Linear Imperfections

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Introduction

Imperfection - sources

Orbit perturbations

Optics perturbations

Coupling between planes

Linear imperfections and geology

Summary
An accelerator is usually built using a number of basic ‘cells’.

The cell layouts of an accelerator come in many subtle variants.

For today we consider a simple FODO cell containing:

- **Dipole magnets** to bend the beams,
- **Quadrupole magnets** to focus the beams,
- **Beam position monitors** (BPM) to measure the beam position,
- **Small dipole corrector magnets** for beam steering.

*Schematic of a ½ cell*
The dipole has two magnetic poles and generates a **homogeneous field** providing a constant force on all beam particles – used to **deflect** the beam.

- A dipole corrector is just a small version of such a magnet, dedicated to steer the beam as we will see later.

**Lorentz force:**

\[
F = q \vec{v} \times \vec{B}
\]

orthogonal to the speed and magnetic field directions
A quadrupole has 4 magnetic poles.

A quadrupole provides a field (force) that **increases linearly** with the distance to the quadrupole center – provides **focussing** of the beam.

- Similar to an optical lens, except that a quadrupole is focussing in one plane, defocussing in the other plane.

\[
F_y = ky \\
F_x = -kx
\]

Force pushes the particle away from the center → **defocussing**

Force pushes the particle towards the center → **focussing**
The LHC arc section are equipped with 107 m long F0D0 cells. Besides our 3 main elements the LHC cell is equipped with other correction (trim) magnets.

- MB: main dipole
- MQ: main quadrupole
- MQT: trim quadrupole
- MQS: skew trim quadrupole
- MO: lattice octupole (Landau damping)
- MSCB: sextupole + orbit corrector dipole
- MCS: Spool piece sextupole
- MCDO: Spool piece 8 / 10 pole
- BPM: Beam position monitor
Recap on beam optics

- There are a few quantities related to a beam optics in a circular accelerator that we will need for the lecture:
  - The **betatron function** ($\beta$) that defines the beam envelope,
    - Beam size / envelope is proportional to $\sqrt{\beta}$
  - The **betatron phase advance** ($\mu$) that defines the phase of an oscillation.

![LHC optics at injection](image)
Consider a particle moving in a section of the accelerator lattice. The focussing elements make it bounce back and forth. Does this not look a bit like a periodic oscillation? This is called a betatron oscillation.
The number of oscillation periods for one turn of the machine is called the machine **tune** ($Q$) or **betatron tune**.

- In this example $Q$ is around 2.75 – 2 periods and $\frac{3}{4}$ of a period.

It is possible to change the **coordinates** (from the longitudinal position in meters to the betatron phase advance in degrees) and transform this ‘rocky’ oscillation into a pure sinusoidal oscillation.

- Very convenient (and simpler) way to analyse the beam motion.
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Summary
The physical units of the machine model defined by the accelerator physicist must be converted into magnetic fields and eventually into currents for the power converters that feed the magnet circuits.

Imperfections (= errors) in the real accelerator optics can be introduced by uncertainties or errors on:
- Beam momentum, magnet calibrations and power converter regulation.

![Example of the LHC main dipole calibration curve](image-url)
From the lab to the tunnel
To ensure that the accelerator elements are in the correct position the alignment must be precise – to the level of micrometres for CLIC!

- At the CERN hadron machines we aim for accuracies of around 0.1 mm.

The alignment process implies:

- Precise measurements of the magnetic axis in the laboratory with reference to the element alignment markers used by the survey group.
- Precise in-situ alignment (position and angle) of the element in the tunnel.

Alignment errors are a common source of imperfections.
Please remember that accelerator components in the CERN tunnels are carefully aligned – please treat with respect!

Use the ladder!
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Summary
The presence of an **unintended deflection** along the path of the beam is a first category of imperfections.

This case is also in general the first one that is encountered when beam is first injected...

The **dipole orbit corrector** is added to the cell to **compensate** the effect of **unintended deflections**.

- With the orbit corrector we can generate a deflection of opposite sign and amplitude that compensates locally the imperfection.

How can an **unintended deflection** appear?
The first source is a **field error** (deflection error) of a *dipole magnet*. This can be due to an **error** in the *magnet current* or in the *calibration table* (measurement accuracy etc).

- The imperfect dipole can be expressed as a perfect one + a small error.

\[
\text{real dipole} = \text{ideal dipole} + \text{small dipole error}
\]

A small **rotation** (misalignment) of a *dipole magnet* has the same effect, but in the other plane.

\[
\text{real dipole} = \text{ideal dipole} + \text{small dipole error}
\]
The second source is a **misalignment** of a *quadrupole* magnet.

- The misaligned quadrupole can be represented as a perfectly aligned quadrupole plus a small deflection.

\[ \text{real quadrupole} = \text{ideal quadrupole} + \text{small dipole error} \]

*Non-zero magnetic field on the beam axis!*

*No magnetic field on the beam axis!*
We set the machine tune to an integer value:
- \( Q = n \in \mathbb{N} \)

When the tune is an integer number, the deflections add up on every turn!
- The amplitudes diverge, the particles do not stay within the accelerator vacuum chamber.

We just encountered our first \textbf{resonance} – the \textbf{integer resonance} that occurs when \( Q = n \in \mathbb{N} \)
We set the machine tune to a half integer value:
- $Q = n+0.5$, $n \in \mathbb{N}$

For half integer tune values, the deflections compensate on every other turn!
- The amplitudes are stable.

This looks like a much better working point for $Q$!
We set the machine tune to a quarter integer value:
- \( Q = n+0.25, n \in \mathbb{N} \)

For quarter tune values, the deflections compensate every four turns!
- The amplitudes are stable.

Also a reasonable working point for Q!
Many turns reveal something

- Let’s plot the 50 first turns on top of each other and change Q.
  - All plots are on the same scale

\[ Q = n + 0.5 \]

\[ Q = n + 0.3 \]

\[ Q = n + 0.1 \]

\[ Q = n \]

\[ Q = n + 0.4 \]

\[ Q = n + 0.2 \]

\[ Q = n + 0.05 \]

- The particles oscillate around a stable mean value \((Q \neq n)! \)
- The amplitude diverges as we approach \(Q = n \) \(\rightarrow\) integer resonance
The closed orbit

- The stable mean value around which the particles oscillate is called the **closed orbit**.
  - Every particle in the beam oscillates around the closed orbit.
  - As we have seen the closed orbit ‘does not exist’ when the tune is an integer value.

- The general expression of the **closed orbit** \( x(s) \) in the presence of a **deflection** \( \theta \) is:

\[
x(s) = \frac{\sqrt{\beta(s)\beta_\theta} \cos(|\mu(s) - \mu_\theta| - \pi Q)}{2\sin(\pi Q)} \theta
\]
Example of the horizontal closed orbit for a machine with tune $Q = 6 + q$. The **kink at the location of the deflection** (→) can be used to localize the deflection (if it is not known) → can be used for orbit correction.
In the example below for the 26.7km long LHC, there is **one undesired deflection**, leading to a perturbed closed orbit.

Where is the location of the deflection?
To make our life easier we divide the position by $\sqrt{\beta(s)}$ and replace the BPM index by its phase $\mu(s)$.

$$\frac{x(s)}{\sqrt{\beta(s)}} = \sqrt{\beta_\theta} \cos(\left| \mu(s) - \mu_\theta \right| - \pi Q) \frac{\theta \propto \cos(\left| \mu(s) - \mu_\theta \right| - \pi Q)}{2\sin(\pi Q)}$$

Can you localize the deflection now?
Now a more realistic orbit with 100’s of deflections.

How do we proceed to correct?
The problem of correcting the orbit deterministically came up a long time ago in the first CERN machines.

B. Autin and Y. Marti published a note in 1973 describing an algorithm that is still in use today (but in JAVA/C/C++ instead of FORTRAN) at ALL CERN machines:

- **MICADO**


* **MInimisation des CArrés des Distortions d'Orbite.**

(Minimization of the quadratic orbit distortions)
The intuitive principle of MICADO is rather simple.

**Preparation:**
- You need a model of your machine,
- You compute for each orbit corrector what the effect (response) is expected to be on the orbit.
MICADO - how does it work?

- MICADO compares the response of every corrector with the raw orbit.
- MICADO then picks out the corrector that has the best match with the orbit, and that will give the largest improvement to the orbit deviation rms.
- The procedure can be iterated until the orbit is good enough (or as good as it can be).
The raw orbit at the LHC can have huge errors, but the correction (based partly on MICADO) brings the deviations down by more than a factor 20.

At the LHC a good orbit correction is vital!
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Summary
What is the impact of a quadrupole gradient error?

Let us consider a particle oscillating in the lattice.

Too strong gradient / lens

The oscillation period is affected \(\rightarrow\) change of tune, here \(Q\) increases!
In a ring a focussing error affects the beam optics and envelope (size) over the entire ring! It also changes the tune.

**Example for LHC: one quadrupole gradient is incorrect**

```
! Image of graph showing nominal and perturbed optics over s [m] range.
```

**Zoom into a subsection**
The local beam optics perturbation… note the oscillating pattern of the error.
The error is easier to analyse and diagnose if one considers the ratio of the betatron function perturbed/nominal.

The ratio reveals an oscillating pattern called the betatron function beating (‘beta-beating’). The amplitude of the perturbation is the same all over the ring!
The beta-beating pattern comes out even more clearly if we replace the longitudinal coordinate with the betatron phase advance.

The result is very similar to the case of the closed orbit kick, the error reveals itself by a kink!

- If you watch closely you will observe that there are two oscillation periods per $2\pi$ (360 deg) phase. The beta-beating frequency is twice the frequency of the orbit!

![Graph showing beta-beating pattern](image-url)
How can one correct such beta-beating?

The correction strategy with MICADO can be applied:
- You can build the response of any gradient change on the optics ($\beta$).
- You can use MICADO to look for the best possible solution.
- The correcting elements are the quadrupole themselves (adjust their current).

For optics corrections more sophisticated and powerful algorithm provide however better correction strategies.
In collision at top energy of 6.5 TeV, the optics is wrong by 100% before correction.

- Can be corrected to a few % residual error with modern correction algorithms.
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Summary
If a quadrupole is rotated by 45° (‘skew quadrupole’) one obtains an element where the force (deflection) in \( x \) depends on \( y \) and vice-versa: the horizontal and vertical planes are **coupled**.

**normal quadrupole**

\[
F_x = -k x \quad \text{No mixing of planes} \\
F_y = k y
\]

**skew quadrupole**

\[
F_x = k y \\ F_y = -k x
\]
- Small quadrupole tilts lead to coupling of the x and y planes.
- The coupling can be corrected by installing dedicated skew quadrupoles to compensate for alignment errors.
The simplest thing to determine if there is coupling is to kick the beam in one plane to generate an oscillation, and then observe the oscillations or the frequency content.

- Or just use the natural beam oscillations if they exist.

If coupling is present, then for a horizontal kick there will be a small vertical oscillation (and vice-versa).
We apply a **Fourier analysis** to the position data to extract the beam oscillation frequencies.

Example: horizontal beam position at a BPM observed turn by turn

The ratio of the vertical to horizontal amplitude measures the amount of coupling → now one can tune the skew quadrupoles until the vertical tune peak disappears.
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Summary
Linear imperfections, geology and celestial bodies
Earth tides

Tide bulge of a celestial body of mass $M$ at a distance $d$

$$\Delta R \sim \frac{M}{2d^3}(3\cos^2\theta - 1)$$

$\theta = \text{angle (vertical, the celestial body)}$

induces surface deformations and affects the water levels of the oceans.

→ impacts the alignment of the entire accelerator!

Such Earth tides alter the accelerator circumference:

- The Moon contributes 2/3, the Sun 1/3.
- Not resonance-driven (unlike Sea tides!).
- Accurate predictions possible (~%).

LEP tide predictions for Nov. 1992

The relative circumference change amounts to $\sim 10^{-8} \sim 1 \text{ mm}$.

Gravitational wave detectors achieve sensitivities of $\sim 10^{-21}$!
At the LHC the beams are ‘captured’ by the RF system which forces the beams to remain synchronous with the RF frequency.

- Because at LHC the speed $\approx c = \text{constant}$, this fixes the length of the orbit.

When the frequency is well adjusted, the length of the orbit $L$ matches the circumference $C$.

If for any reason $C$ varies, the beam has to move radially if $L$ is kept constant.

A mismatch between $C$ and $L$ can be observed on the mean radial orbit using the BPMs that move with the ring.

- As a side effect it also changes very slightly the beam energy (level of 0.01%).

\[ L = C \]

\[ L < C \]
Tides are observed very clearly on the LHC circumference by measuring the mean radial (=horizontal) beam position.

Tide observations (from orbit changes) over one week at 4 TeV in 2016 (expressed in energy change $\Delta p/p$)

Earthquake in New Zealand
The pressure waves induce a modulation of the circumference

$10^{-4}$
Waves from earthquakes

Different types of body (Pressure, Shear) and surface waves (Raleigh, Love), multiple paths and reflections produce a complex signature of earthquakes at seismic measurement stations – also at the LHC.
A magnitude 7.6 earthquake in Costa Rica (05/09/2012 @ 14:42:10 UTC) ‘struck’ the LHC in fill 3032 during stable colliding beams.

- Arrival of the first waves at CERN ~15:06 UTC.

The arrival of the different waves can be observed on the radial beam position – equivalent to largest tides.
We have seen that magnetic field errors and misalignments of accelerator components induce:

- Errors on the beam orbit,
- Errors on the optics and the tune,
- Coupling between the horizontal and vertical planes.

The errors are often sufficiently large (for sure at LHC) that the machine operates poorly or not at all.

Since the 1970’s ever improving tools and algorithms have been developed to correct for such errors.

However to minimize the imperfections from the start we need:

- well measured calibration curves of magnets,
- precise power converters,
- the best possible machine alignment.
What value for the tunes?

- Various collider tune working points.