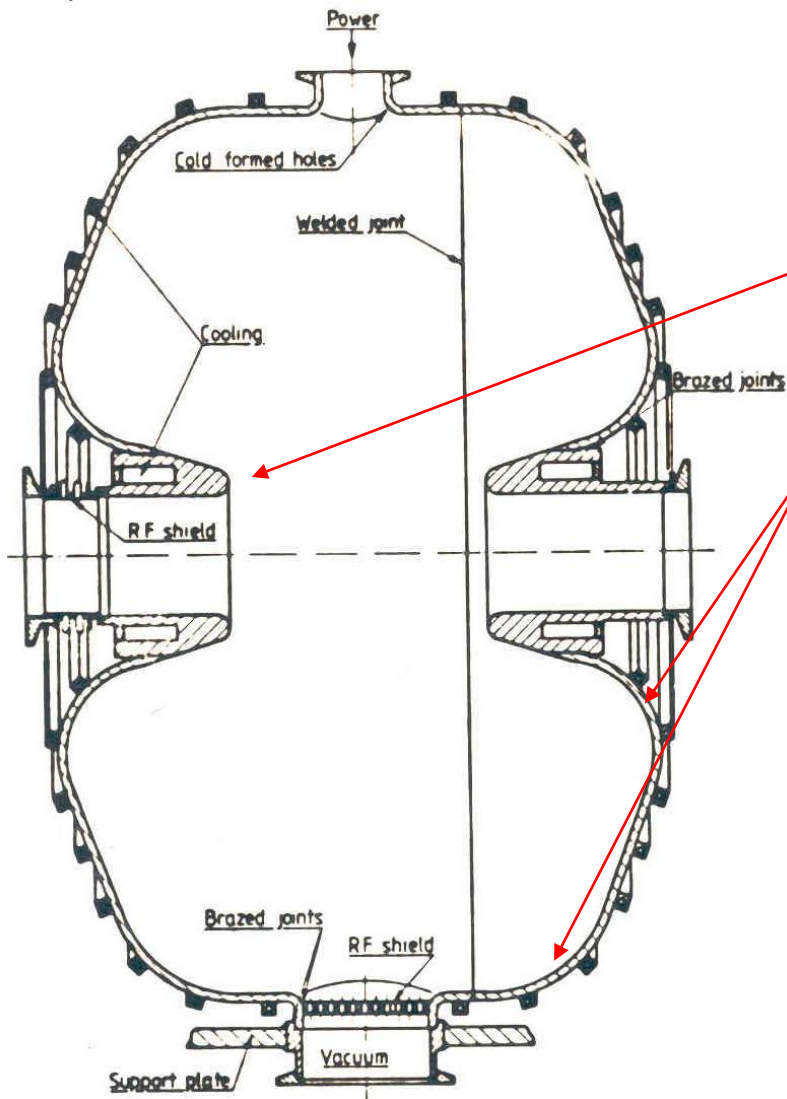
magnetic field

## Some real “pillbox” cavities

CERN PS 200 MHz cavities



# The Pill Box Cavity -> Real Cavity



The design of a cavity can be sophisticated in order to **improve** its **performances**:

- A **nose cone** can be introduced in order to concentrate the electric field around the axis

- **Round** shaping of the **corners** allows a better distribution of the magnetic field on the surface and a reduction of the Joule losses.

It also prevents from multipactoring effects (e- emission and acceleration).

A good cavity efficiently transforms the RF power into accelerating voltage.

**Simulation codes** allow precise calculation of the properties.



# Transit time factor

The accelerating **field varies during** the **passage** of the particle  
 $\Rightarrow$  particle does not always see maximum field  $\Rightarrow$  **effective acceleration smaller**

Transit time factor  
 defined as:

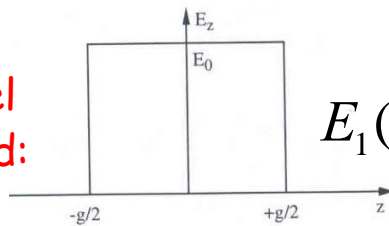
$$T_a = \frac{\text{energy gain of particle with } v = bc}{\text{maximum energy gain (particle with } v \rightarrow \infty)}$$

In the general case, the transit time factor is:

for  $E(s, r, t) = E_1(s, r) \times E_2(t)$

$$T_a = \frac{\int_{-\infty}^{+\infty} E_1(s, r) \cos \left( \frac{\omega_{RF}}{c} s \right) \frac{ds}{v}}{\int_{-\infty}^{+\infty} E_1(s, r) ds}$$

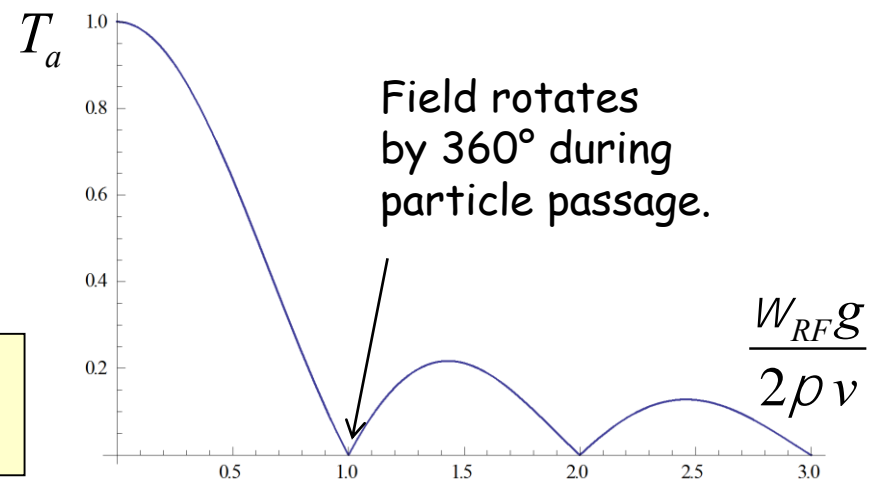
Simple model  
 uniform field:



$$E_1(s, r) = \frac{V_{RF}}{g}$$

follows:  $T_a = \left| \sin \frac{\omega_{RF} g}{2v} \right| / \left| \frac{\omega_{RF} g}{2v} \right|$

$0 < T_a < 1$ ,  $T_a \rightarrow 1$  for  $g \rightarrow 0$ , smaller  $\omega_{RF}$   
**Important for low velocities (ions)**

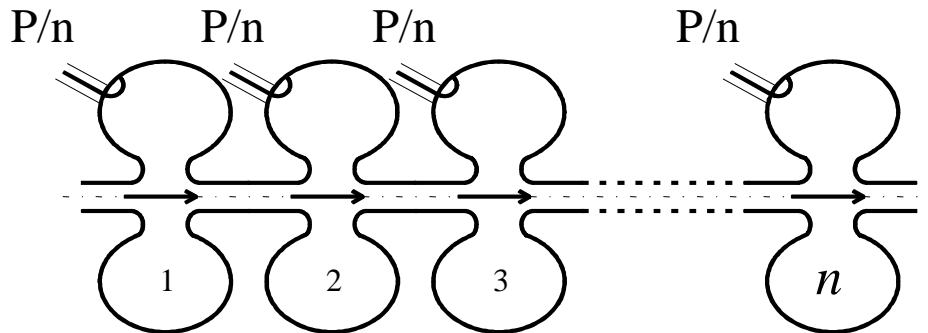


# Multi-Cell Cavities

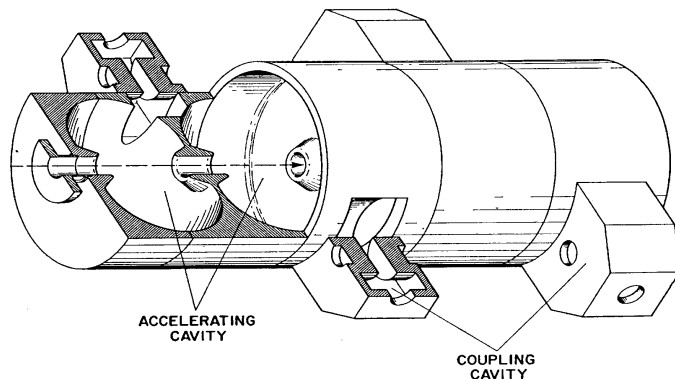
Acceleration of one cavity limited => **distribute power over several cells**

Each cavity receives  $P/n$

Since the field is proportional  $\sqrt{P}$ , you get  $E_i \propto \sqrt{P/n} = \sqrt{n} E_0$



Instead of distributing the power from the amplifier, one might as well **couple the cavities**, such that the power automatically distributes, or have a **cavity with many gaps** (e.g. drift tube linac).





# Multi-Cell Cavities - Modes

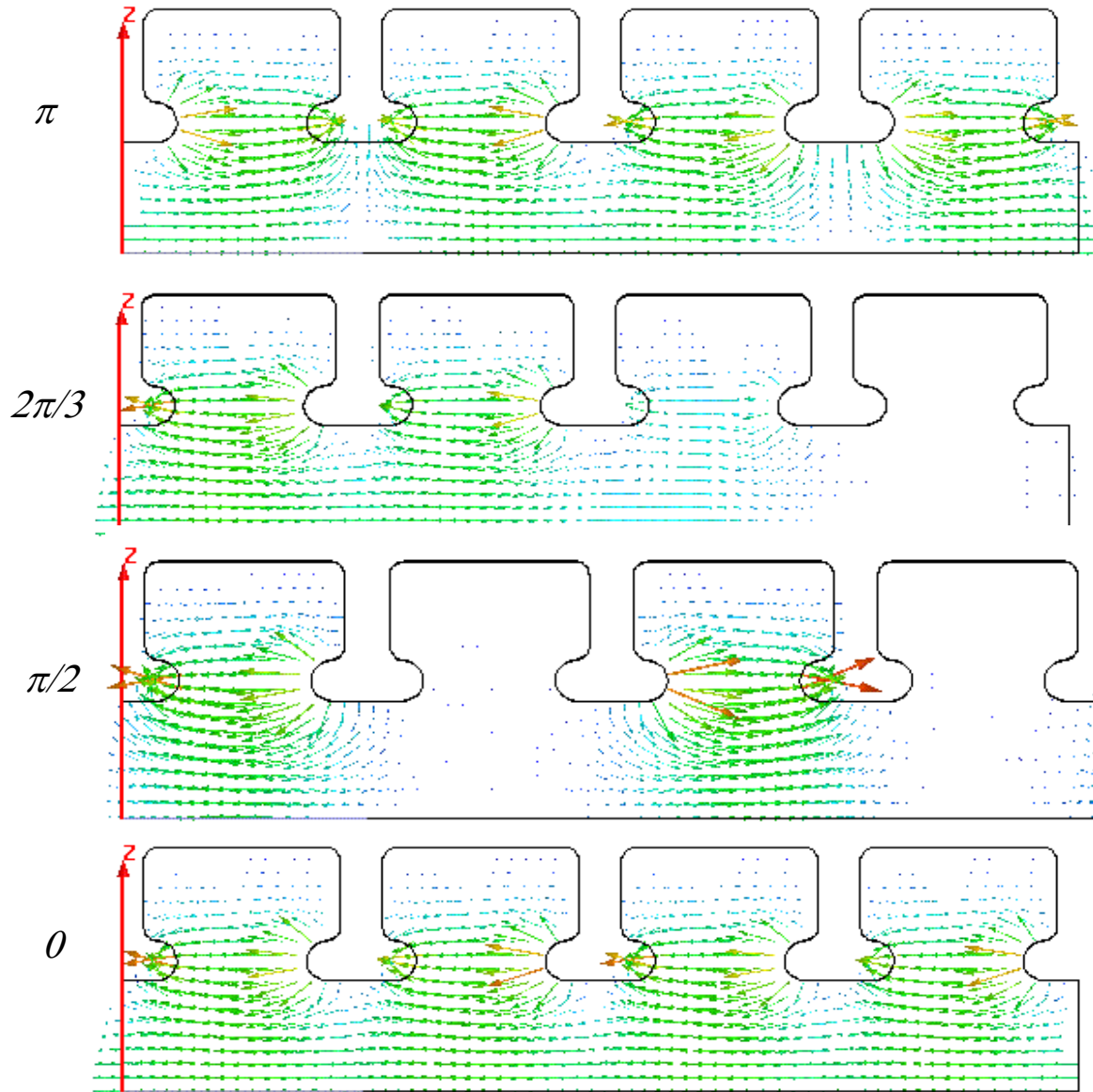
The **phase relation** between gaps is **important!**

Coupled harmonic oscillator

=> **Modes**, named after the **phase difference** between adjacent cells.

Relates to different synchronism conditions for the cell length  $L$

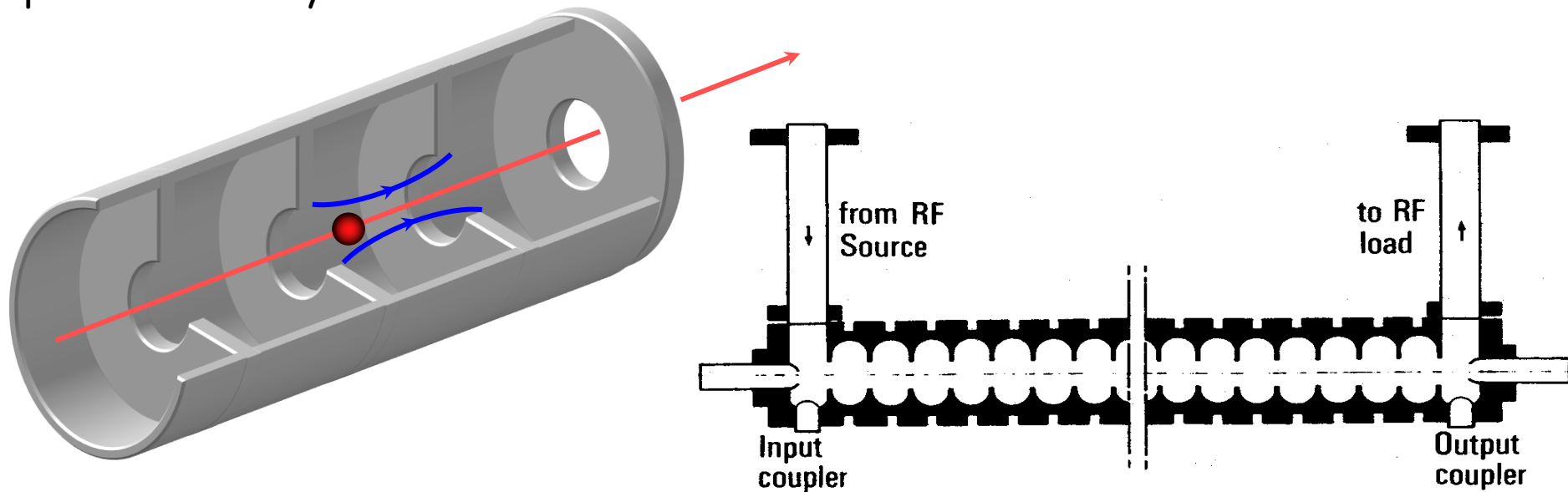
Mode	$L$
0 ( $2\pi$ )	$\beta\lambda$
$\pi/2$	$\beta\lambda/4$
$2\pi/3$	$\beta\lambda/3$
$\pi$	$\beta\lambda/2$



# Disc-Loaded Traveling-Wave Structures

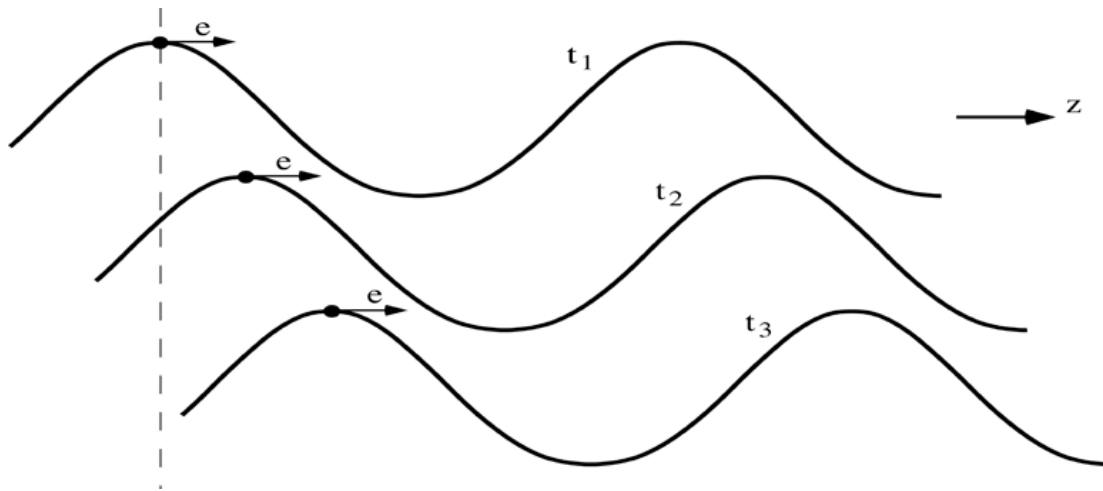
When particles get **ultra-relativistic** ( $v \sim c$ ) the drift tubes become very long unless the operating frequency is increased. Late 40's the development of radar led to high power transmitters (klystrons) at very high frequencies (3 GHz).

Next came the idea of suppressing the drift tubes using **traveling waves**. A wave guide has always a phase velocity  $v_\phi > c$ . However to get a continuous acceleration the phase velocity of the wave needs to be adjusted to the particle velocity.



**solution: slow wave guide with irises** ==> iris loaded structure

# The Traveling Wave Case



The particle travels along with the wave, and  $k$  represents the wave propagation factor.

$$E_z = E_0 \cos(W_{RF}t - kz)$$

$$k = \frac{W_{RF}}{v_j} \quad \text{wave number}$$

$$z = v(t - t_0)$$

$v_\phi$  = phase velocity

$v$  = particle velocity

$$E_z = E_0 \cos\left(W_{RF}t - W_{RF} \frac{v}{v_j} t - \Phi_0\right)$$

If synchronism satisfied:  $v = v_\phi$  and  $E_z = E_0 \cos \Phi_0$   
 where  $\Phi_0$  is the RF phase seen by the particle.

## Cavity Parameters: Quality Factor Q

The **total energy stored** is  $W = \iiint_{cavity} \left( \frac{\epsilon}{2} |\vec{E}|^2 + \frac{\mu}{2} |\vec{H}|^2 \right) dV.$

- **Quality Factor Q** (caused by wall losses) defined as

$$Q_0 = \frac{\omega_0 W}{P_{loss}}$$

Ratio of stored energy  $W$   
and dissipated power  $P_{loss}$   
on the walls in one RF cycle

The Q factor determines the maximum energy the cavity can fill to with a given input power.

Larger Q => less power needed to sustain stored energy.

The Q factor is  $2\pi$  times the number of rf cycles it takes to dissipate the energy stored in the cavity (down by  $1/e$ ).

- function of the geometry and the **surface resistance of the material**:  
superconducting (niobium) :  $Q = 10^{10}$   
normal conducting (copper) :  $Q = 10^4$

# Important Parameters of Accelerating Cavities

## - Accelerating voltage $V_{acc}$

$$V_{acc} = \int_{-\infty}^{\infty} E_z e^{-i\frac{\omega z}{\beta c}} dz$$

Measure of the acceleration

## - R upon Q

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{2\omega_0 W}$$

Relationship between acceleration  $V_{acc}$  and stored energy  $W$

independent from material!

Attention: Different definitions are used!

## - Shunt Impedance R

$$R = \frac{|V_{acc}|^2}{2P_{loss}}$$

Relationship between acceleration  $V_{acc}$  and wall losses  $P_{loss}$

depends on

- material
- cavity mode
- geometry

# Important Parameters of Accelerating Cavities (cont.)

## - Fill Time $t_F$

### - standing wave cavities:

$$P_{loss} = -\frac{dW}{dt} = \frac{\omega}{Q} W$$

Exponential decay of the stored energy  $W$  due to losses

$$t_F = \frac{Q}{\omega}$$

time for the field to decrease by  $1/e$  after the cavity has been filled  
measure of how fast the stored energy is dissipated on the wall

Several fill times needed to fill the cavity!

### - travelling wave cavities:

time needed for the electromagnetic energy to fill the cavity of length  $L$

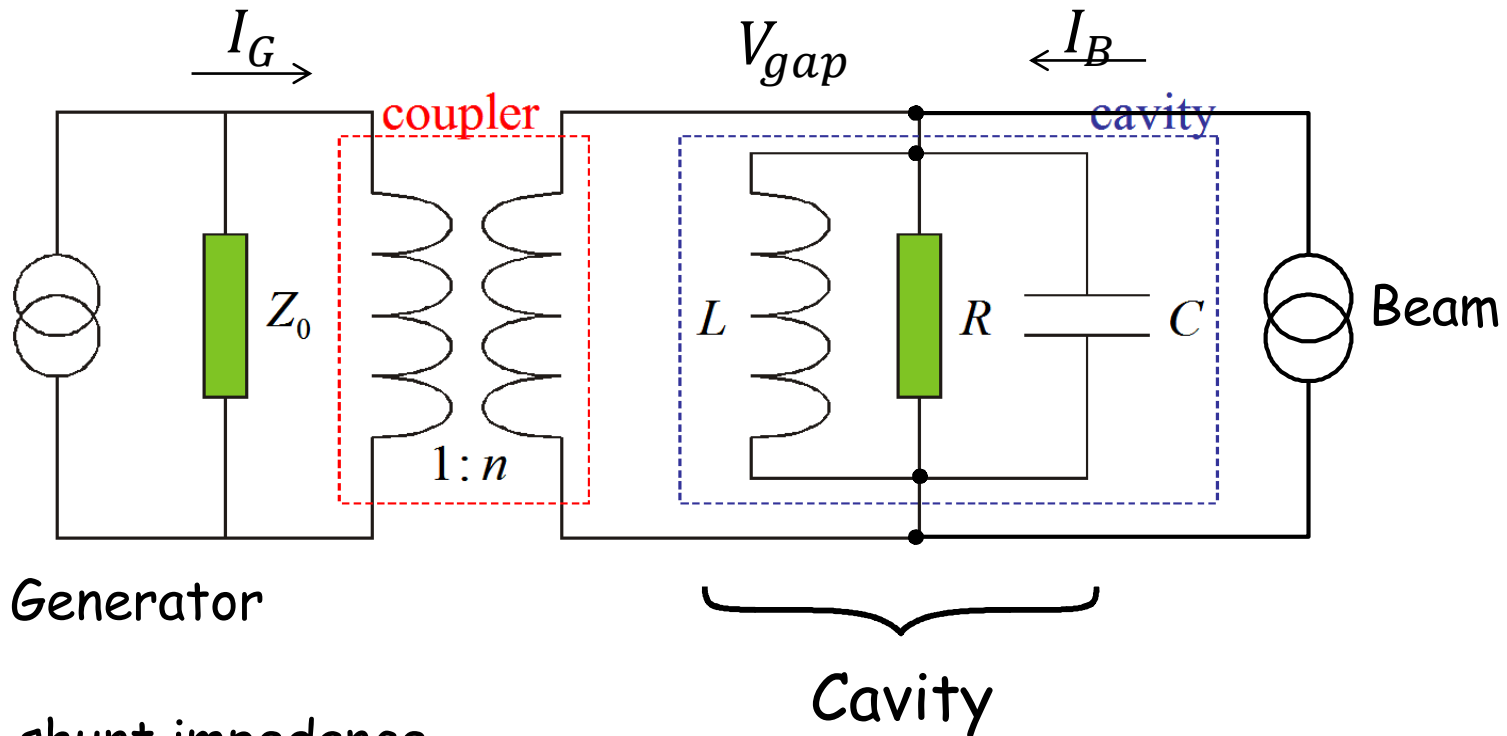
$$t_F = \int_0^L \frac{dz}{v_g(z)}$$

$v_g$ : velocity at which the energy propagates through the cavity

Cavity is completely filled after 1 fill time!

# SW Cavity resonator - equivalent circuit

Simplification: single mode

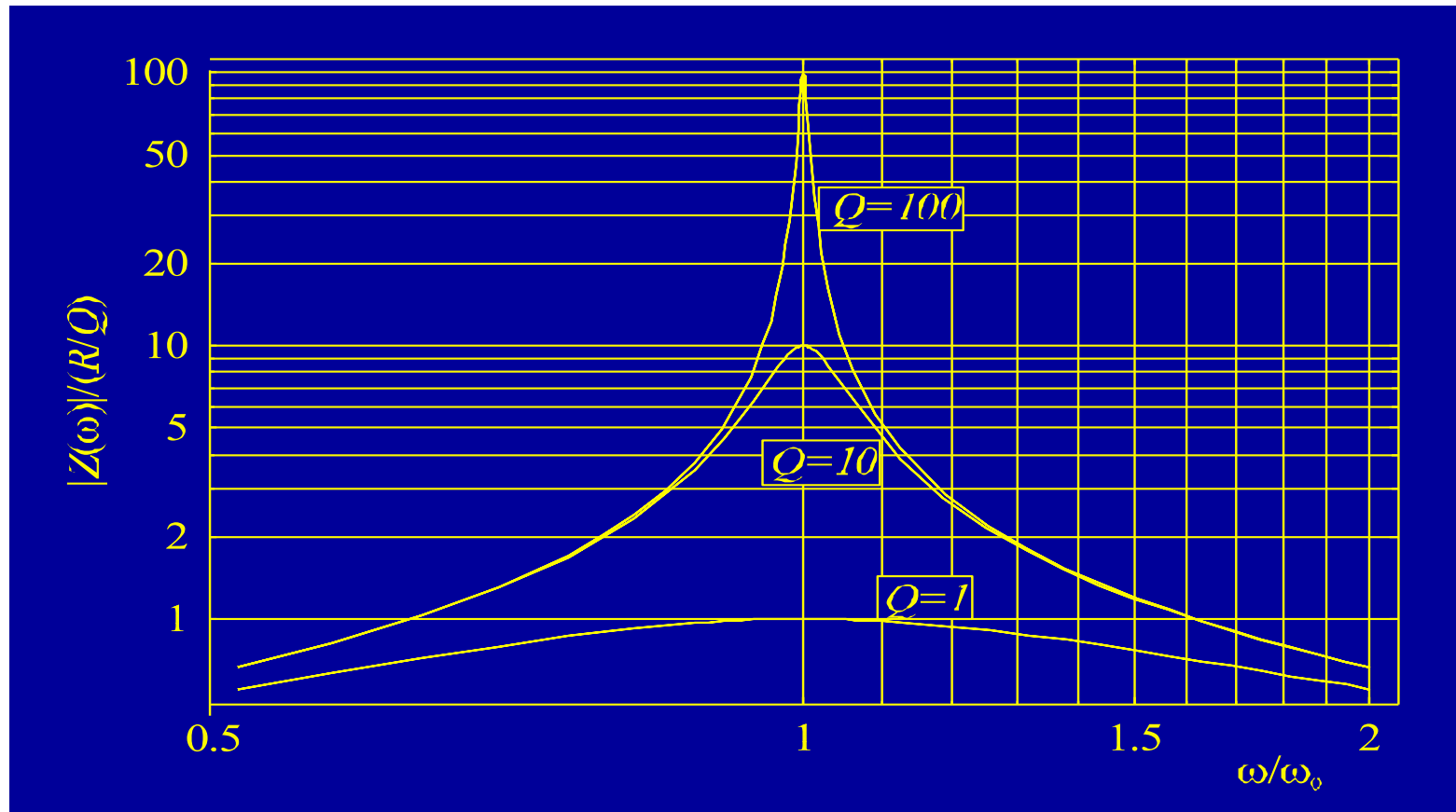


$R$ : shunt impedance  
 $\sqrt{L/C} = \frac{R}{Q}$ : R-upon-Q

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$$



# Resonance



A high  $Q_0$ : small wall losses  $\Rightarrow$  less power needed for the same voltage.  
But the bandwidth becomes very narrow.

Note: a 1 GHz cavity with a  $Q_0$  of  $10^{10}$  has a natural bandwidth of 0.1 Hz!  
... to make this manageable,  $Q_{ext}$  is chosen much smaller!

## Power coupling - Loaded $Q$

Note that the generator inner impedance also loads the cavity  
- for very large  $Q_0$  more than the cavity wall losses.

To calculate the **loaded  $Q$**  ( $Q_L$ ), the losses have to be added:

$$\frac{1}{Q_L} = \frac{P_{loss} + P_{ext} + \dots}{\omega_0 W} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} + \frac{1}{\dots}$$

The **coupling factor  $\beta$**  is the ratio  $P_{ext}/P_{loss}$ .  $\beta = \frac{P_{ext}}{P_{loss}} = \frac{Q_0}{Q_{ext}}$   
With  $\beta$ , the loaded  $Q$  can be written

$$Q_L = \frac{Q_0}{1 + \beta}.$$

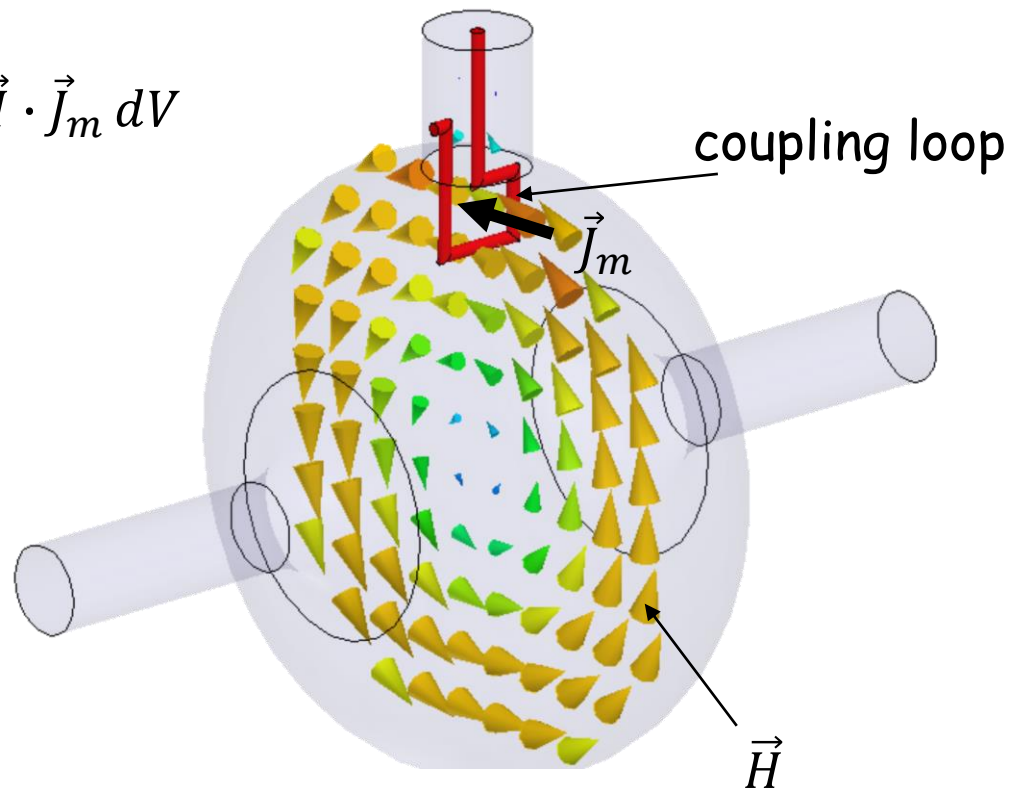
For NC cavities, often  $\beta = 1$  is chosen (power amplifier matched to empty cavity); for SC cavities,  $\beta = \mathcal{O}(10^4 \dots 10^6)$ .

# Magnetic (loop) coupling

The magnetic field of the cavity main mode is intercepted by a coupling loop

The coupling can be adjusted by changing the size or the orientation of the loop.

$$\text{Coupling: } \propto \iiint \vec{H} \cdot \vec{J}_m dV$$



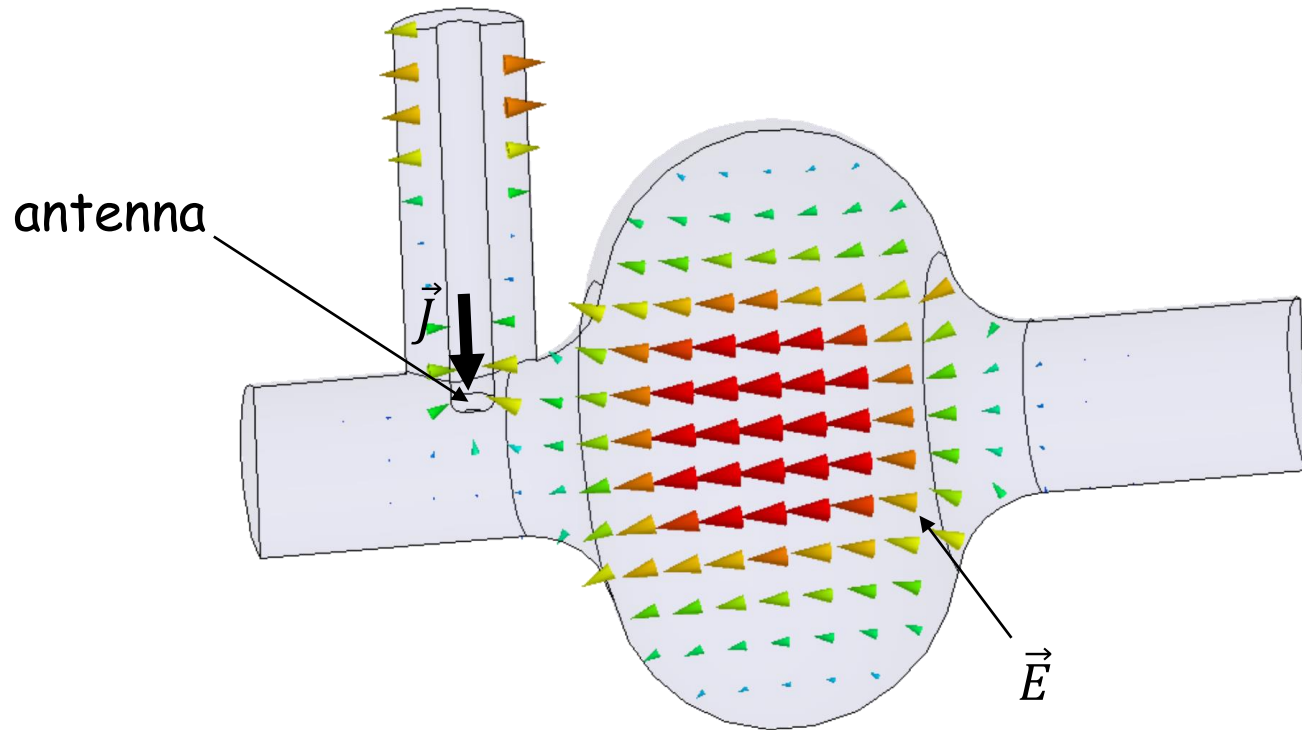
courtesy: David Alesini/INFN

# Electric (antenna) coupling

The inner conductor of the coaxial feeder line ends in an antenna penetrating into the electric field of the cavity.

The coupling can be adjusted by varying the penetration.

$$\text{Coupling} \propto \iiint \vec{E} \cdot \vec{J} dV$$



courtesy: David Alesini/INFN

# Cavity parameters

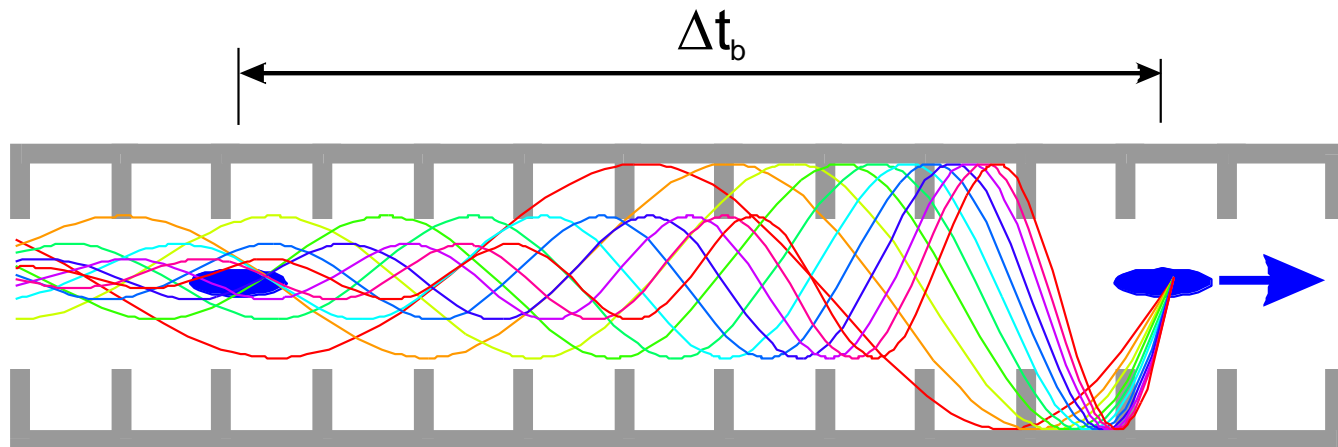
Resonance frequency	$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$	
Transit time factor	$TT = \frac{\left  \int E_z e^{i \frac{\omega}{\beta c} z} dz \right }{\left  \int E_z dz \right }$	
$Q$ factor	$\omega_0 W = Q P_{loss}$	
	<b><i>Circuit definition</i></b>	<b><i>Linac definition</i></b>
Shunt impedance	$ V_{gap} ^2 = 2 R P_{loss}$	$ V_{gap} ^2 = R P_{loss}$
$R/Q$ (R-upon-Q)	$\frac{R}{Q} = \frac{ V_{gap} ^2}{2 \omega_0 W} = \sqrt{L/C}$	$\frac{R}{Q} = \frac{ V_{gap} ^2}{\omega_0 W}$
Loss factor	$k_{loss} = \frac{\omega_0 R}{2 Q} = \frac{ V_{gap} ^2}{4W} = \frac{1}{2C}$	$k_{loss} = \frac{\omega_0 R}{4 Q} = \frac{ V_{gap} ^2}{4W}$

# Wakefields and Beam Loading

The cavities' electric field accelerates the beam.

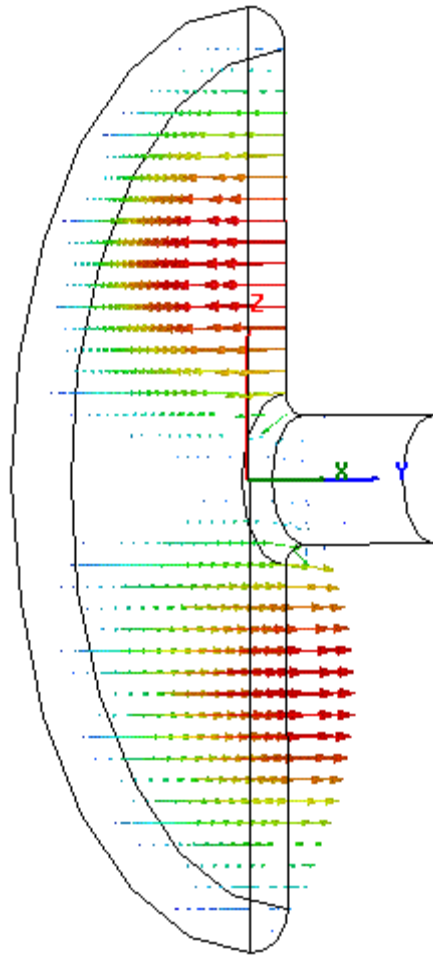
But the beam will also act on the fields inside the cavities

- **Accelerating field** will be **reduced** (energy conservation!)  
=> Beam Loading (longitudinal wakefield)
- Beam can excite perturbing cavity modes (Higher Order Modes - HOM) and deflect following bunches  
=> (transverse) **Wakefields**

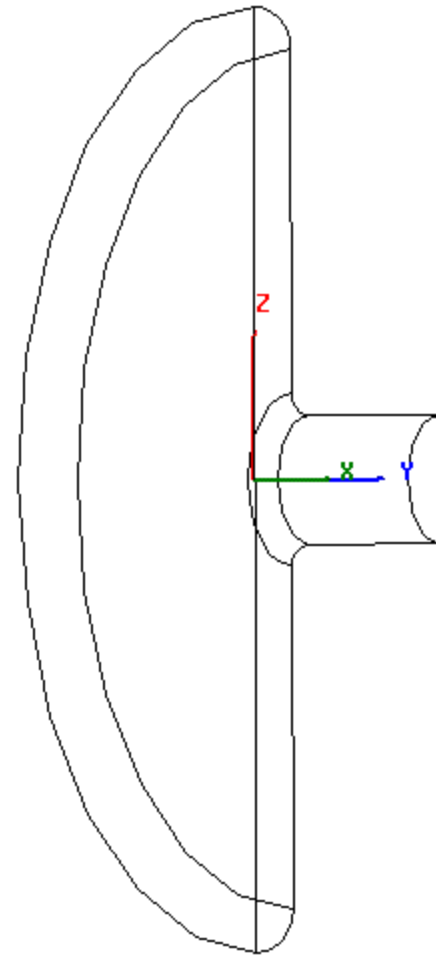


# Dipole mode in a pillbox

$TM_{110}$ -mode (only 1/4 shown)



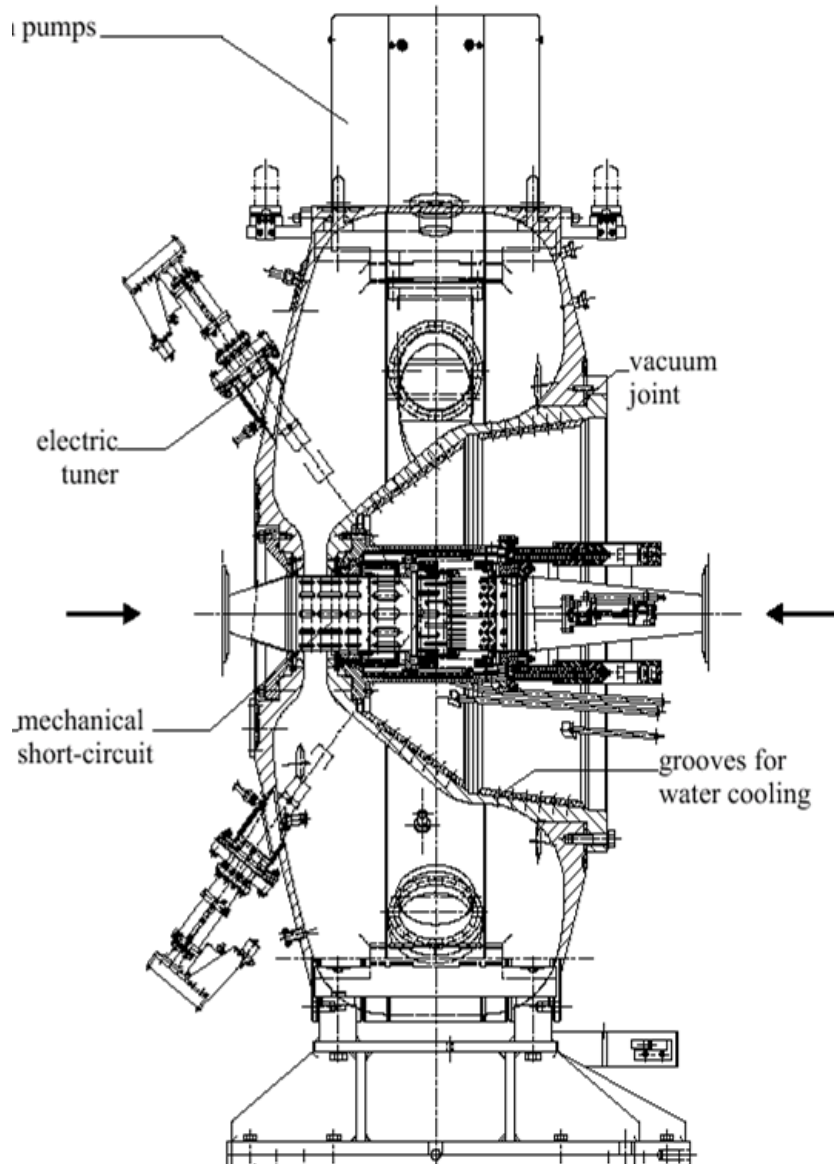
electric field



magnetic field



# CERN/PS 80 MHz cavity (for LHC)

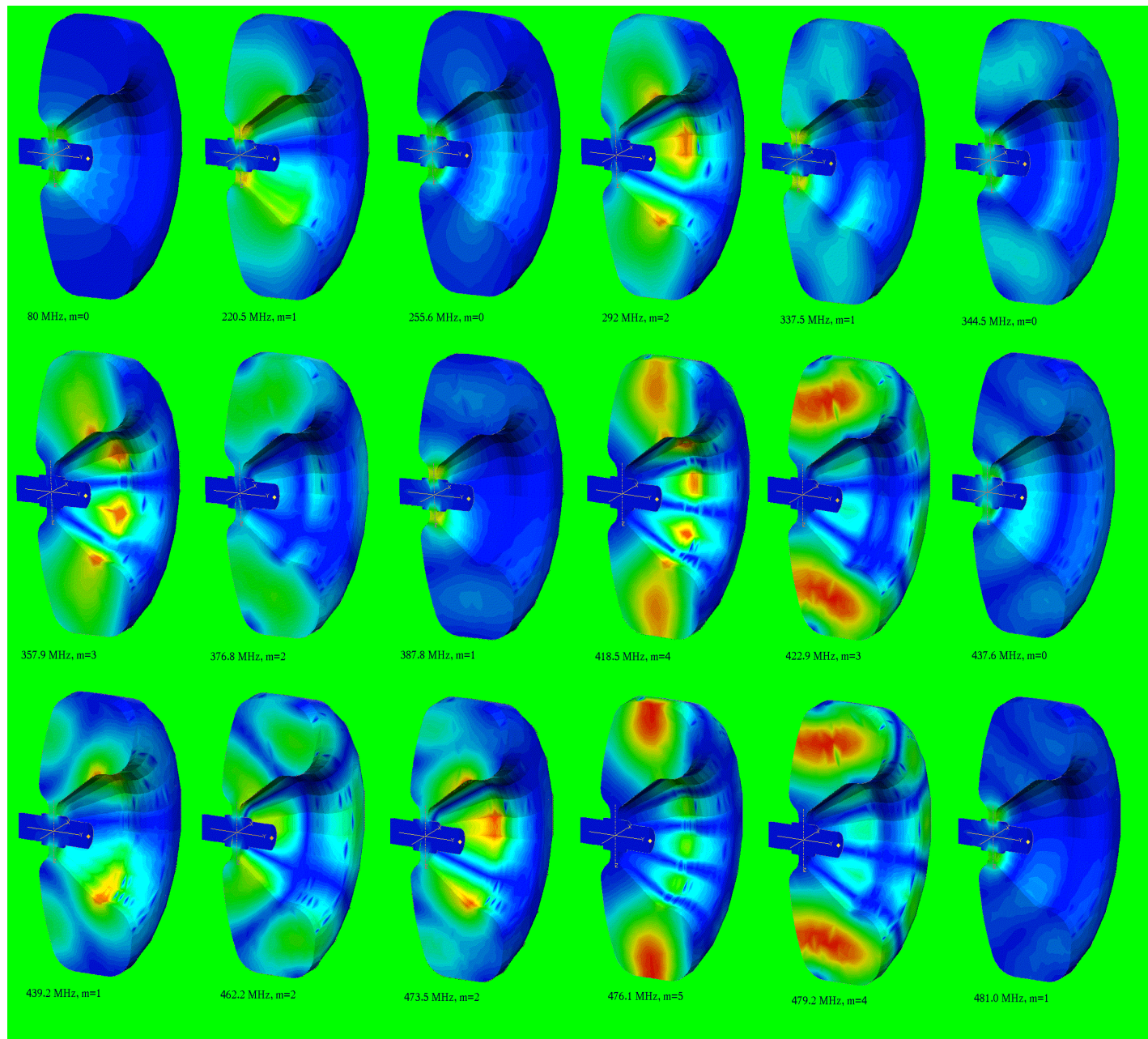
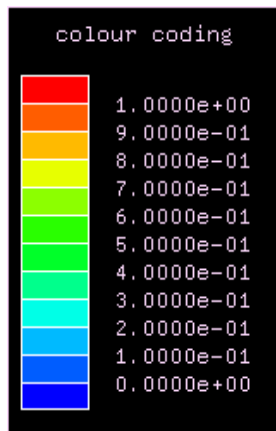


# HOM's

Example  
shown:

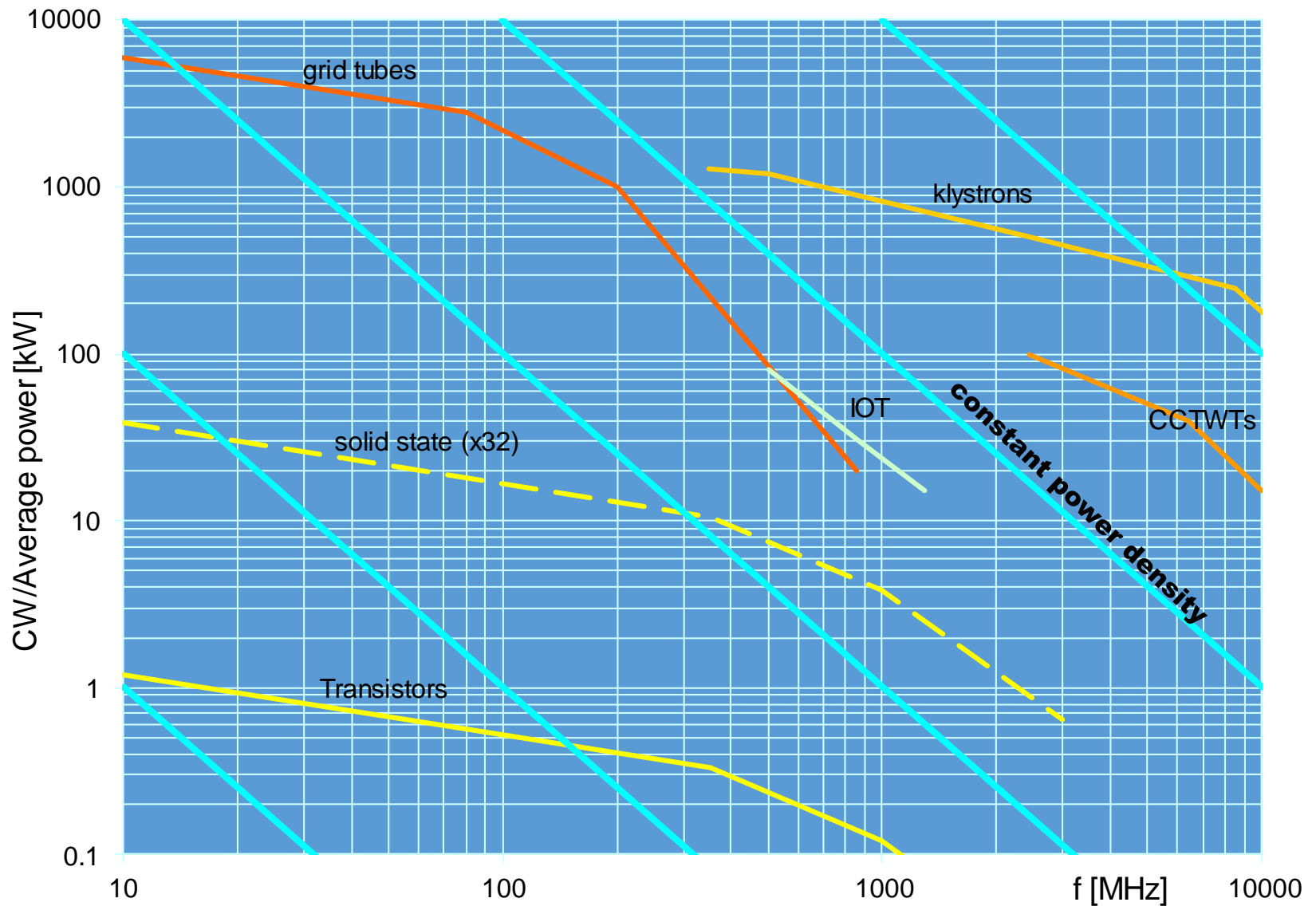
CERN/PS  
80 MHz  
cavity

Colour coding:  $|\vec{E}|$



# RF power sources

Typical ranges (commercially available)

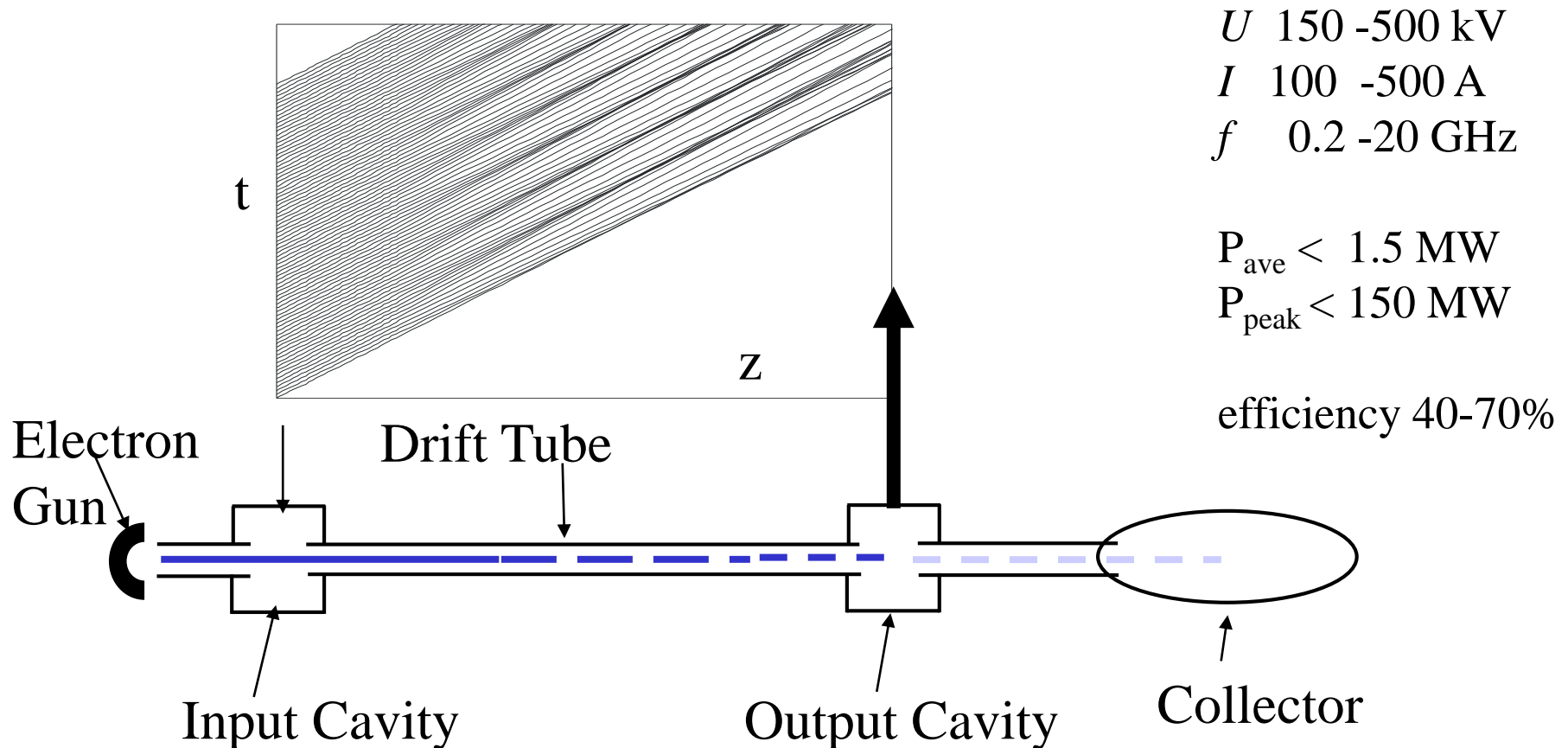


# Klystrons

low-power RF signal at the design frequency excites input cavity

**Velocity modulation** of electron beam  $\rightarrow$  **density modulation**

Bunched beam excites output cavity





# Klystrons



**CERN CTF3 (LIL):**  
3 GHz, 45 MW,  
4.5  $\mu$ s, 50 Hz,  $\eta$  45 %

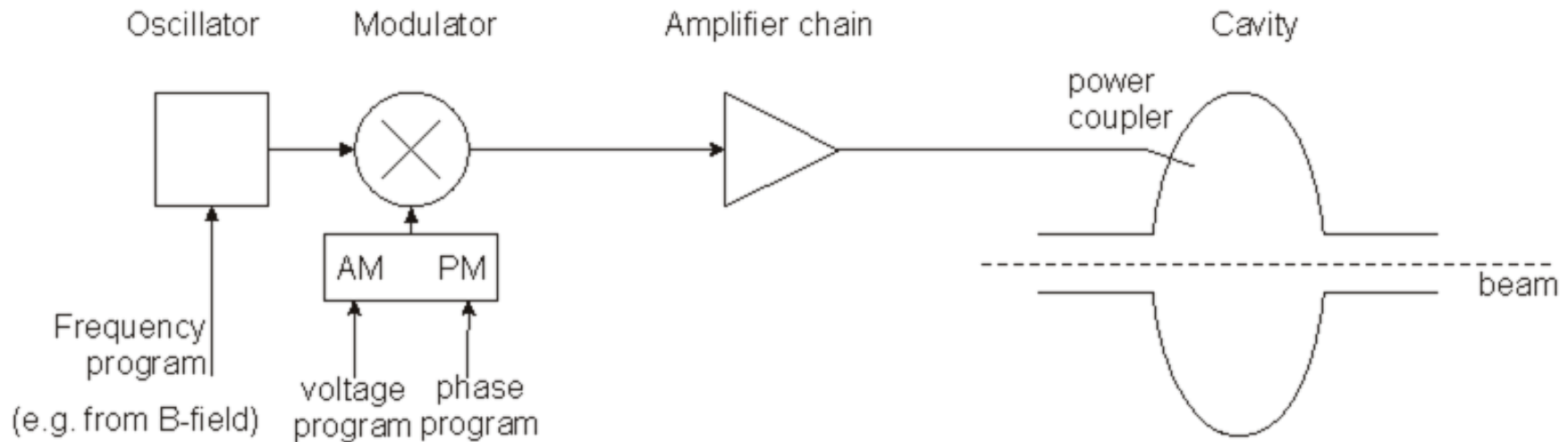


**CERN LHC:**  
400 MHz, 300 kW,  
CW,  $\eta$  62 %

# Minimal RF system (of a synchrotron)

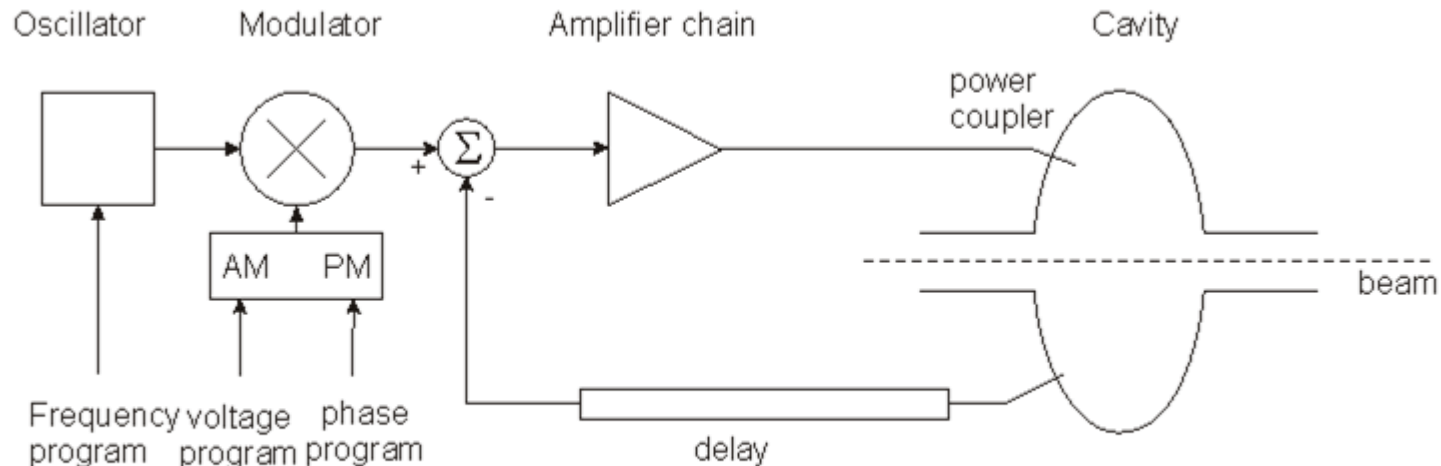
Low-level RF

High-Power RF



- The frequency has to be controlled to follow the magnetic field such that the beam remains in the centre of the vacuum chamber.
- The voltage has to be controlled to allow for capture at injection, a correct bucket area during acceleration, matching before ejection; phase may have to be controlled for transition crossing and for synchronisation before ejection.

# RF Feed-back loops



- Compares actual RF voltage and phase with desired and corrects.
- Limited by total group delay (path lengths) (some 100 ns).
- Works also to keep voltage at zero for strong beam loading, i.e. it reduces the beam impedance.
- Voltage control loop (AVC)
- Beam phase loop
- 1-turn feedback
- Radial loop (measure orbit and change  $f$  to keep beam centred)
- Synchronisation loop (to other machines at extraction)



# CERN PS RF Systems



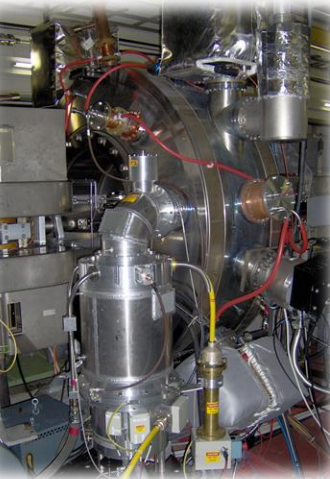
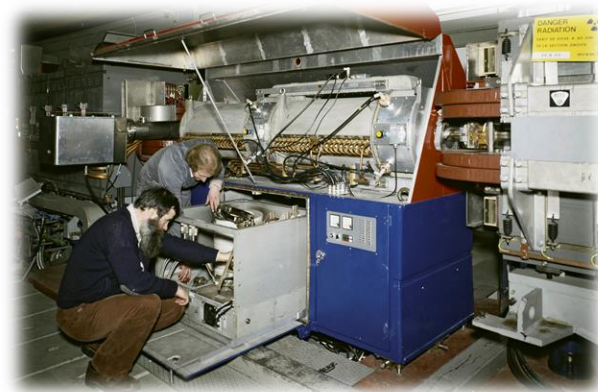
10 MHz system,  $h=7...21$



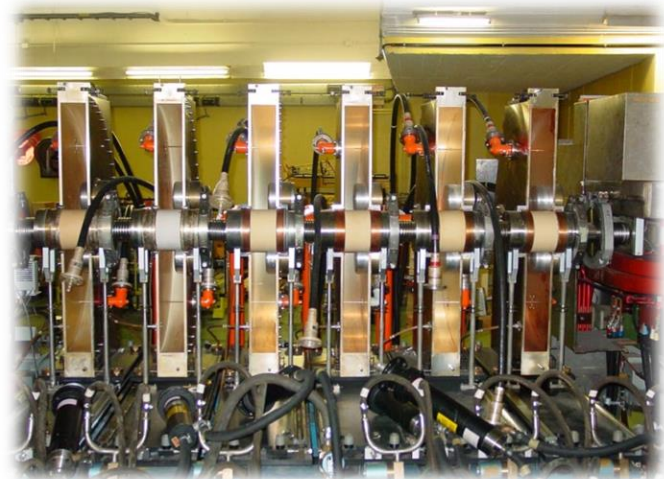
13/20 MHz system,  $h=28/42$



40 MHz system,  $h=84$



80 MHz system,  $h=168$



200 MHz system



# Acknowledgements

I would like to thank everyone for the material that I have used.

In particular (hope I don't forget anyone):

- **Erk Jensen** (from whom I inherited the course)
- Joël Le Duff
- Graeme Burt
- David Alesini
- Fu-Kwun Hwang and Lookang Lawrence Wee

...

**Homework:**

Try this in your bathtub!

