Luminosity and Beam-Beam Effects in the Large Hadron Collider (LHC)

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Acknowledgements: material and discussions W. Herr, S. White, W. Kotzanecki
Circular Accelerators: acceleration occurs at every turn!

E. Lawrence 1930
Circular Accelerators:
acceleration occurs at every turn!

Two Beams of
\[ E_1, \vec{p}_1, E_2, \vec{p}_2, m_1 = m_2 = m \]

\[ E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2} \]

Beam 2 is a Target \( \rightarrow \) \[ \vec{p}_2 = 0 \]

\[ E_{cm} = \sqrt{2m^2 + 2E_1 m} \]

7 TeV proton beam against fix target \( \rightarrow \) 115 GeV
Colliders: higher energy

Anello di Accumulazione AdA
B. Touschek 1960
Colliders: higher energy

Two Beams of
$E_1, \vec{p}_1, E_2, \vec{p}_2, m_1 = m_2 = m$

$E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}$

Beam 2 is a counter rotating beam

$\vec{p}_1 = \vec{p}_2$

$E_{cm} = E_1 + E_2$

7 TeV proton beam colliding $\rightarrow$ 14 TeV

Anello di Accumulazione AdA
B. Touschek 1960
The Large Hadron Collider

27 Km length
Protons
Maximum 14 TeV center of mass energy
4 Interaction Regions for Experiments
Circular colliders: Luminosity

Collider Luminosity $L$ is the proportionality factor between the cross section $\sigma_{\text{event}}$ and the number of events per second $\frac{dR}{dt}$.

\[
\frac{dR}{dt} = L \times \sigma_{\text{event}}
\]

Units: $cm^{-2} s^{-1}$

\[
\int L(t) dt = 10^{39} cm^{-2}
\]

RUN1 1400 Higgs events with 30 $fb^{-1}$
RUN2 we are now around 120 $fb^{-1}$

Luminosity is a machine parameter

→ Independent of the physical reaction
→ Reliable procedure to compute and measure
Luminosity calculation

The overlap integral of two bunches crossing each other head-on is proportional to the luminosity and it is given by:

\[
\mathcal{L} \propto KN_1N_2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho_1(x, y, s, -s_0)\rho_2(x, y, s, +s_0) dx dy ds s_0
\]

\[s_0 = c \cdot t\]

\[K = \sqrt{\left(\vec{v}_1 - \vec{v}_2\right)^2 - \left(\vec{v}_1 \times \vec{v}_2\right)^2} / c^2\]
Luminosity formula

\[ \mathcal{L} \propto K N_1 N_2 \int \int \int \int_{-\infty}^{+\infty} \rho_1(x, y, s, -s_0) \rho_2(x, y, s, +s_0) \, dx \, dy \, ds \, ds_0 \]

Uncorrelated densities in all planes

→ Factorize the distribution density as:

\[ \rho_1(x, y, s, -s_0) = \rho_{1x}(x) \rho_{1y}(y) \rho_{1s}(s - s_0) \]

For head-on collisions where

→ “Kinematic Factor” \( K = 2 \)

To have the luminosity per second

→ Needs to multiple by revolution frequency \( f \)

In the presence of many bunches \( n_b \)

\[ \mathcal{L} = 2 \cdot N_1 N_2 \cdot f \cdot n_b \cdot \int \int \int \int_{-\infty}^{+\infty} \rho_{1x}(x) \rho_{1y}(y) \rho_{1s}(s - s_0) \cdot \rho_{2x}(x) \rho_{2y}(y) \rho_{2s}(s + s_0) \, dx \, dy \, ds \, ds_0 \]
Closed solution for Gaussian distributions

Simplest case assumptions:

- Gaussian distributions
- No dispersion at the collision point
- Head-on collision

\[ \rho_{i,z}(z) = \frac{1}{\sigma_z \sqrt{2\pi}} \exp\left(\frac{z^2}{2\sigma_z^2}\right) \]

\[ \sigma_{x,y} = \sqrt{\epsilon \cdot \beta_{x,y}^*} \]

\[ K = 2 \]

\[ \mathcal{L} = \frac{2N_1 N_2 f n_b}{(\sqrt{2\pi})^6 \sigma_s^2 \sigma_x^2 \sigma_y^2} \int \int \int \int e^{-\frac{x^2}{\sigma_x^2}} e^{-\frac{y^2}{\sigma_y^2}} e^{-\frac{s^2}{\sigma_s^2}} e^{-\frac{s_0^2}{\sigma_s^2}} \, dx dy ds ds_0 \]

Equal Transverse beams “Round” beams

\[ \sigma_{1x} = \sigma_{2x} \]
\[ \sigma_{1y} = \sigma_{2y} \]

Un-Equal Transverse beams “Flat” beams or optics

\[ \sigma_{1x} \neq \sigma_{2x} \]
\[ \sigma_{1y} \neq \sigma_{2y} \]
The LHC design parameters

\[ \mathcal{L} = \frac{N_1 N_2 f n_b \gamma}{4\pi \epsilon_{x,y} \beta^*} \]

**LHC Design**
- \( N_1 = N_2 = 1.15 \times 10^{11} \) protons per bunch
- \( \sigma_x = \sigma_y = 16.6 \ \mu m \)
- \( \beta^* = 55 \ \text{cm} \)
- \( \Rightarrow \mathcal{L} = 10^{34} \ \text{cm}^{-2}\text{s}^{-1} \)

**LHC Record**
- \( N_1 = N_2 = 1.15 \times 10^{11} \) protons per bunch
- \( \sigma_x = \sigma_y = 9.5 \ \mu m \)
- \( \beta^* = 30 \ \text{cm} \)
- \( \Rightarrow \mathcal{L} = 2 \times 10^{34} \ \text{cm}^{-2}\text{s}^{-1} \)

**High Luminosity Upgrade of LHC**
- \( N_1 = N_2 = 2.2 \times 10^{11} \) protons per bunch
- \( \sigma_x = \sigma_y = 7.0 \ \mu m \)
- \( \beta^* = 64 \rightarrow 15 \ \text{cm} \)
- \( \Rightarrow \mathcal{L} = (10-20) \times 10^{34} \ \text{cm}^{-2}\text{s}^{-1} \)
Different types of collisions

- They occur when two beams get closer and collide

- Two types
  - High energy collisions between two particles (wanted)
  - Distortions of beam by electromagnetic forces (unwanted)

- Unfortunately: usually both go together...
  - 0.001% (or less) of particles collide
  - 99.999% (or more) of particles are distorted
Proton Beams ➔ Electro Magnetic potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges

Force on itself (space charge) and opposing beam (beam-beam effects)

Focusing quadrupole

Opposite Beam
Beam is a collection of charges
Beam is an electromagnetic potential for other charges

Force on itself: space charge effects goes with $1/\gamma^2$ factor for high energy colliders this contribution is negligible (i.e. force scales LHC $1/\gamma^2 = 1.8 \times 10^{-8}$)

**Proton Beams $\rightarrow$ Electro Magnetic potential**

![Electric field lines](image)
Proton Beams $\rightarrow$ Electro Magnetic potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges

Electromagnetic force from opposing beam (beam-beam effects)

Single particle motion and whole bunch motion distorted

A beam acts on particles like an electromagnetic lens, but...
Beam-beam Force derivation

General approach in electromagnetic problems Reference[5] already applied to beam-beam interactions in Reference[1,3, 4]

\[ \Delta U = -\frac{1}{\varepsilon_0} \rho(x, y, z) \]

Derive potential from Poisson equation for charges with distribution \( \rho \)

\[
U(x, y, z, \sigma_x, \sigma_y, \sigma_z) = \frac{1}{4\pi \varepsilon_0} \int \int \int \frac{\rho(x_0, y_0, z_0)dx_0dy_0dz_0}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}
\]

Solution of Poisson equation

\[ \vec{E} = -\nabla U(x, y, z, \sigma_x, \sigma_y, \sigma_z) \]

Then compute the fields

\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \]

From Lorentz force one calculates the force acting on test particle with charge \( q \)

Making some assumptions we can simplify the problem and derive analytical formula for the force...
Beam-Beam Force for Round Gaussian distributions

Gaussian distribution for charges
Round beams:
Very relativistic, Force has only radial component:

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r, s, t) \, dt$$

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

$$\sigma_x = \sigma_y = \sigma$$

$$\beta \approx 1 \quad r^2 = x^2 + y^2$$

Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)

Only radial component in relativistic case

How does this force look like?
Beam-beam Force

\[ F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]
Can we quantify the beam-beam strength?

For small amplitudes: linear force (quadrupole) \[ F \propto -\xi \cdot r \]

The slope of the force gives you the beam-beam parameter \( \xi \)

\( \xi \) Quantifies the strength of the force but does NOT reflect the nonlinear nature of the force.
Beam-Beam Parameter

\[ \Delta r' = \frac{1}{mc\beta \gamma} \int F_r(r, s, t) \, dt \]

For small amplitudes: linear force

\[ \Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]

\[ \Delta r' = \frac{2N_p r_0}{\gamma} \cdot \frac{1}{r} \cdot \left[ 1 - \left( 1 - \frac{r^2}{2\sigma^2} + \ldots \right) \right] \]

\[ \Delta r'|_{r \to 0} = \frac{N r_0 r}{\gamma \sigma^2} = + f \cdot r \]

\[ \xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2} \]

For non-round beams:

\[ \xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi \gamma \sigma_{x,y} (\sigma_x + \sigma_y)} \]
Beam-Beam Parameter

\[ \Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r, s, t) \, dt \]

\[ \xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2} \]

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For non-round beams:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>LHC TDR</th>
<th>LHC 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity ( N_{p,e} )/bunch</td>
<td>1.15 ( 10^{11} )</td>
<td>1.8 ( 10^{11} )</td>
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<tr>
<td>Energy GeV</td>
<td>7000</td>
<td>4000</td>
</tr>
<tr>
<td>Beam size H</td>
<td>16.6 ( \mu \text{m} )</td>
<td>16.6 ( \mu \text{m} )</td>
</tr>
<tr>
<td>Beam size V</td>
<td>16.6 ( \mu \text{m} )</td>
<td>16.6 ( \mu \text{m} )</td>
</tr>
<tr>
<td>( \beta_{x,y}^* ) m</td>
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<td>0.60</td>
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<tr>
<td>Crossing angle ( \mu \text{rad} )</td>
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<td>290</td>
</tr>
<tr>
<td>( \xi_{\text{bb}} )</td>
<td>0.0037</td>
<td>0.007</td>
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<table>
<thead>
<tr>
<th>HL-LHC</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>2.2 ( 10^{11} )</td>
<td></td>
</tr>
<tr>
<td>Energy GeV</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>Beam size H</td>
<td>14 ( \mu \text{m} )</td>
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</tr>
<tr>
<td>Beam size V</td>
<td>14 ( \mu \text{m} )</td>
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<tr>
<td>( \beta_{x,y}^* ) m</td>
<td>0.64-0.15</td>
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<td>Crossing angle ( \mu \text{rad} )</td>
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<td></td>
</tr>
<tr>
<td>( \xi_{\text{bb}} )</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>
Why do we care?

Pushing for luminosity means stronger beam-beam effects

\[ \mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b \]

\[ F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[ 1 - e^{-\frac{r^2}{2\sigma^2}} \right] \]

Strongest non-linearity in a collider YOU CANNOT AVOID!

Strong non-linear electromagnetic distortion

→ impact on beam quality
   (particle losses and emittance blow-up)

→ luminosity reduction
Crossing angle operation

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4 \pi \sigma_x \sigma_y} \]

Num. of maximum bunches \( n_b = 2808 \)

Multi Bunch operations brings un-wanted interactions left and right of the 4 Experiments

A finite crossing angle needed to avoid multiple collision points
Two type of interactions:

Other beam passing in the center force

→ **HEAD-ON** beam-beam interaction

→ LHC has 4 experiments:
  → ATLAS and CMS colliding head-on
  → ALICE and LHCB with transverse offset

Other beam passing at an offset $r$

→ **LONG-RANGE** beam-beam interaction

→ LHC has up to 120 LR interactions
Multiple bunch Complications

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4 \pi \sigma_x \sigma_y} \]

Num. of bunches : \( n_b = 2808 \)

Due to the train structure of the beams \( \rightarrow \) different bunches will experience a different number of interactions!
Long-Range separations

Multi Bunch operations brings unwanted interactions left and right of the 4 Experiments

\[ \beta(s) = \beta^* + \frac{s^2}{\beta^*} \]

\[ d_{lr} \propto \sqrt{\frac{\beta^*\alpha^2\gamma}{\epsilon_n}} \]
Luminosity Geometric reduction factor

Due to the crossing angle the overlap integral between the two colliding bunches is reduced!

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \cdot S \]

\[ S = \frac{1}{\sqrt{1 + \left(\frac{\sigma_x}{\sigma_s} \tan \frac{\phi}{2}\right)^2}} \cdot \frac{1}{\sqrt{1 + \left(\frac{\sigma_s}{\sigma_x} \tan \frac{\phi}{2}\right)^2}} \]

\[ \sigma_s \gg \sigma_{x,y} \]

Always valid for LHC and HL-LHC
\[ \sigma_x = 17-7 \mu m, \sigma_s = 7.5 \text{ cm} \]

\[ S \approx \frac{1}{\sqrt{1 + \left(\frac{\sigma_s}{\sigma_x} \frac{\phi}{2}\right)^2}} \]

LHC design: \( \phi = 285 \mu \text{rad}, \sigma_x = 17 \mu \text{m}, \sigma_s = 7.5 \text{ cm}, S=0.84 \)
LHC 2018: \( \phi = 320 \mu \text{rad}, \sigma_x = 9.3 \mu \text{m}, \sigma_s = 7.5 \text{ cm}, S=0.61 \)
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Due to the crossing angle the overlap integral between the two colliding bunches is reduced!

\[
\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \cdot S
\]

\[
S = \left( \frac{1}{\sqrt{1 + \left( \frac{\sigma_x}{\sigma_s} \tan \frac{\phi}{2} \right)^2}} \right) \left( \frac{1}{\sqrt{1 + \left( \frac{\sigma_s}{\sigma_x} \tan \frac{\phi}{2} \right)^2}} \right)
\]

\[
\sigma_s \gg \sigma_x, \sigma_y
\]

Always valid for LHC and HL-LHC

\[\sigma_x = 17-7 \, \mu m, \sigma_s = 7.5 \, cm\]

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\[ \sigma_s \gg \sigma_x, y \]

Always valid for LHC and HL-LHC
\[ \sigma_x = 17-7 \, \mu m, \sigma_s = 7.5 \, cm \]

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LHC 2018: \( \phi = 320 \, \mu rad, \sigma_x = 9.3 \, \mu m, \sigma_s = 7.5 \, cm, S=0.61 \)
LHC operates at finite crossing angle

HL-LHC will have bunches of $2.2 \times 10^{11}$ protons per bunch.

$\phi = 590 \, \mu \text{rad}$, $\sigma_x = 9.3 \, \mu \text{m}$, $\sigma_s = 7.5 \, \text{cm}$, $S=0.26 \rightarrow 73\%$ of luminosity lost!

\[
S \approx \frac{1}{\sqrt{1 + \left(\frac{\sigma_s}{\sigma_x} \cdot \frac{\phi}{2}\right)^2}}
\]

Crab Cavities used to tilt the bunches longitudinally and compensate for the crossing angle at the collision point!

Testing of crab cavities on-going in SPS!
Beam-Beam Force: single particle head-on collision

Lattice defocusing quadrupole

For small amplitudes: linear force

For large amplitude: very non-linear

The beam will act as a strong non-linear electromagnetic lens!
Linear Tune shift due to head-on collision

For small amplitude particles beam-beam can be approximated as linear force as a quadrupole

\[ F \propto -\xi \cdot r \]

Focal length is given by the beam-beam parameter:

\[ \frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*} \]

Beam-beam matrix:

\[
\begin{pmatrix}
1 & 0 \\
-\frac{\xi \cdot 4\pi}{\beta^*} & 1
\end{pmatrix}
\]

Equivalent to tune shift
Perturbed one turn matrix

For small amplitudes beam-beam can be approximated as linear force as a quadrupole

\[ F \propto -\xi \cdot r \]

Focal length:

\[ \frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*} \]

Beam-beam matrix:

\[
\begin{pmatrix}
1 & 0 \\
-\frac{\xi \cdot 4\pi}{\beta^*} & 1
\end{pmatrix}
\]

Perturbed one turn matrix with perturbed tune \( \Delta Q \) and beta function at the IP \( \beta^* \):

\[
\begin{pmatrix}
\cos(2\pi(Q + \Delta Q)) & \beta^* \sin(2\pi(Q + \Delta Q)) \\
-\frac{1}{\beta^*} \sin(2\pi(Q + \Delta Q)) & \cos(2\pi(Q + \Delta Q))
\end{pmatrix}
\]

\[
= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(2\pi Q) & \beta^*_0 \sin(2\pi Q) \\ -\frac{1}{\beta^*_0} \sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}
\]
Tune shift and dynamic beta

Solving the one turn matrix one can derive the tune shift $\Delta Q$ and the perturbed beta function at the IP $\beta^*$:

Tune is changed

$$cos(2\pi(Q + \Delta Q)) = cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi \xi}{\beta^*} sin(2\pi Q)$$

...how does the tune changes?
Tune shift due to beam-beam interactions

Tune shift as a function of tune

- Larger $\xi$
- Strongest variation with $Q$
- Effects of multiple Interaction Points does not add linearly (phase advance between IP..)

HiLumi LHC 3 IPs
HL-LHC
LHC design
Tune shift in 2 dimensional case equally charged beams and tunes far from integer and half

Zero amplitude particle will fill an extra defocusing term

\[ \xi_{bb} = 0.02 \]

\[ \Delta Q \approx \xi_{bb} \]
A beam is a collection of particles

Beam-beam force

Beam 2 passing in the center of force produce by Beam 1
Particles of Beam 2 will experience different ranges of the beam-beam forces

Tune shift as a function of amplitude (detuning with amplitude or tune spread)
A beam will experience all the force range

Beam-beam force

Second beam passing in the center
HEAD-ON beam-beam interaction

Beam-beam force

Second beam displaced offset
LONG-RANGE beam-beam interaction

Different particles will see different force
Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam is related to the derivative of the force with respect to the amplitude.

\[ \Delta Q \propto \frac{\delta F}{\delta r} \]

For small amplitude test particle, linear tune shift.

\[ \Delta Q_{quad} = \text{const} \]
\[ \Delta Q_{bb} \approx \text{const} \]

\[ \lim_{r \to 0} \Delta Q(r) = -\frac{N r_0 \beta^*}{4\pi \gamma \sigma^2} = \xi \]
Beam with many particles this results in a tune spread

\[ \Delta Q \propto \frac{\delta F}{\delta r} \]


**Detuning with Amplitude for head-on**

\[ \Delta Q_{quad} = \text{const} \]

\[ \Delta Q(x) = \frac{Nr_0\beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(x/2)^2} \cdot (\exp -\left(\frac{x}{2}\right)^2 I_0\left(\frac{x}{2}\right)^2 - 1) \]

Head-on detuning with amplitude

$\Delta Q \propto \frac{\delta F}{\delta r}$

1-D plot of detuning with amplitude for opposite and equally charged beams

Maximum tune shift for small amplitude particles
Zero tune shift for very large amplitude particles

And in the other plane? THE SAME DERIVATION
Head-on detuning with amplitude and footprints

1-D plot of detuning with amplitude

Footprint

2-D mapping of the detuning with amplitude of particles
Long Range detuning with amplitude
1-D plot of detuning with amplitude for opposite and equally charged beams

Maximum tune shift for **large amplitude particles**
Smaller tune shift detuning for **zero amplitude particles and opposite sign**
2-D Long Range detuning with amplitude

Tune shift as a function of separation in horizontal plane
In the horizontal plane long range tune shift
In the vertical plane opposite sign!

Long range tune shift scaling for distances
\[ d > 6\sigma \]
\[ \Delta Q_{lr} \propto -\frac{N}{d^2} \]
Beam-beam tune shift and tune spread

Head-on and Long range interactions detuning with amplitude

Footprints depend on:
- number of interactions (124 per turn)
- Type (Head-on and long-range)
- Separation
- Plane of interaction

Very complicated depending on collision scheme

Pushing luminosity increases this area while we need to keep it small to avoid resonances and preserve the stability of particles

Strongest non-linearity in a collider
Beam-beam tune shift and spread

Higher Luminosity $\rightarrow$ increases this area
We need to keep it small to avoid resonances and preserve the long term stability of particles

The footprint from beam-beam sits in the tune diagram
LHC Footprints and multiple experiments

LHC 2012 example

...operationally it is even more complicated!
...different intensities, emittances...
Dynamical Aperture and Particle Losses

Dynamic Aperture: area in amplitude space with stable motion
Stable area of particles depends on beam intensity and crossing angle

Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, $\beta^*$, intensity) is the result of careful study of different effects!
Dynamical Aperture and Particle Losses

Beam-beam linear dependency with Intensity

Our goal: keep dynamical aperture as large as possible \(\rightarrow\) all particles not lost over long tracking time (\(10^6\) turns in simulation) equivalent to 1 minute of collider

Example collider collision time: 24 hours
Round optics 15 cm, 590μrad: intensity scan

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]

\[ \xi = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2} \]
Round optics 15 cm, 590μrad: intensity scan

\[ L = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]

\[ \xi = \frac{Nr_0 \beta^*}{4\pi \gamma \sigma^2} \]
Round optics 15 cm, 590μrad: intensity scan

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]

\[ \xi = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2} \]
AT high intensity the beam-beam force gets too strong and makes particles unstable and eventually are lost.
Round 15cm, 2.2E11, 690μrad

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]

Smaller beam-beam separation at parasitic long-range encounters stronger non linearities \( \rightarrow \) smaller stable area \( \rightarrow \) losses
Round 15cm, 2.2E11, 650μrad

\[ L = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]

\[ d_{lr} \propto \sqrt{\frac{\beta \alpha^2 \gamma}{\varepsilon_n}} \]
Round 15cm, 2.2E11, 590μrad

\[ d_{lr} \propto \sqrt{\frac{\beta \alpha^2 \gamma}{\epsilon_n}} \]

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]
Round 15cm, 2.2E11, 540μrad

\[ \beta = 15 \text{cm} \quad \text{Beam Charge} = 2.2 \cdot 10^{11} \text{p/bunch} \]

\[ d_{lr} \propto \sqrt{\frac{\beta \alpha^2 \gamma}{\epsilon_n}} \]

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]
Round 15cm, 2.2E11, 490μrad

\[ d_{lr} \propto \sqrt{\frac{\beta \alpha^2 \gamma}{\epsilon_n}} \]

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]
Round 15cm, 2.2E11, 440μrad

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]

\[ d_{lr} \propto \sqrt{\frac{\beta \alpha^2}{\epsilon_n}} \]
Round 15cm, 2.2E11, 390μrad

\[ \beta = 15\text{cm} \quad \text{Beam Charge}=2.2 \times 10^{11} \text{p/bunch} \]

\[ d_{lr} \propto \sqrt{\frac{\beta \alpha^2 \gamma}{\epsilon_n}} \]

\[ \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \]
Round 15cm, 2.2E11, 390μrad

Crossing angle changes the separation and the strength of BB-LR that strongly affect the dynamics of particles tails first then if too strong core

At small separation particles gets unstable and eventually lost
How does it look like in the LHC?

Particle losses follow number of Long range interactions

Beam-Beam separation at first LR

\[ d_{sep} = \alpha \cdot \sqrt{\frac{\gamma \cdot \beta^*}{\epsilon}} \]

Small crossing angle = small separation

If separation of long range too small particles become unstable and are lost proportionally to the number of long range encounters

Particle losses follow number of Long range interactions
Do we see the particle losses?

Particle losses follow number of Long range interactions
Machine protection implication and beam lifetimes gets worse...

Best performance of collider always a trade off between beam-beam and luminosity

Regular Physics Fill of 2012 RUN LHC

ill No. 2710 Bunch by bunch luminosity decay rate 0-1 hours LRs comparison

Beam-Beam separation at first LR

\[ d_{sep} = \alpha \cdot \sqrt{\frac{\gamma \cdot \beta^*}{\epsilon}} \]

Small crossing angle = small separation

Luminosity decays following the long range numbers... higher number of long range interactions larger losses
Long-range Beam-Beam effects: orbit

Long Range Beam-beam interactions lead to several effects...

Long range angular kick

\[ \Delta x'(x + d, y, r) = -\frac{2N r_0 (x + d)}{\gamma r^2} [1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)] \]

For well separated beams

\[ d \gg \sigma \]

The force has several components at first order we have an amplitude independent contribution: ORBIT KICK

\[ \Delta x' = \frac{\text{const}}{d} \left[ 1 - \frac{x}{d} + O\left(\frac{x^2}{d^2}\right) + \ldots \right] \]

In simple case (1 interaction) one can compute it analytically
Orbit effect as a function of separation

Angular Deflections:

\[ \theta_y + i\theta_x = \frac{2r_p}{\gamma} N_p F_0(x, y, \Sigma) \]

Closed Orbit effect:

\[ Orb_{x,y} = \theta_{x,y} \cdot \beta_{x,y} \cdot \frac{1}{2 \tan(\pi \cdot Q_{x,y})} \]
Orbit effect as a function of separation

Angular Deflections:
\[ \theta_y + i\theta_x = \frac{2r_p}{\gamma} N_p F_0(x, y, \Sigma) \]

Closed Orbit effect:
\[ Orb_{x,y} = \theta_{x,y} \cdot \beta_{x,y} \cdot \frac{1}{2\tan(\pi \cdot Q_{x,y})} \]

Orbit can be corrected but we should remember PACMAN effects
LHC orbit effects

Many long range interactions could become important effect! Holes in bunch structure leads to PACMAN effects this cannot be corrected!

\[ L = L_0 \cdot e^{-\frac{d^2}{4\sigma^2}} \]

1-2% Luminosity loss due to beam-beam orbit effects
Summary

- Head-on Interactions
- Long-Range Effects
- Beam-Beam parameter
- Tune spread
- Dynamic Beta
- Particle Losses
- Orbit effects
- Emittance Increase
- Crossing angles
- \( \mathcal{L} = \frac{N_1 N_2 f n_b \gamma}{4\pi \epsilon_{x,y} / \beta^*} \cdot F \)
- \( L \) measurements uncertainties
...not covered here...

- *Beam-Beam compensation schemes*
- *Landau damping and beam-beam*
- *Beam-Beam coherent effects*
- *Asymmetric beams effects*
- *Noise on colliding beams*
- *Luminosity hourglass effect*
- *Measuring Luminosity: Van der Meer scans*
- *Pile-up and leveling luminosity*
- *....*
Pile-up and Luminosity leveling

Experiments might need luminosity control
• if too high can cause high voltage trips then impact efficiency of the detectors
• might have event size or bandwidth limitations in read-out
• too many simultaneous event cause loss of resolution

...experiments also care about the average number of inelastic interactions per bunch crossing

\[ \mathcal{L} = \frac{N_1 N_2 f n_b \gamma}{4 \pi \epsilon_x, y \beta^*} \cdot F \]

• \( \beta^* \) leveling
• Offset leveling
• Crossing angle leveling

ALICE and LHCb level with transverse offset

\[ L = L_0 \cdot e^{-\frac{d^2}{4 \sigma_x^2}} \]
Pile-up and Luminosity leveling

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ALICE and LHCb level with transverse offset

\[ L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}} \]
Thank you!

Questions?
References:

[*] R. Assmann et al., “Results of long-range beam-beam studies - scaling with beam separation and intensity ”

...much more on the LHC Beam-beam webpage:
http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/
Luminosity Basics

\[ N_{\text{events}} = L \times \sigma_{\text{event}} \]

Mean number of inelastic interactions per Bunch crossing

\[ \mu_{\text{vis}} = \varepsilon \mu = \text{Mean number of interactions per Bunch crossing seen by detector} \]

\[ \mathcal{L} = \frac{\mu n_b f_r}{\sigma_{\text{inel}}} = \frac{\mu_{\text{vis}} n_b f_r}{\sigma_{\text{vis}}} \]

Inelastic cross section (unknown)

Cross section seen by detector

\[ \sigma_{\text{vis}} \text{ is determined in dedicated fills based on beam parameters} \]

W. Kozanecki

Van der Meer Scans

- Luminosity in terms of beam densities $\rho_1$ and $\rho_2$ in machine:

Luminosity in general

$$\mathcal{L} = n_b f_r n_1 n_2 \int \rho_1(x, y) \rho_2(x, y) dx dy$$

Gaussian beams and uncorrelated x & y components no crossing angle:

$$L_0 = \frac{N_1 N_2 f n_b}{2\pi \sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}}$$
Calibrating $\sigma_{\text{vis}}$ during van der Meer Scans

\[ \mathcal{L} = \frac{\mu n_b f_r}{\sigma_{\text{inel}}} = \frac{\mu_{\text{vis}} n_b f_r}{\sigma_{\text{vis}}} \]

**Measured in VdM scan**

**Detector independent**

\[ \sigma_{\text{vis}} = \mu_{\text{vis}}^{\text{Max}} \frac{2\pi \sum_x \sum_y}{n_1 n_2} \]

**Detector dependent**

**Measured by beam instrumentation**

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Gaussian fit of Lumi scans to extrapolate $\mu_{\text{vis}}^{\text{Max}}$ and $\Sigma_x$

LHC fill: 2520

$\nu S = 8\text{TeV}$

W. Kozanecki
Van der Meer scans and Beam-beam

Beam-Beam force

$$\theta_y + i\theta_x = \frac{2 r_p}{\gamma} N_p F_0(x, y, \Sigma)$$

Beam-beam angular kick produces orbit change

Dynamic beta effects: beam sizes affected by beam-beam

Uncertainties corrected for during Van der Meer calibration scans
Impact of long-range encounters on $L$ scans: data

May 2011

vdM scan

Total # Long-Range Encounters

Orbit drift

W. Kozanecki