Injection, extraction and beam transfer

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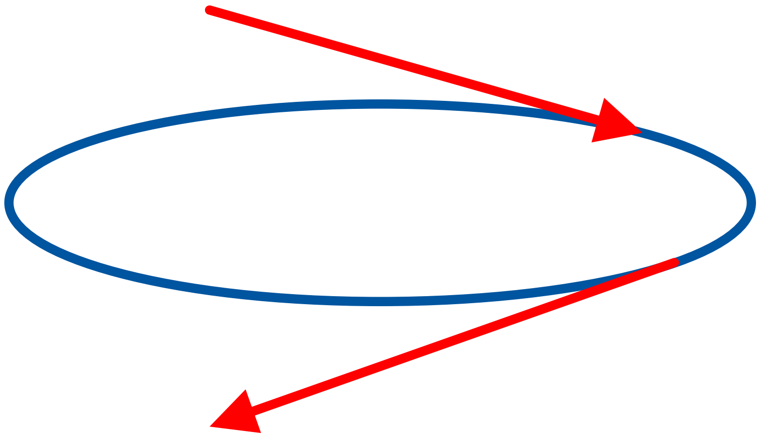
Basics of Accelerator Science and Technology at CERN

CAS@ESI 2018

Based on lectures by Brennan Goddard, Matthew Alexander Fraser, J. S. Schmidt, Rende Steerenberg

Outline

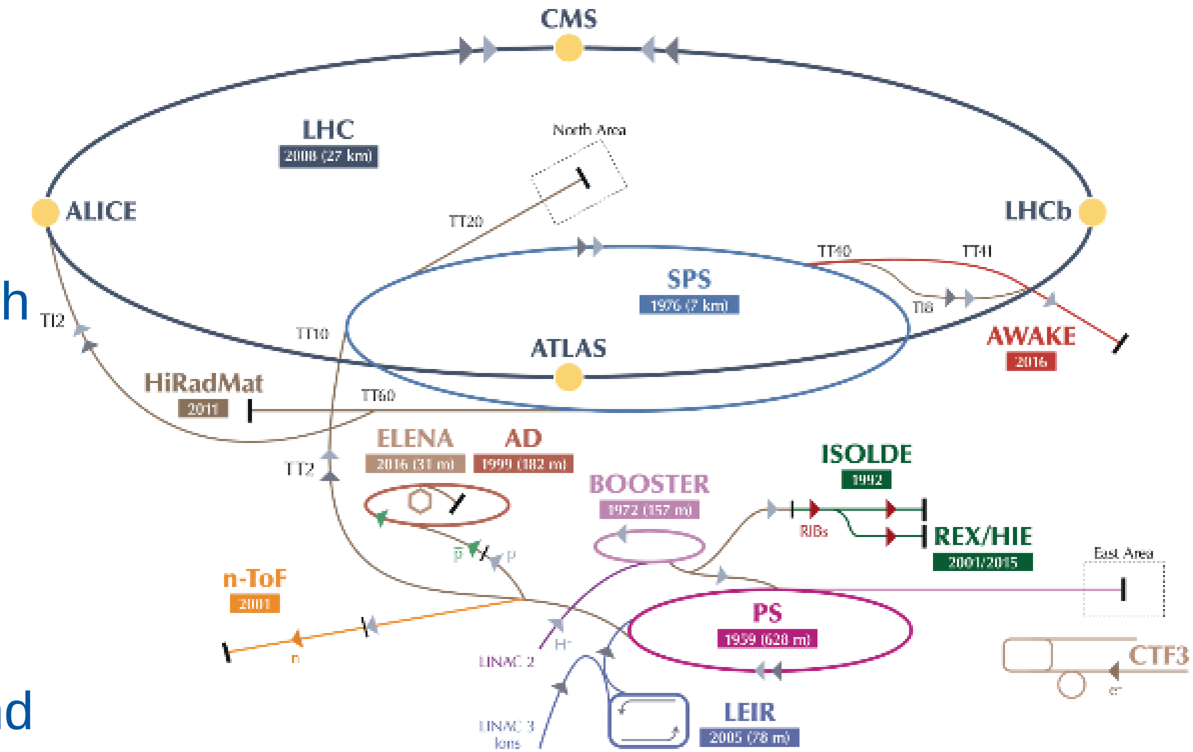


- **Introduction**
- Single-turn methods
  - Injection
  - Fast extraction
- Multi-turn methods
  - Multi-turn hadron injection
  - Charge-exchange H- injection
  - Multi-turn extraction
- Resonant slow extraction
- Summary

Injection, extraction and transfer

- An accelerator has limited dynamic range
- Chain of stages is needed to reach high energy
- Periodic re-filling of storage rings, like LHC
- External facilities and experiments:
  - e.g. ISOLDE, HiRadMat,…

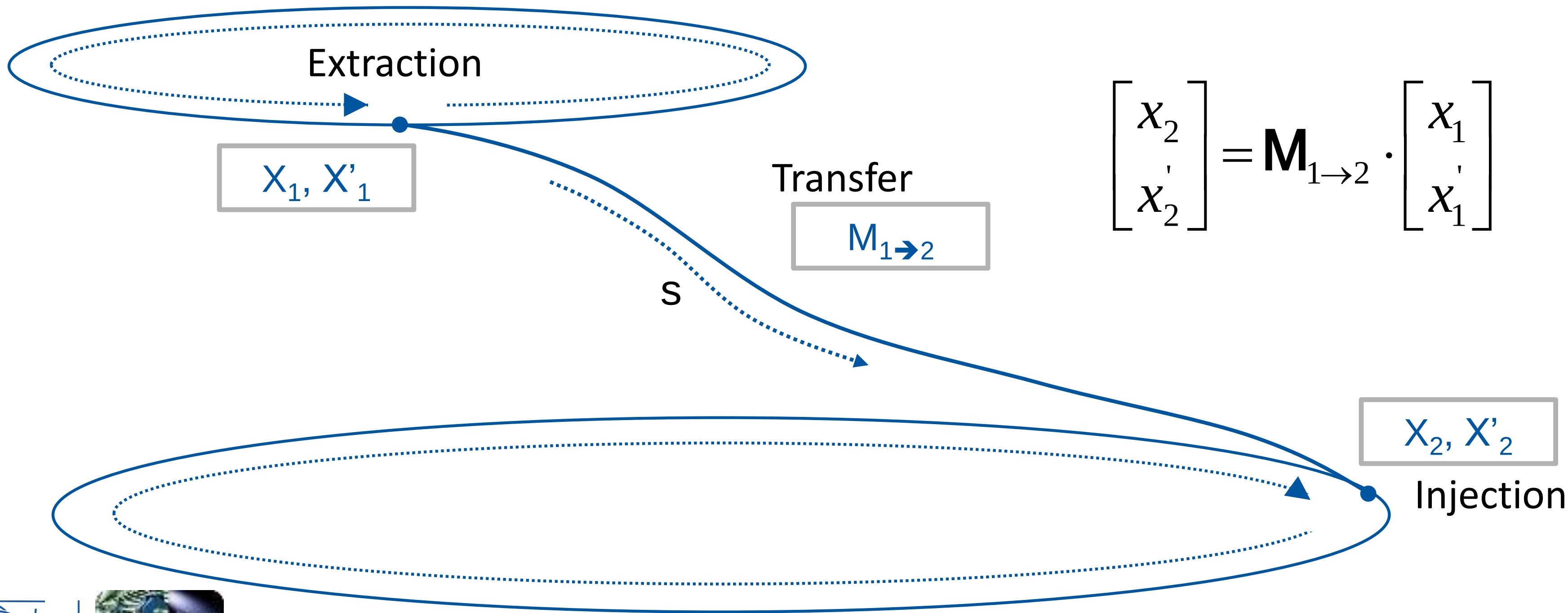
CERN Accelerator Complex



Beam transfer (into, out of, and between machines) is necessary.

Linking Machines

1. **Extract** a beam out of one machine
   ➔ initial beam parameters X_1, X'_1
2. **Transport** this beam towards the following machine (or experiment)
   ➔ apply transfer matrix
3. **Inject** this beam into a following machine with a predefined acceptance
   ➔ produce required beam parameter for matching X_2, X'_2



$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$$

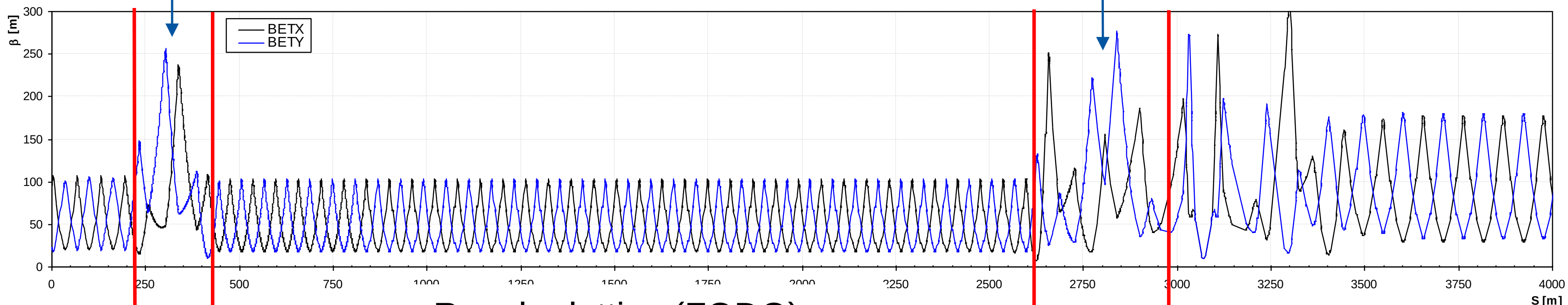
Optics Matching

Initial matching section

Independently powered (tuneable) quadrupoles

Final matching section

Independently powered (tuneable) quadrupoles and (in this case) passive protection devices



Regular lattice (FODO)
(elements all powered in series with same strengths)

SPS

LHC

SPS to LHC Transfer Line (3 km)

Extraction point

Injection point

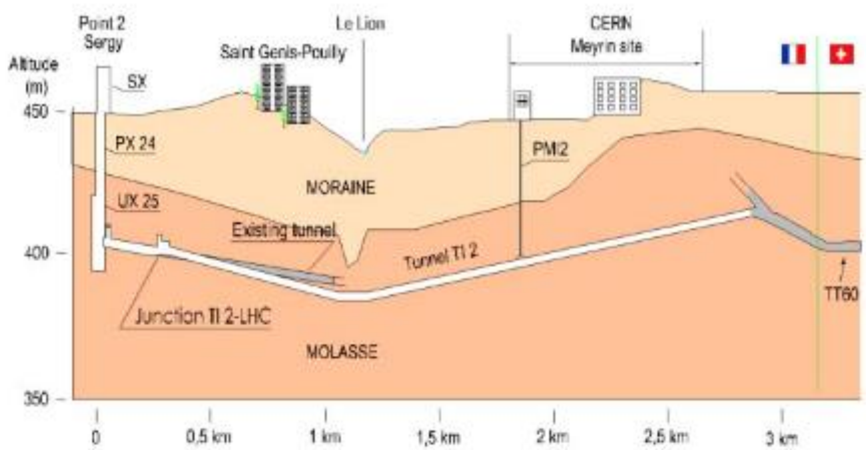
Need to “match” 8 variables (TWISS parameter) for propagation from start to end of a transfer

Linking Machines – constraints

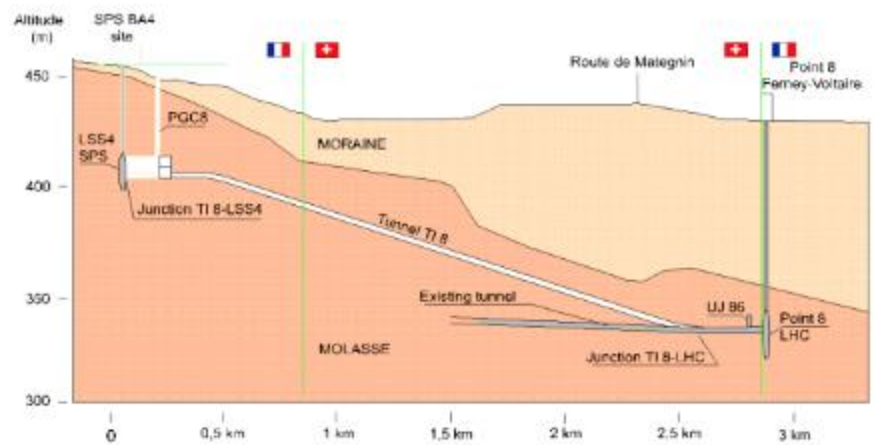
- Apertures of beam line elements define limitations for maximum β and dispersion values

$$\text{Envelope(S)} = \sqrt{\epsilon_{geo} \cdot \beta(S)} + \text{Dispersion(S)} \cdot \frac{\Delta p}{p} + \text{mechanical alignment} + \text{orbit error} \cdot \sqrt{\frac{\beta(S)}{\beta_{max}}}$$

- Minimum bend radius, maximum quadrupole gradient, magnet aperture, cost, geology or other obstacles, etc.
- Insertions for special equipment (like stripping foils)



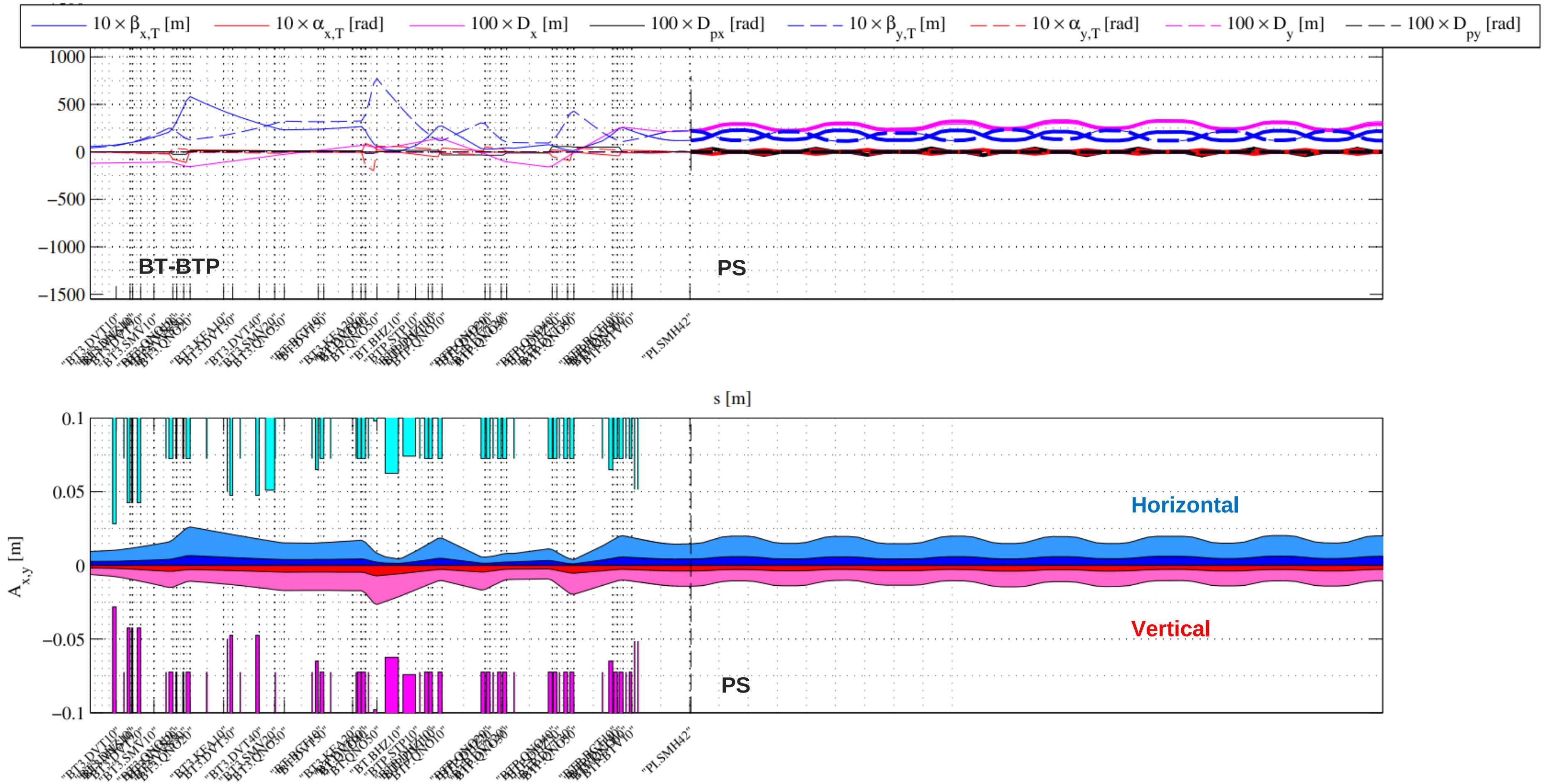
SPS to LHC transfer tunnel TI2 (Beam 1)



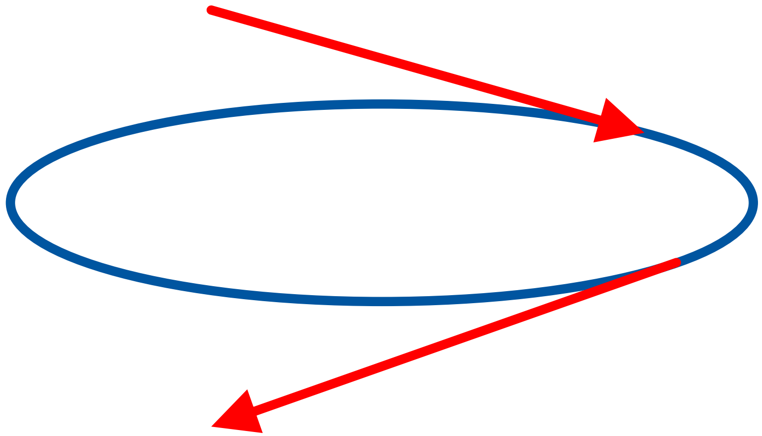
SPS to LHC transfer tunnel TI8 (Beam 2)

Aperture example

Example: Test optics for matching PSB-to-PS transfer line



Outline



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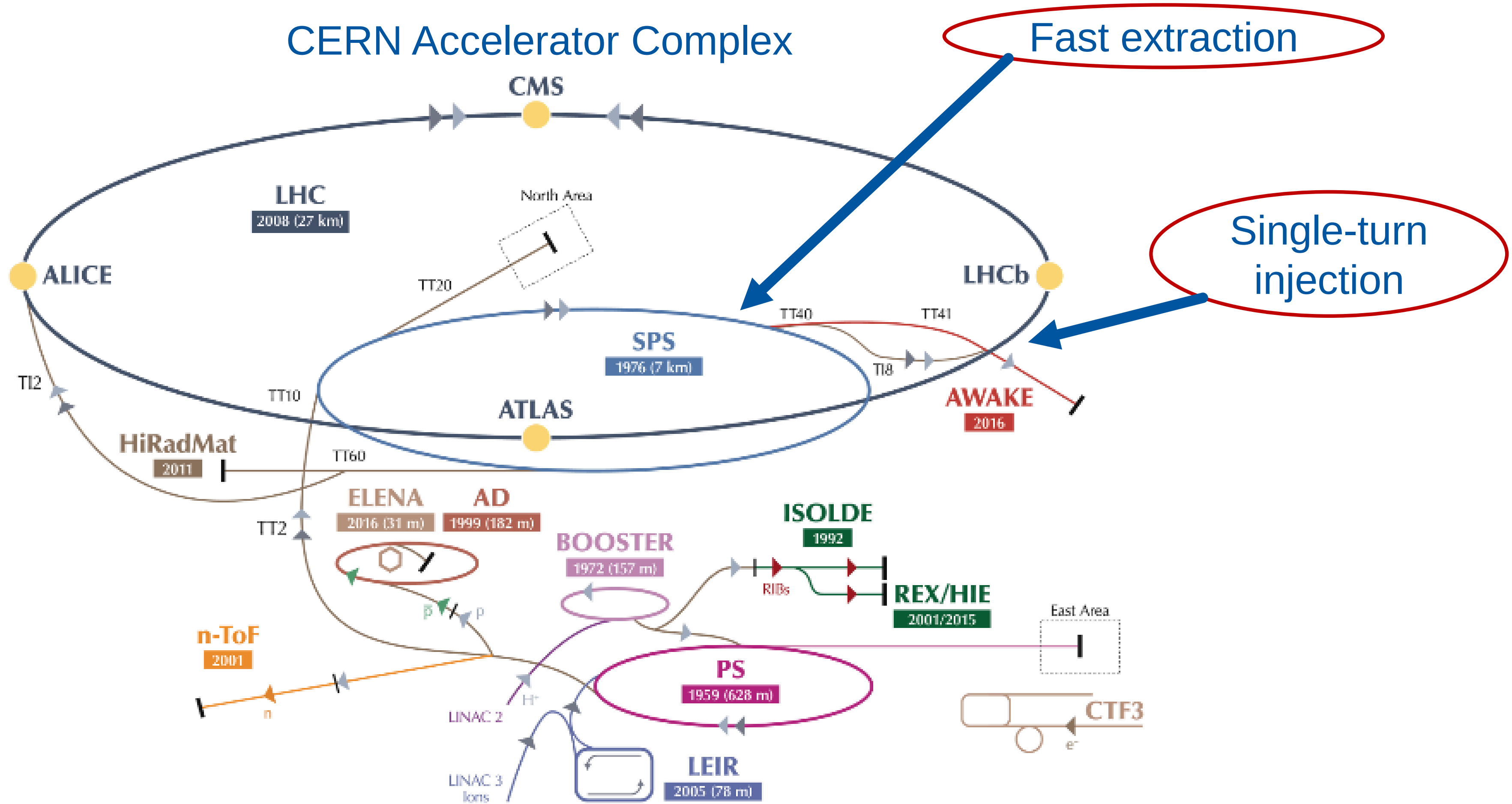
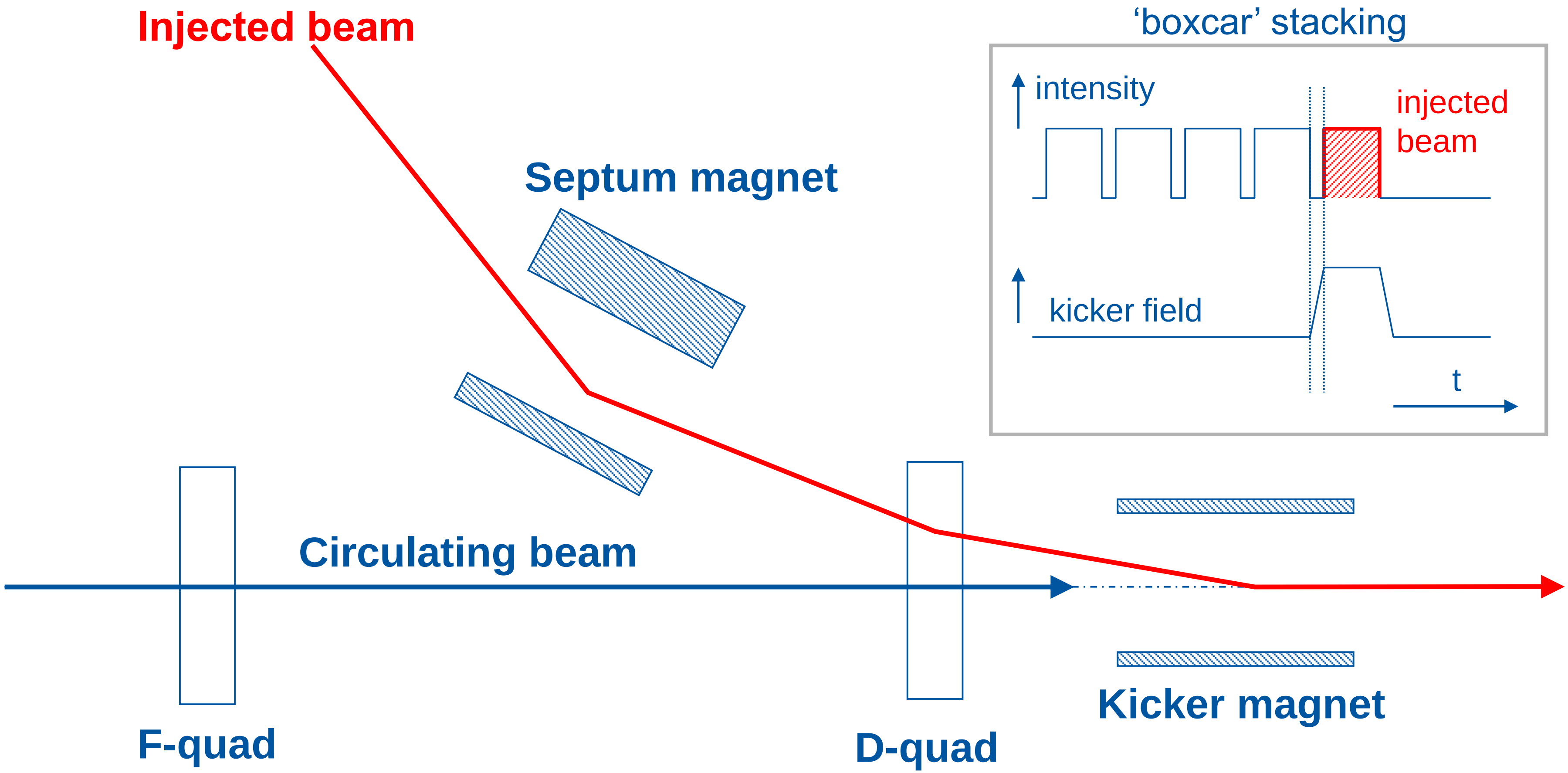
Single-turn methods



Injection / extraction of a single bunch or a bunch train within one turn.

Single-turn injection – same plane

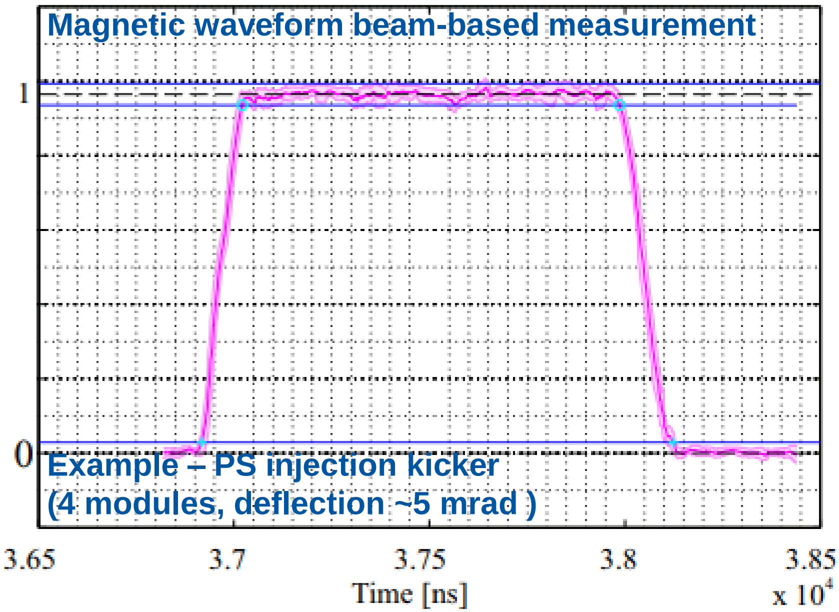


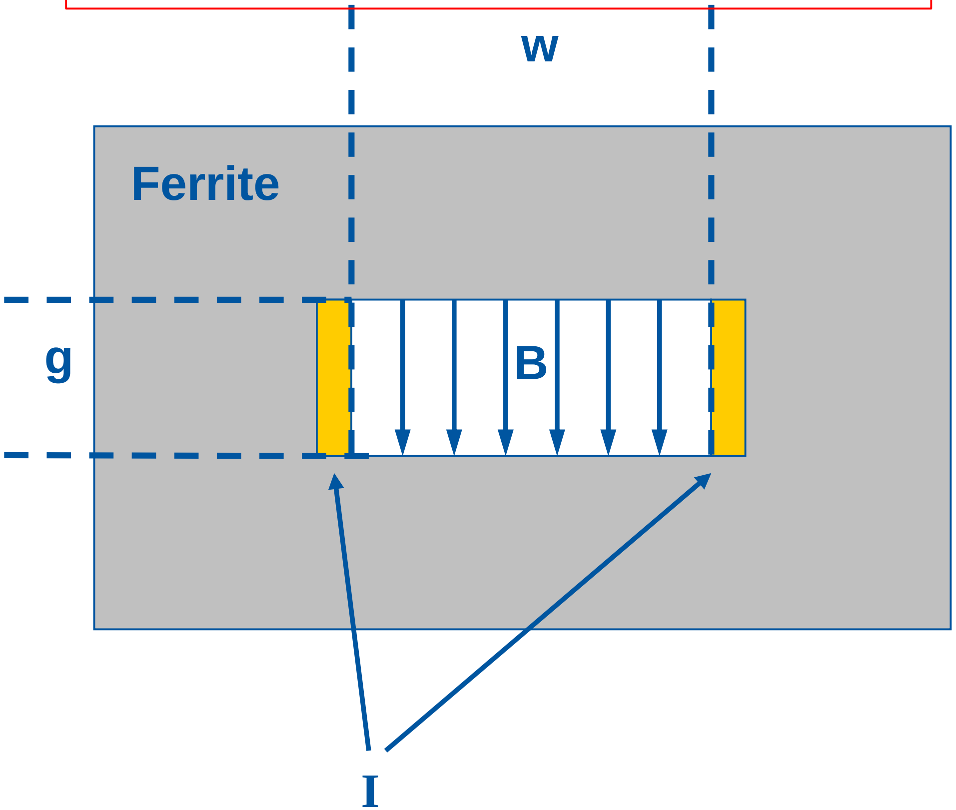
- Septum deflects the beam onto the closed orbit at the centre of the kicker
- Kicker compensates for the remaining angle
- Septum and kicker either side of D quad to minimize kicker strength

Kicker magnet

More details in T. Kramer's talk later

- Pulsed magnet with very fast rise time (100 ns – few μs)
- Typically 3 kA in 1 μs rise time





$B = \mu_0 I / g$

L [per unit length] $= \mu_0 w / g$

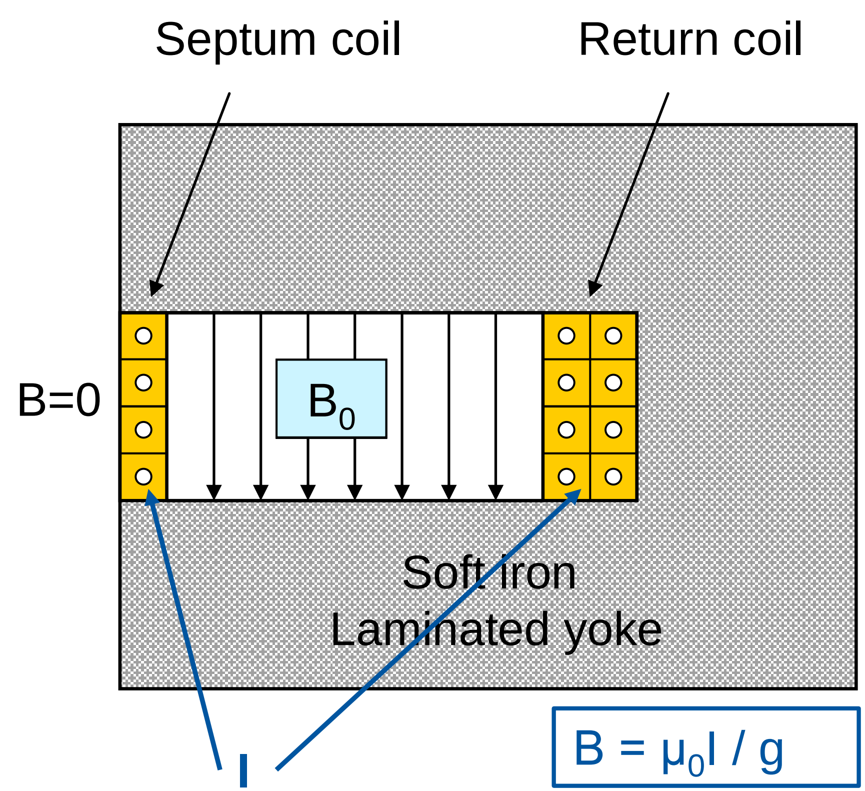
$dI/dt = V / L$

Example – SPS fast extraction
5 kicker with 0.5 mrad total deflection

Magnetic septum

More details in T. Kramer's talk later

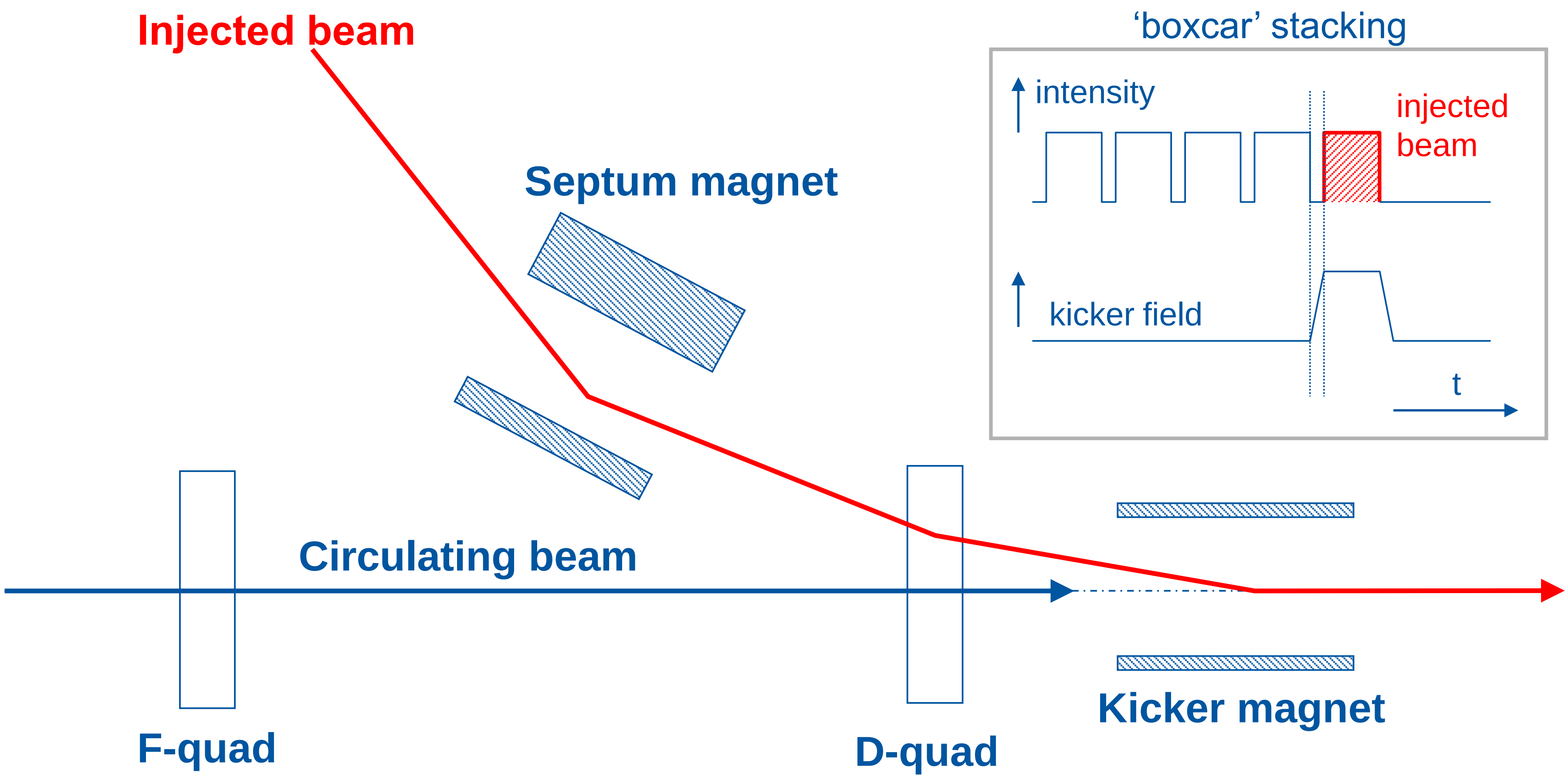
- Pulsed or DC magnet with thin (2 – 20 mm) Septum between zero field and high field region
- Typically
  - ~10x more deflection given by magnetic septa, compared to kickers
  - I ~ 5 - 25 kA



$B = \mu_0 I / g$

Example – SPS fast extraction
2.25 mrad total deflection

Single-turn injection – same plane



- Septum deflects the beam onto the closed orbit at the centre of the kicker
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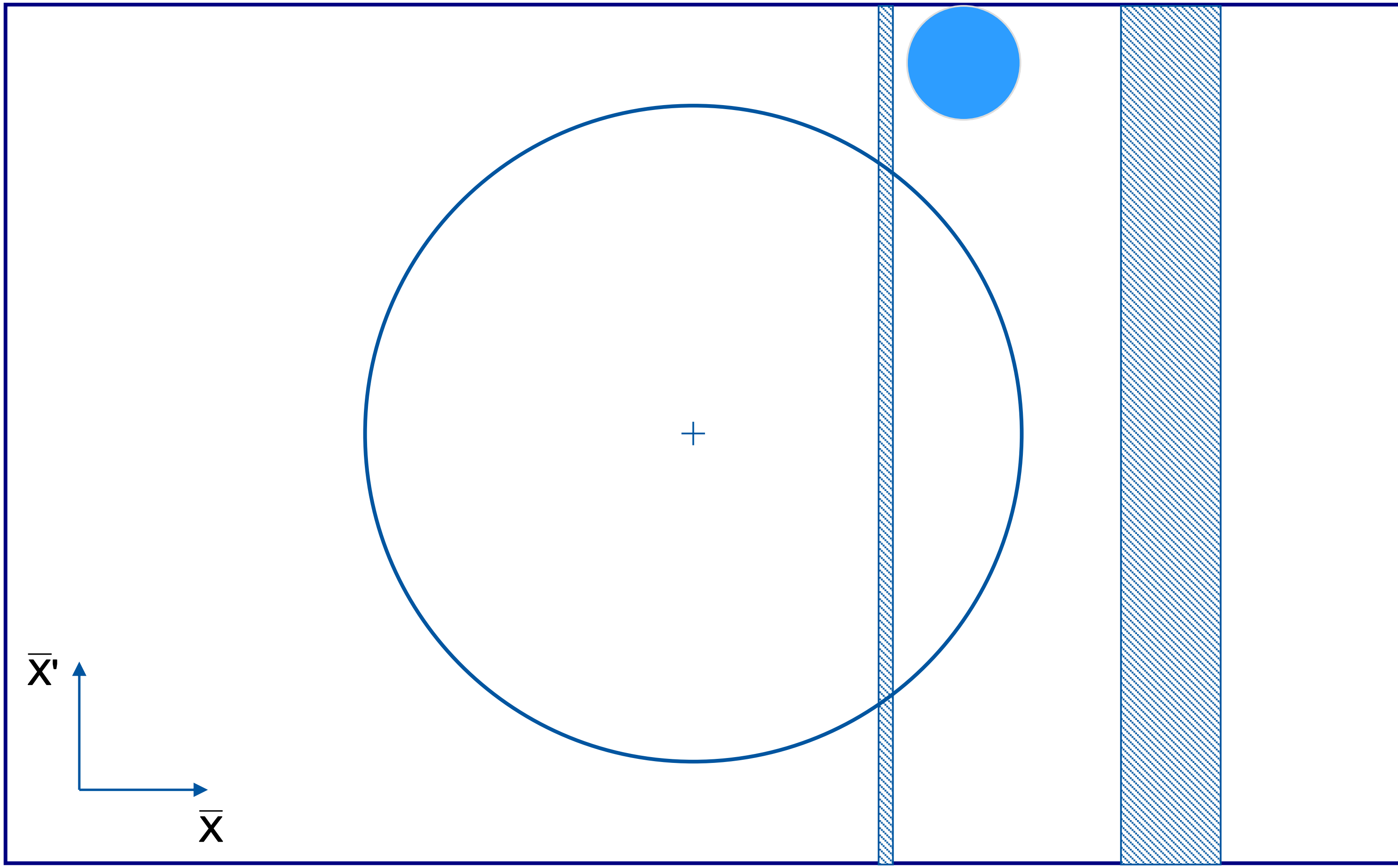
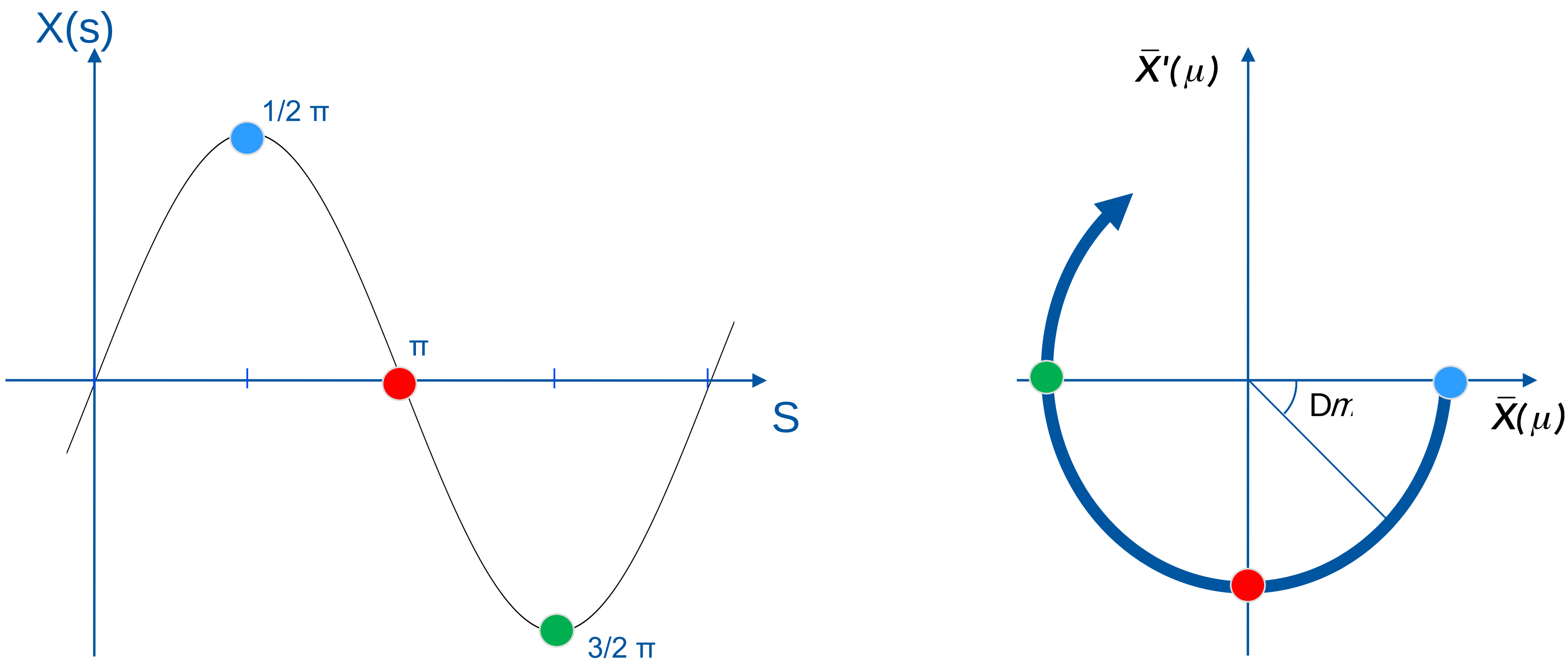
Single-turn injection

Normalised phase space at centre of idealised septum

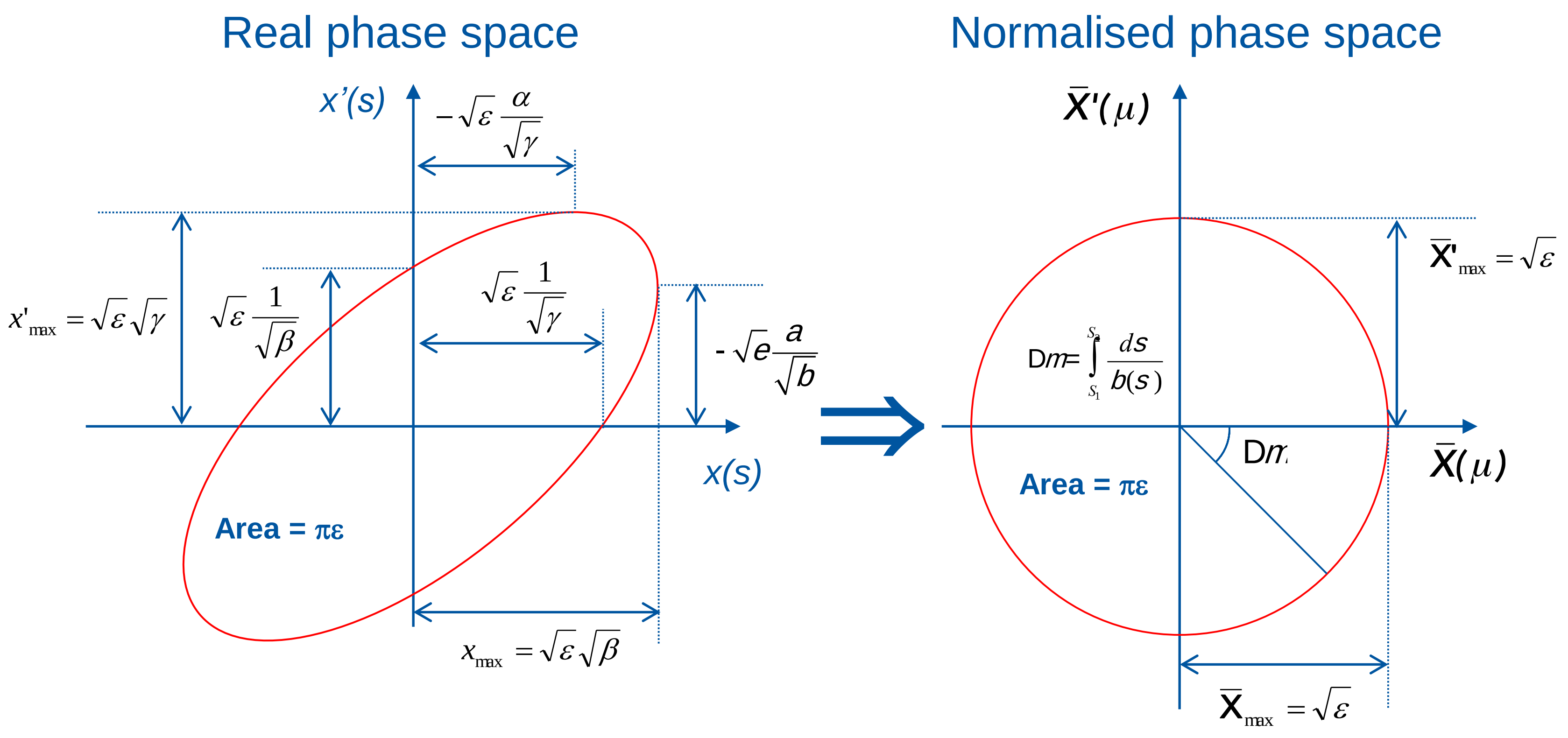


Normalised phase space



An oscillation in the longitudinal coordinate S can be translated into a rotation in phase.

Normalised phase space

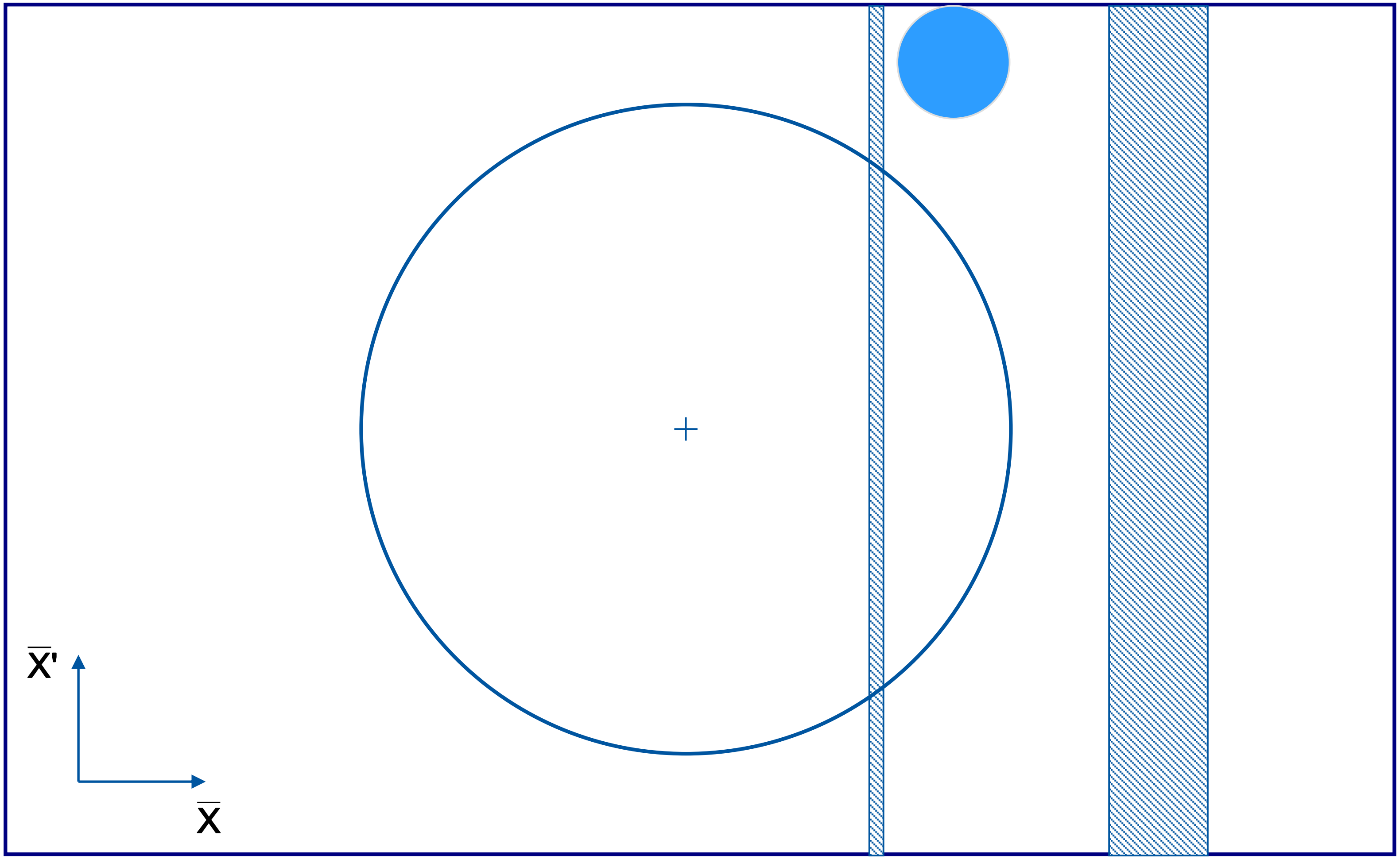


$$\varepsilon = \gamma \cdot x^2 + 2\alpha \cdot x \cdot x' + \beta \cdot x'^2$$

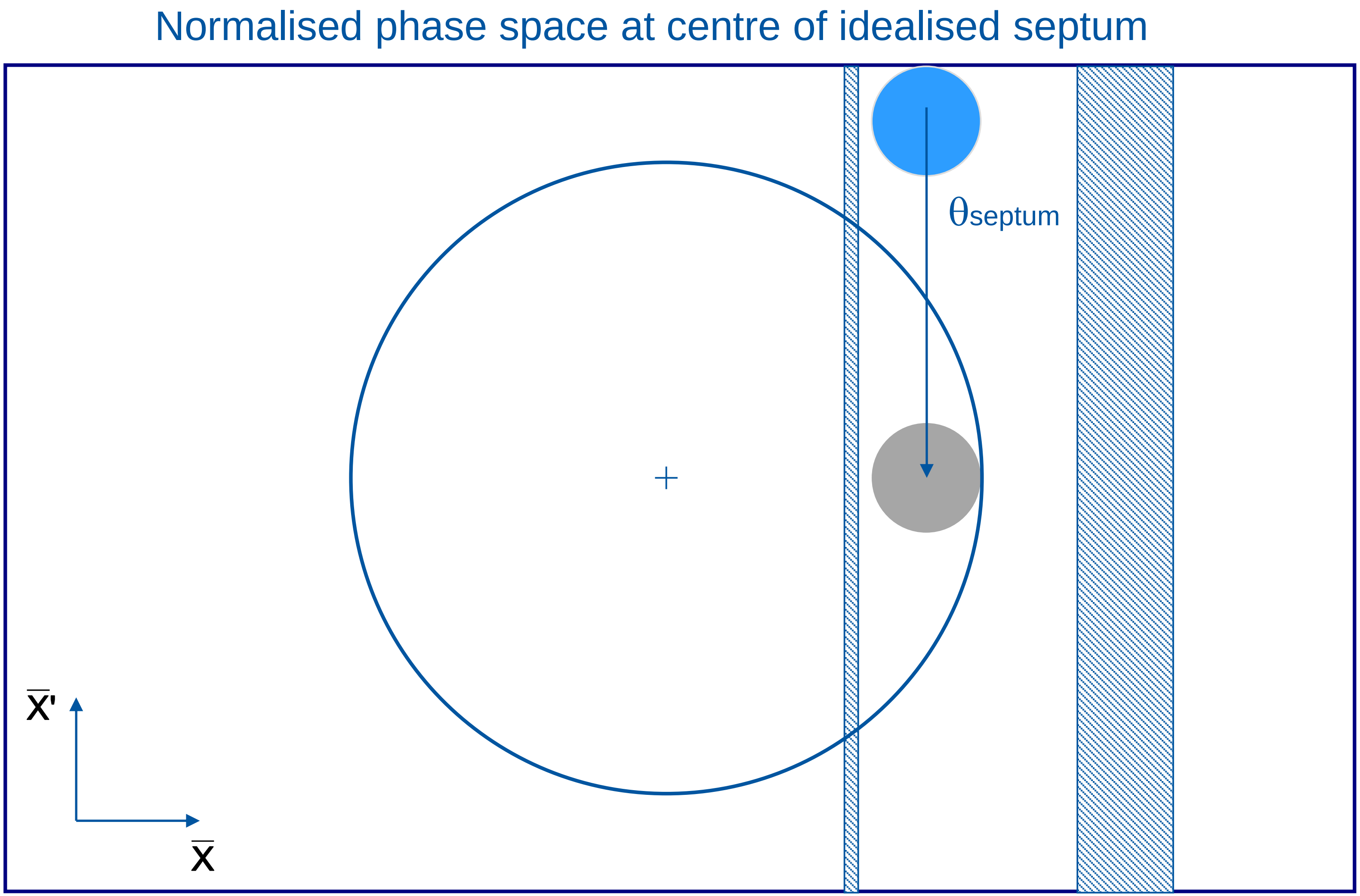
$$\varepsilon = \overline{X}^2 + \overline{X}'^2$$

Single-turn injection

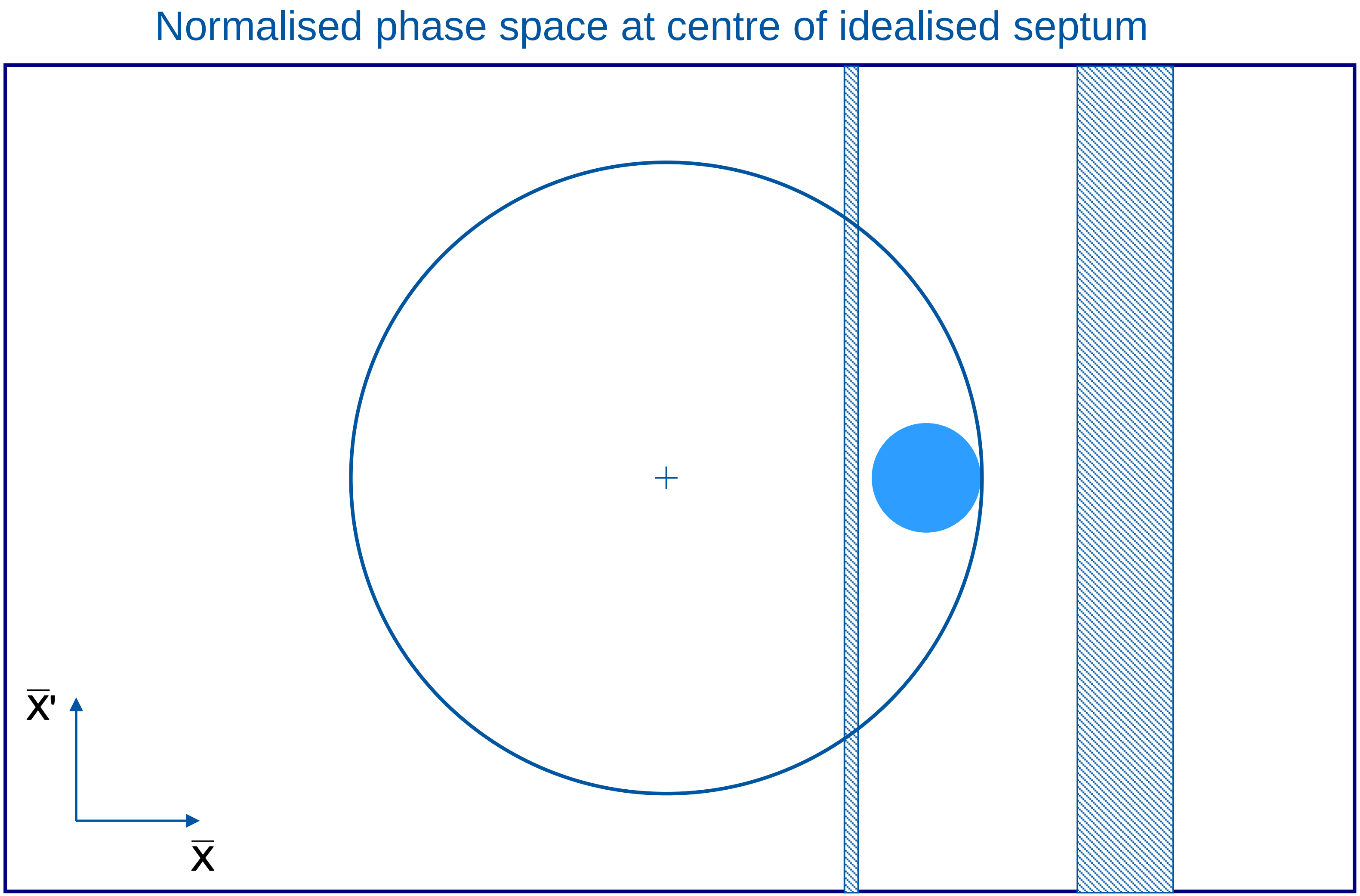
Normalised phase space at centre of idealised septum



Single-turn injection

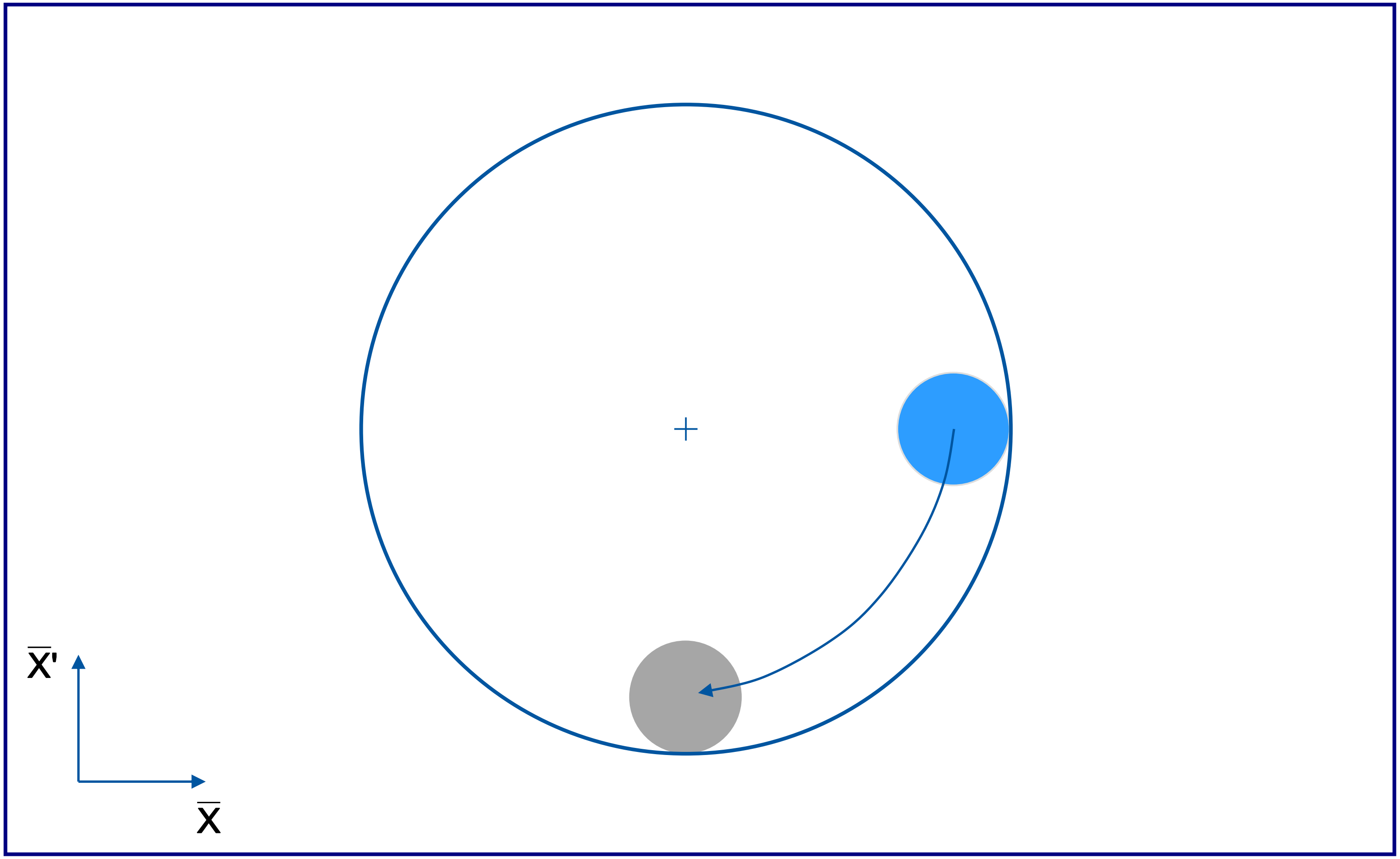


Single-turn injection



Single-turn injection

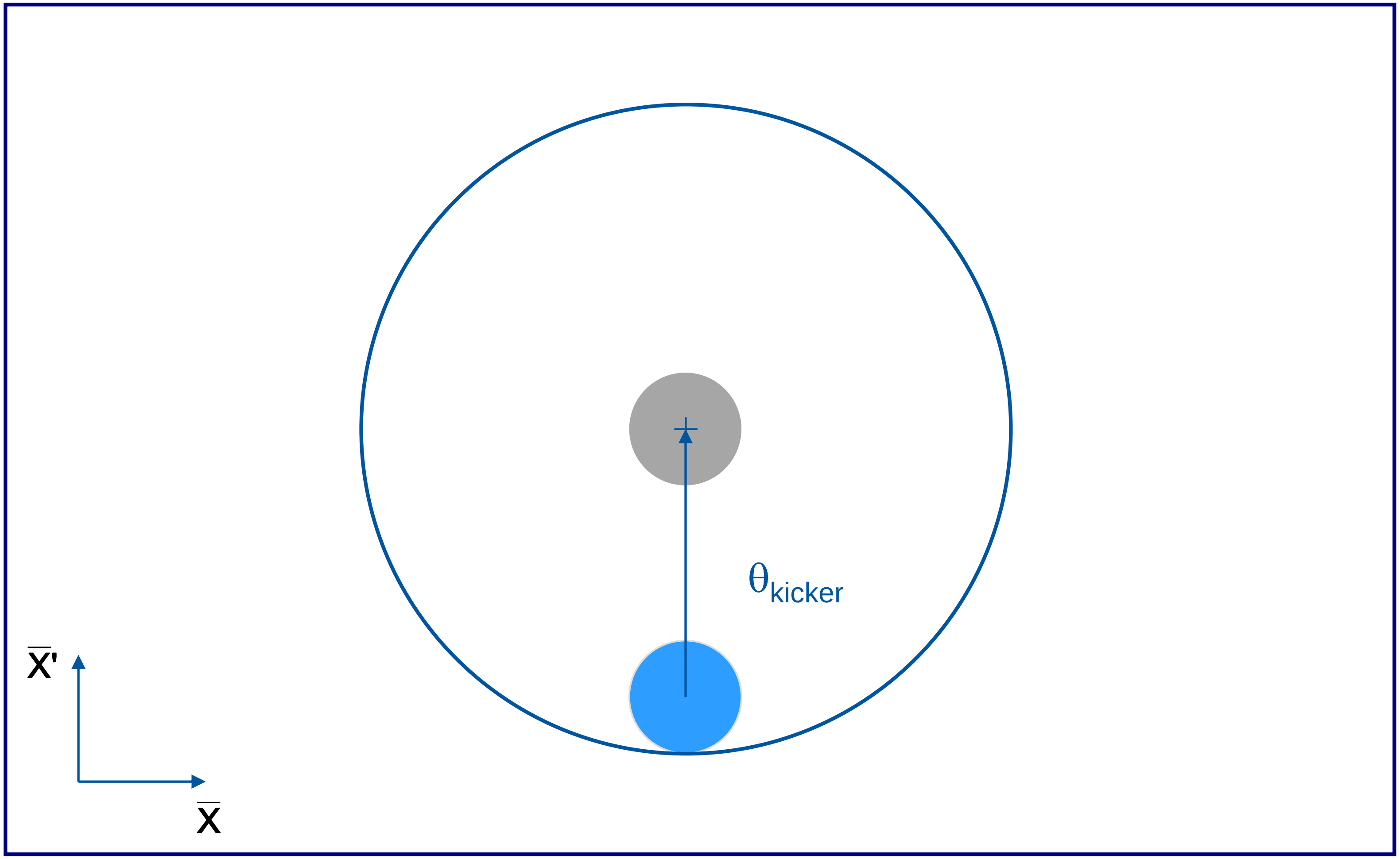
π/2 phase advance to kicker location



Single-turn injection

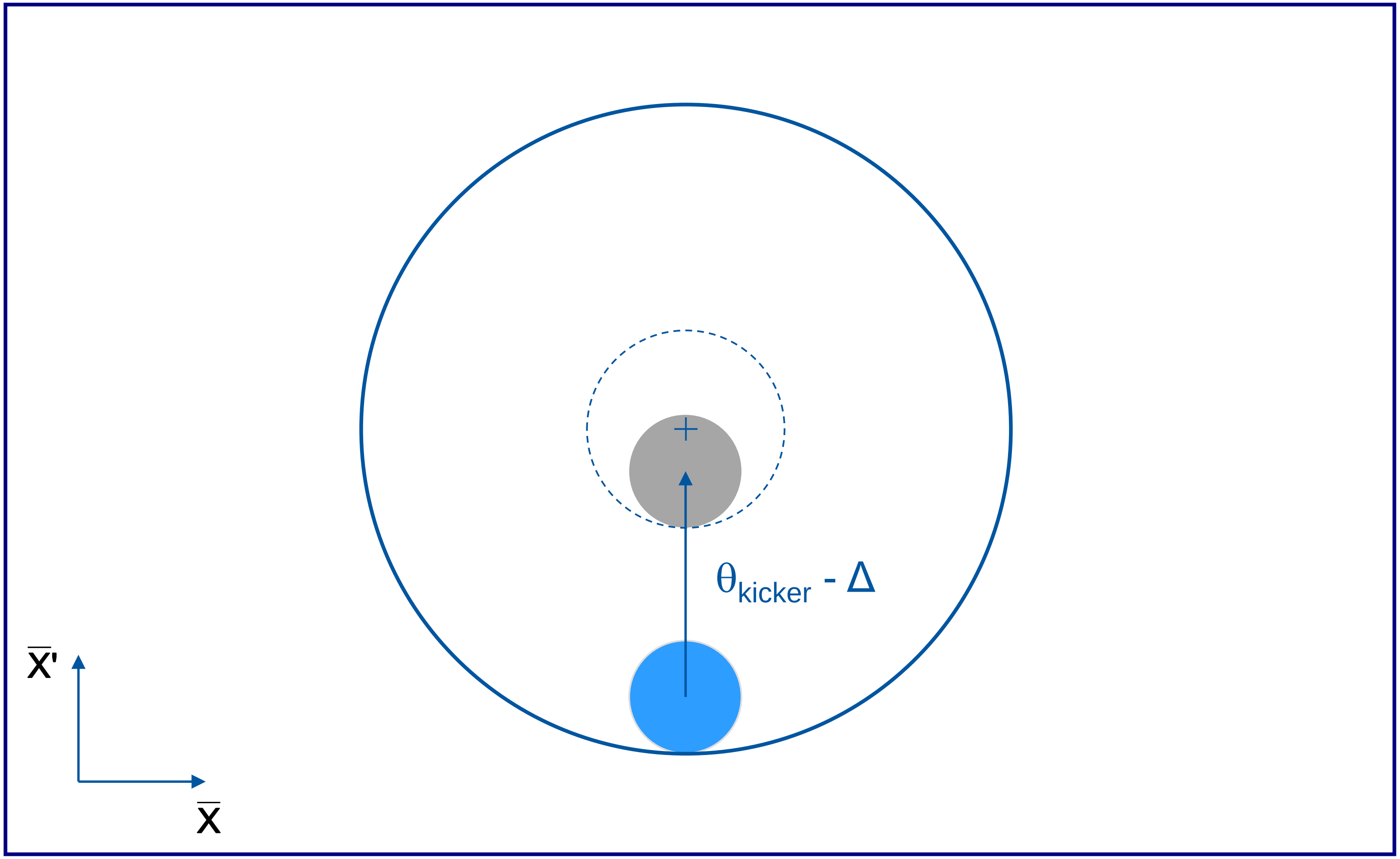
Normalised phase space at centre of idealised septum

Kicker deflection places beam on central orbit:



Injection oscillations

For imperfect injection the beam oscillates around the central orbit, e.g. kick error, Δ:



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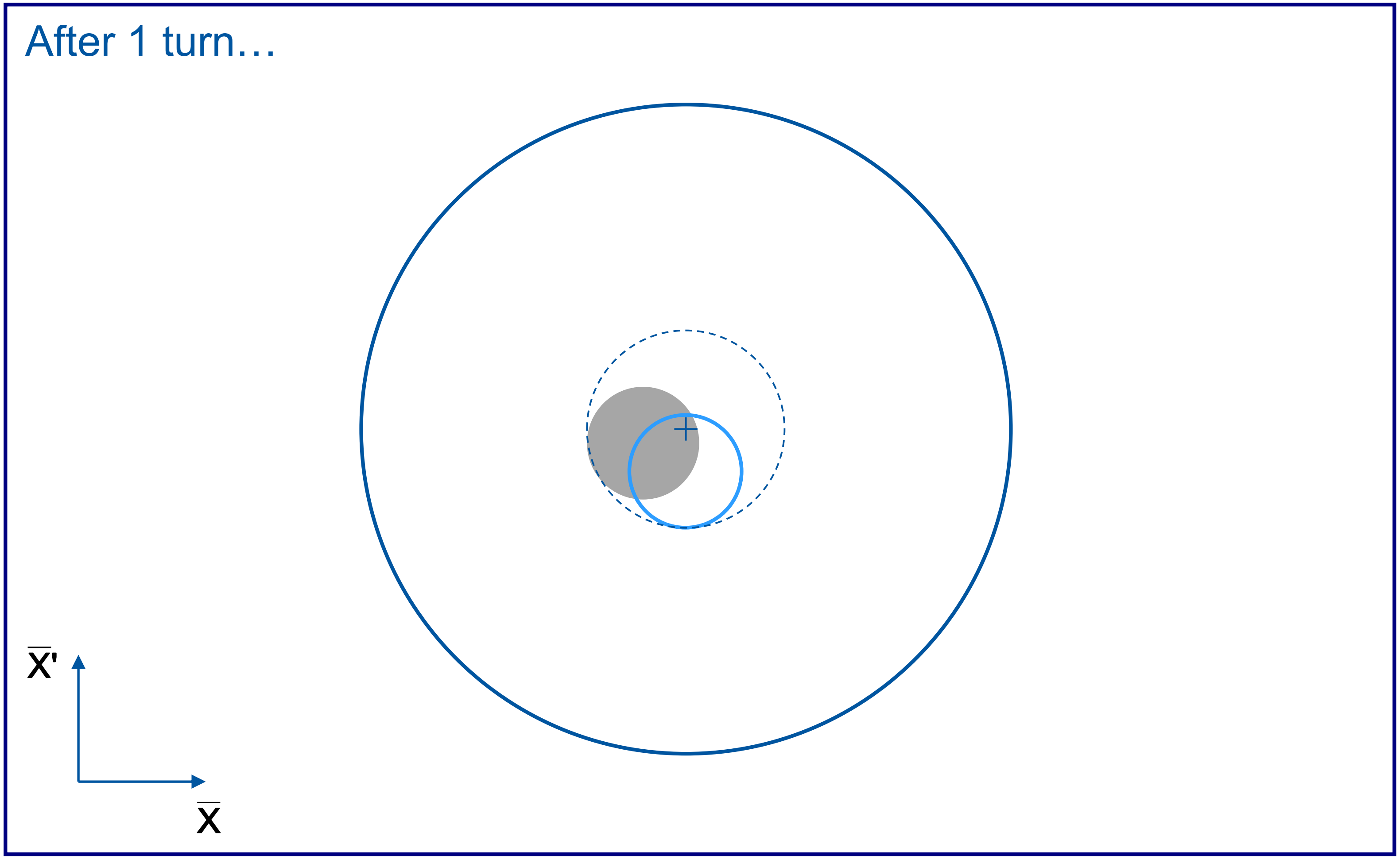

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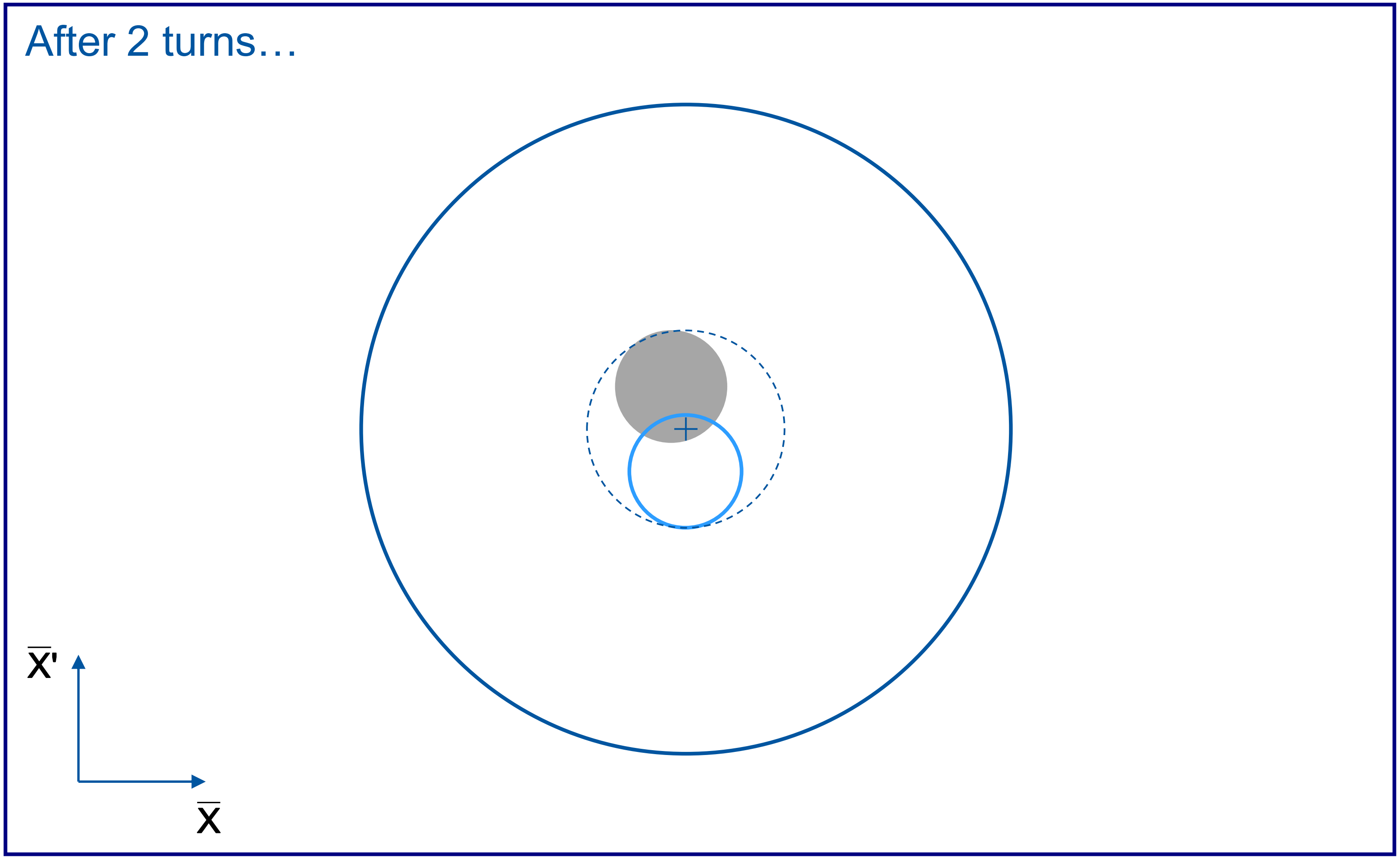
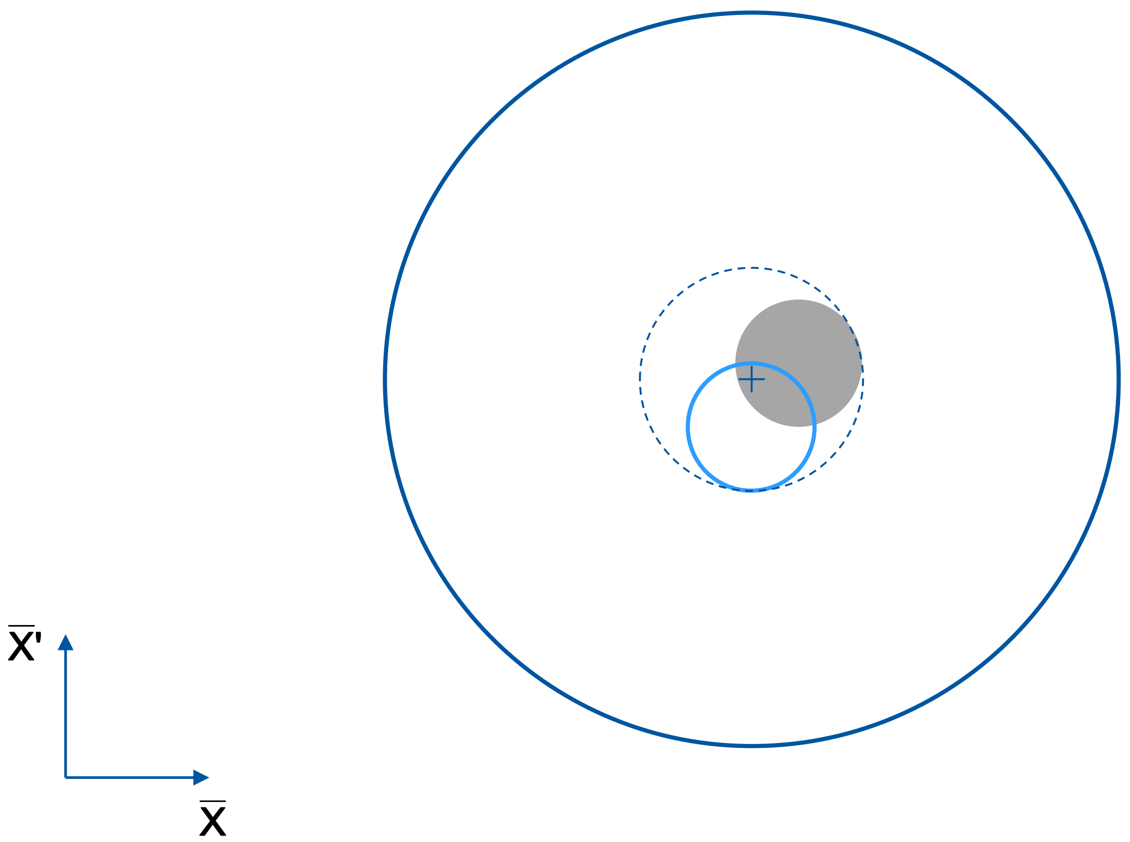


Injection oscillations

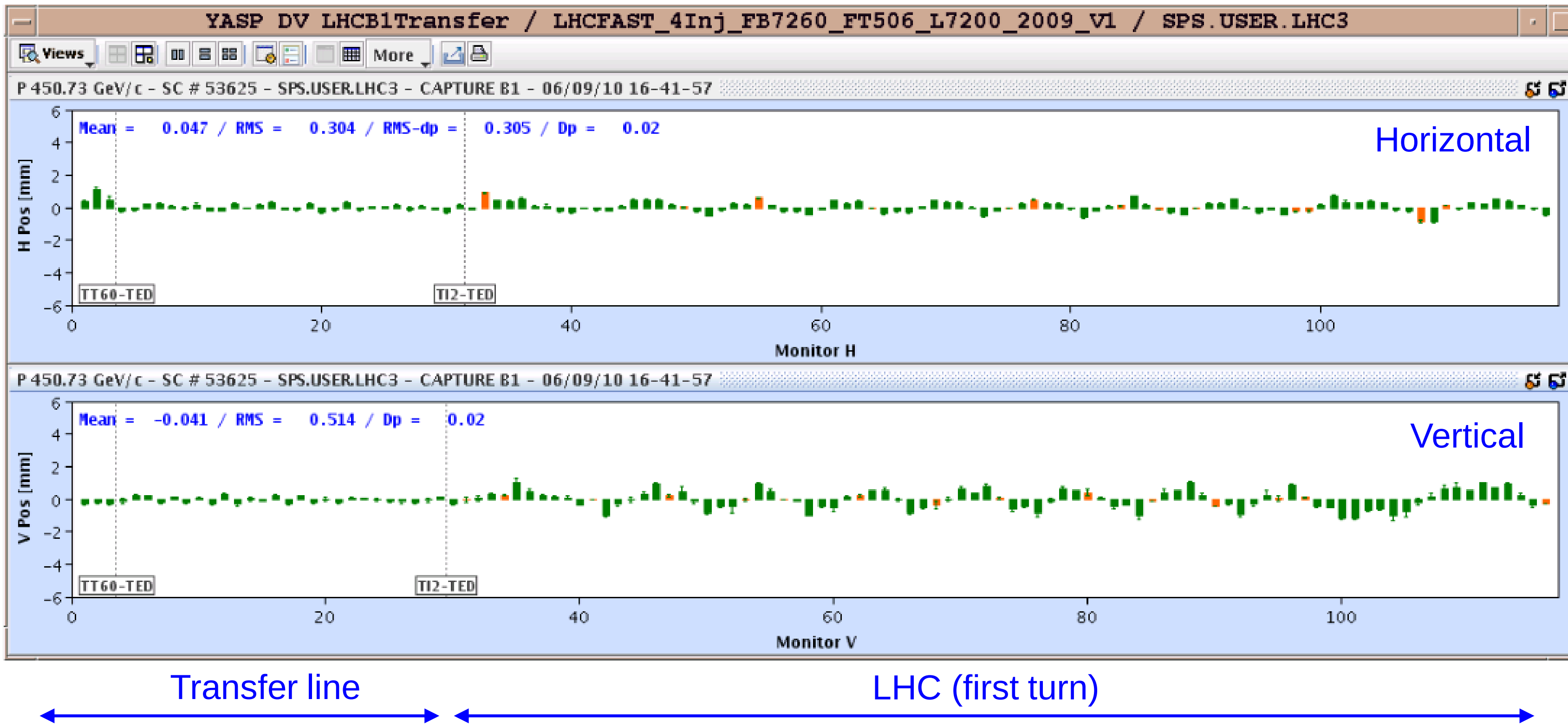
For imperfect injection the beam oscillates around the central orbit, e.g. kick error, Δ:

After 3 turns etc…

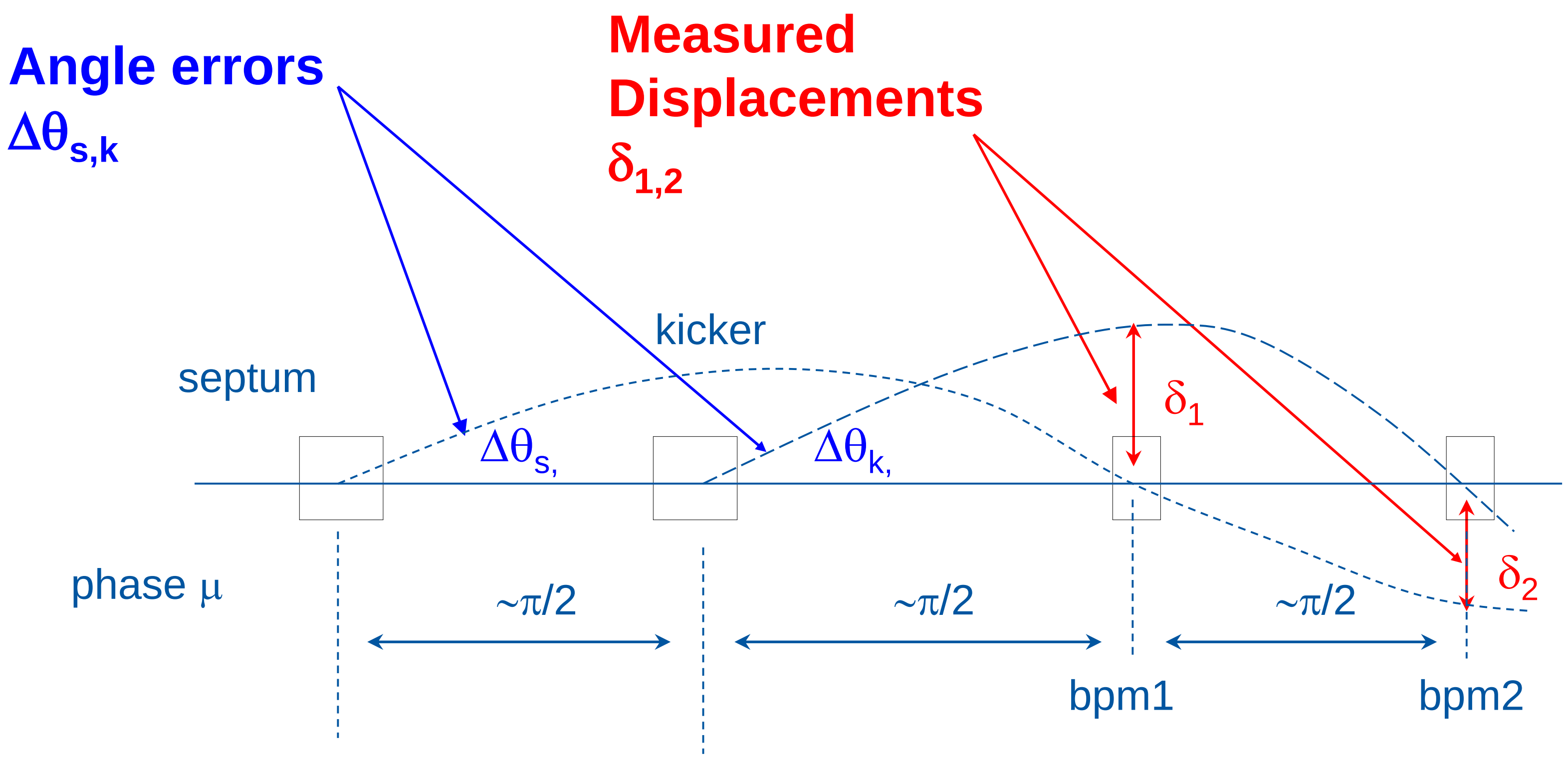


Injection oscillations

Betatron oscillations with respect to the Closed Orbit:



Injection errors



$$\delta_1 = \Delta\theta_s \sqrt{(\beta_s\beta_1)} \sin(\mu_1 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k\beta_1)} \sin(\mu_1 - \mu_k)$$

$$\approx \Delta\theta_k \sqrt{(\beta_k\beta_1)}$$

$$\delta_2 = \Delta\theta_s \sqrt{(\beta_s\beta_2)} \sin(\mu_2 - \mu_s) + \Delta\theta_k \sqrt{(\beta_k\beta_2)} \sin(\mu_2 - \mu_k)$$

$$\approx -\Delta\theta_s \sqrt{(\beta_s\beta_2)}$$

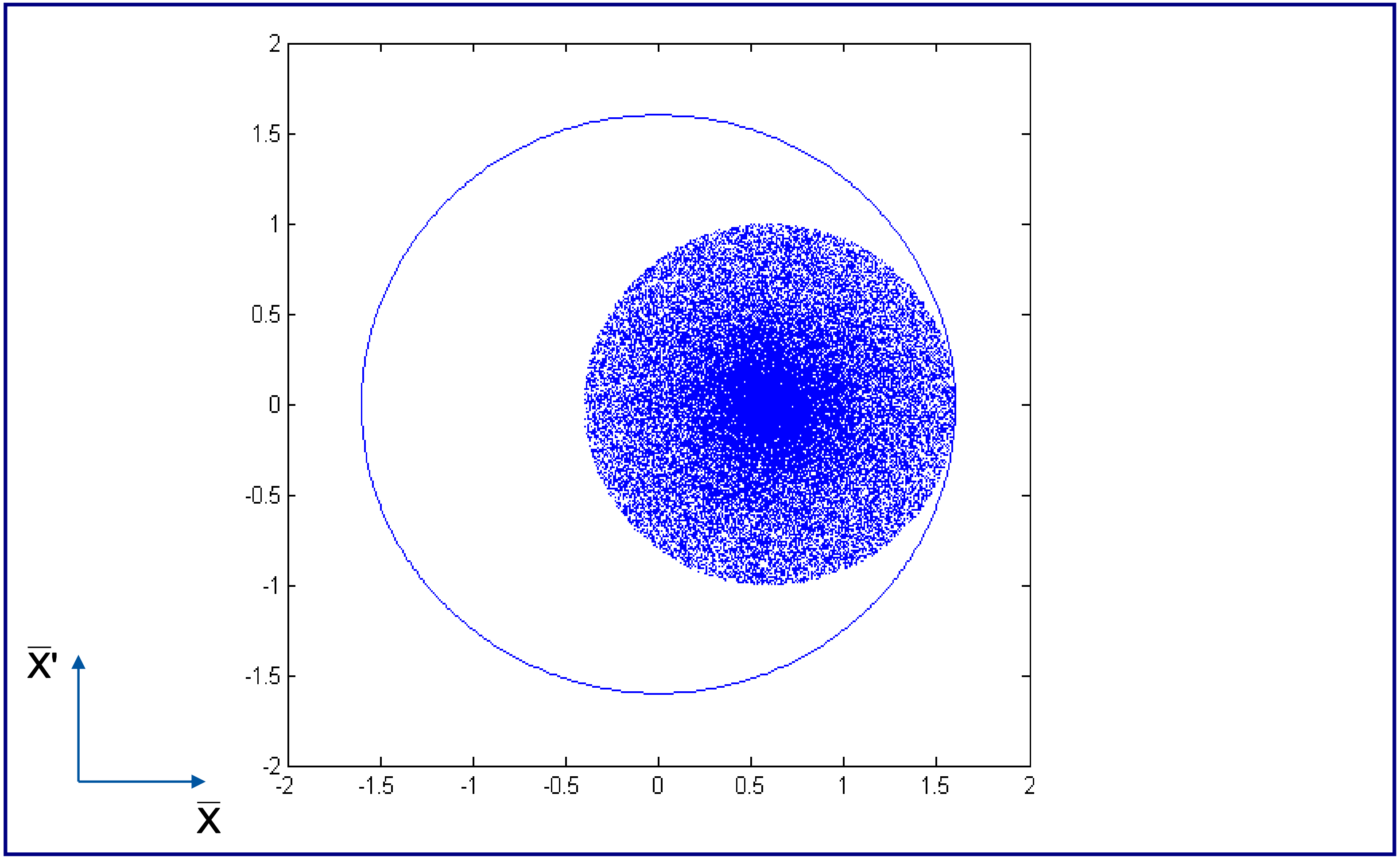
Filamentation

- **Non-linear effects** (e.g. higher-order field components) introduce amplitude-dependent effects into particle motion.

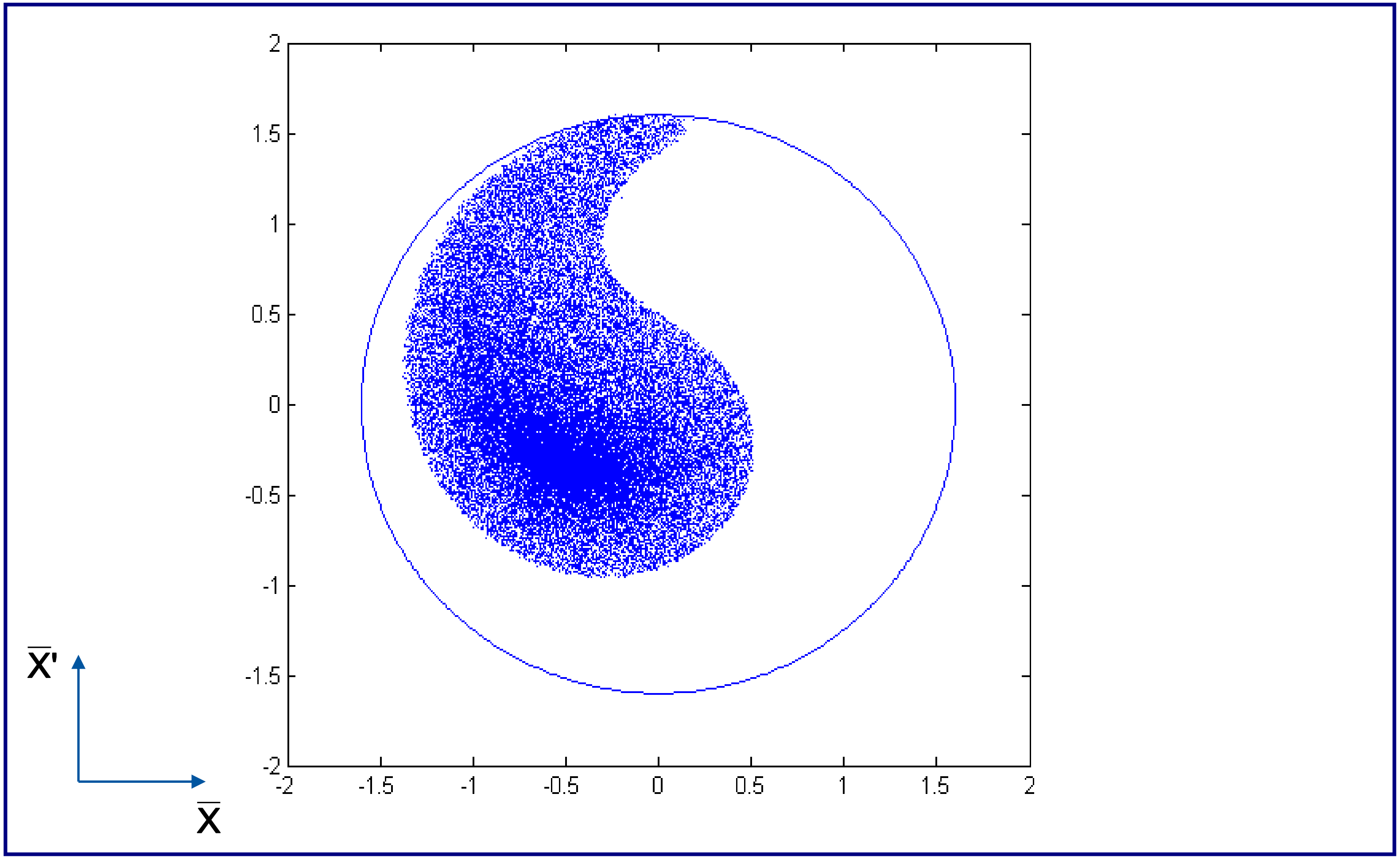
- Over many turns, a phase-space oscillation is transformed into an emittance increase.
- So any residual transverse oscillation will lead to an emittance blow-up through filamentation

- Remark:
  - Chromaticity coupled with a non-zero momentum spread at injection can also cause filmentation, often termed *chromatic decoherence*.
  - “Transverse damper” systems are used to damp injection oscillations - bunch position measured by a pick-up, which is linked to a kicker

Filamentation



Filamentation



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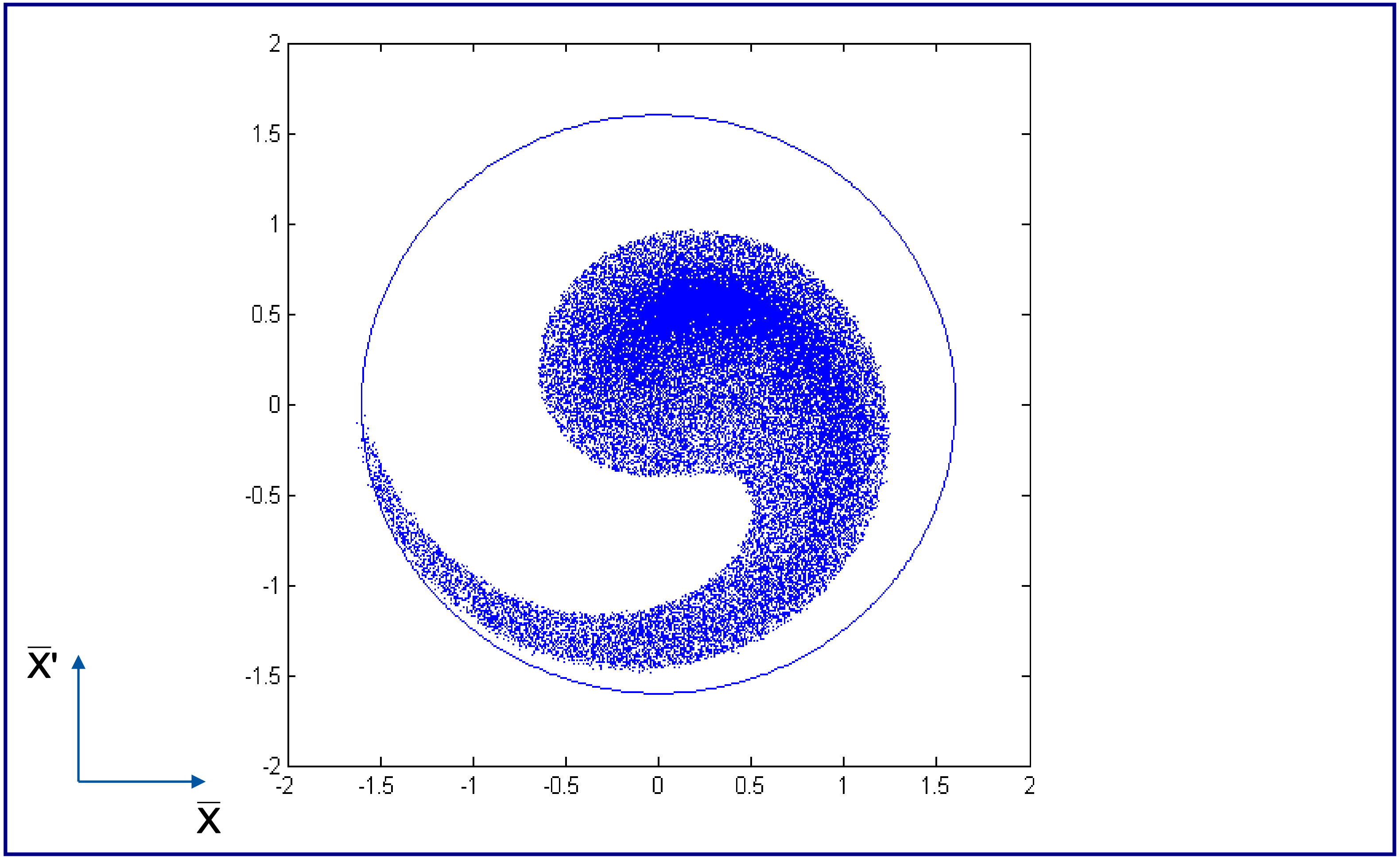

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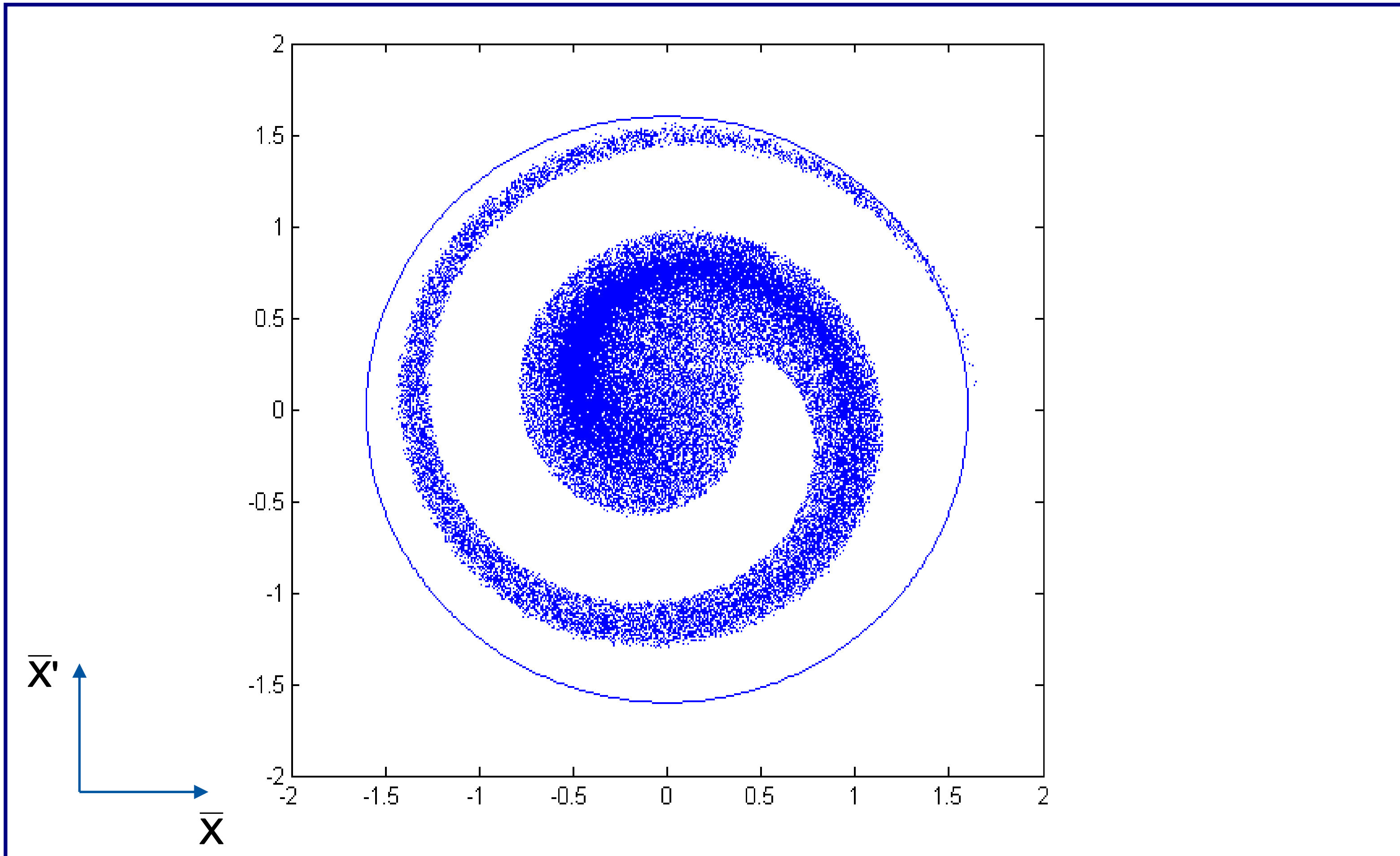

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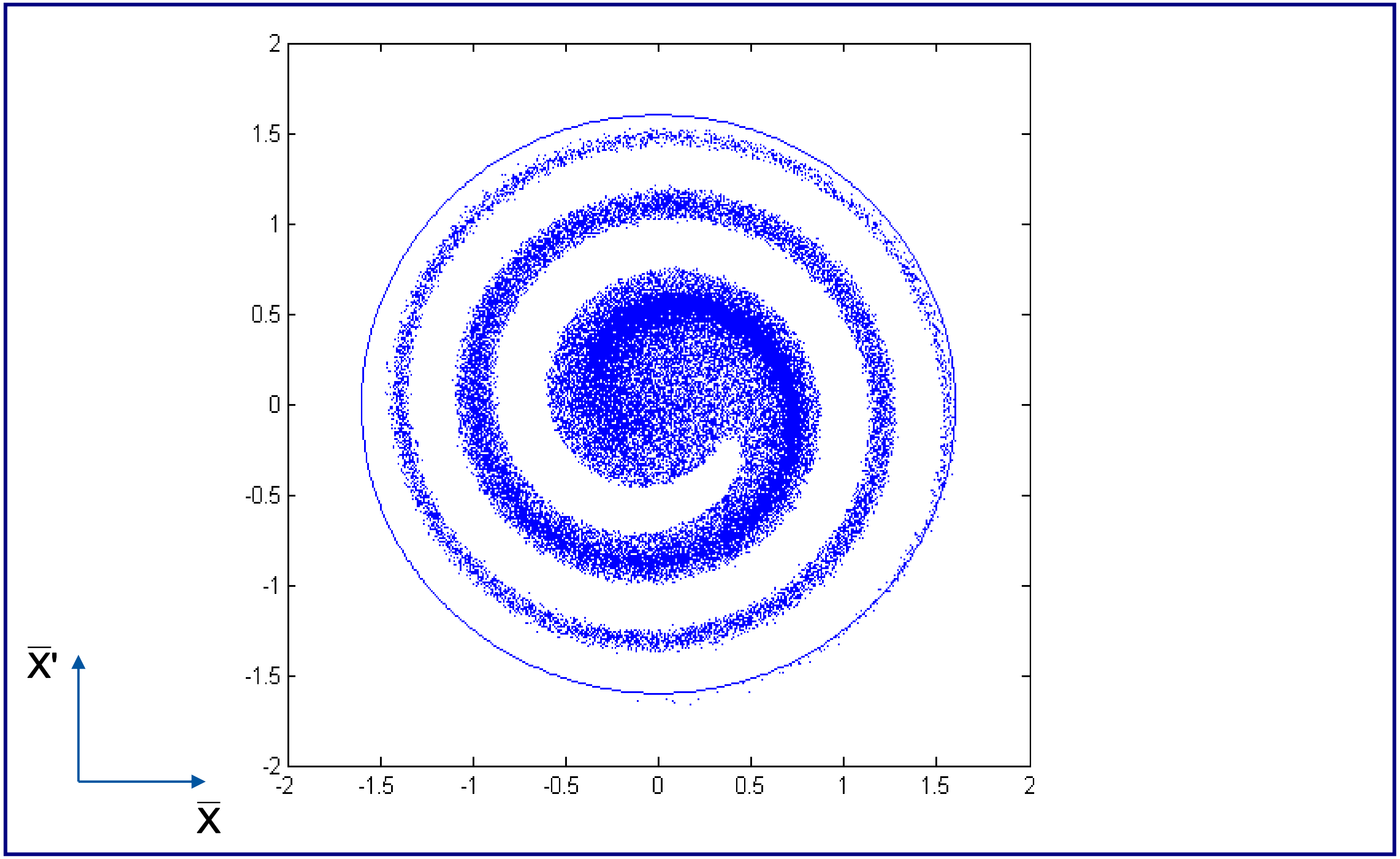

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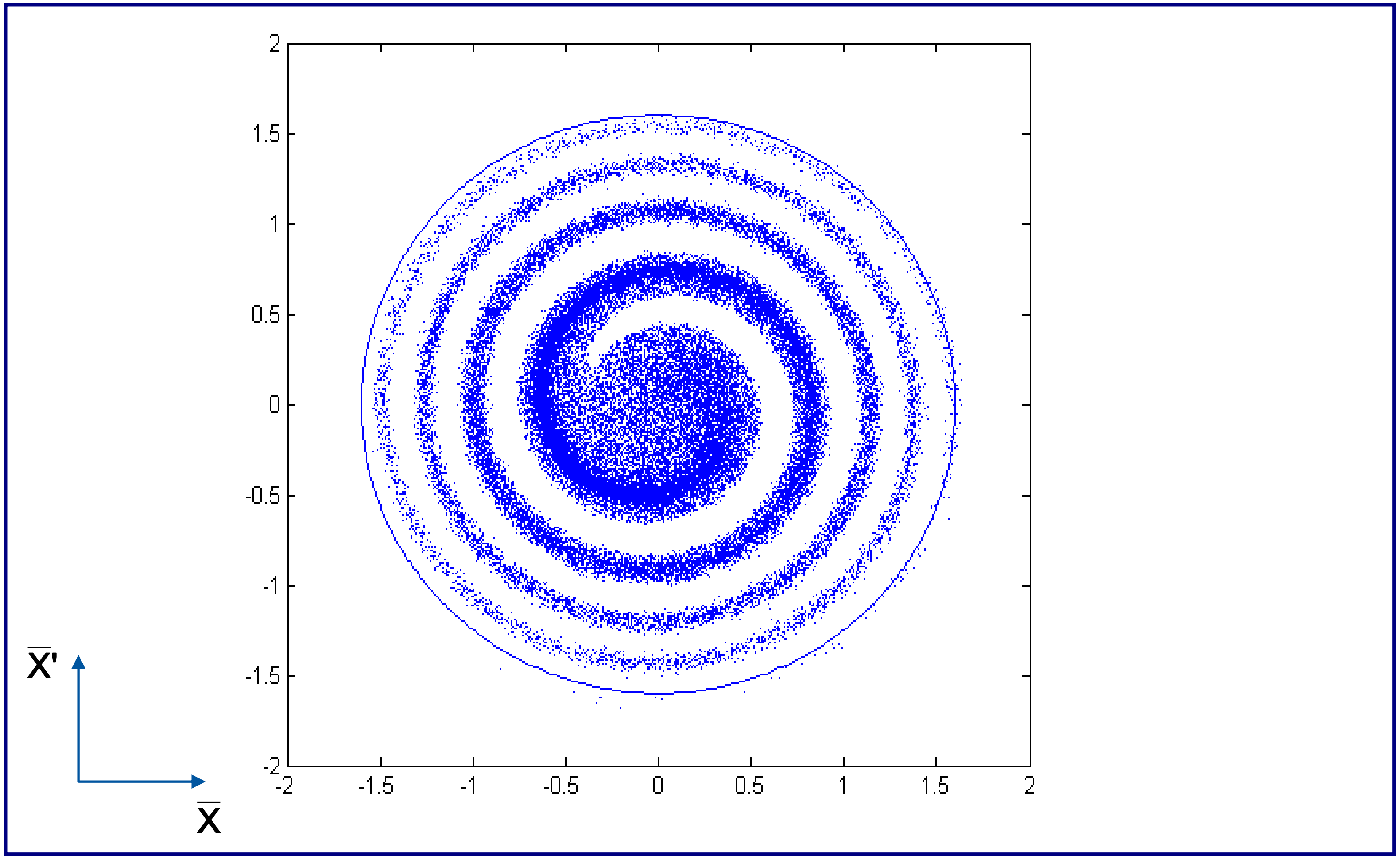

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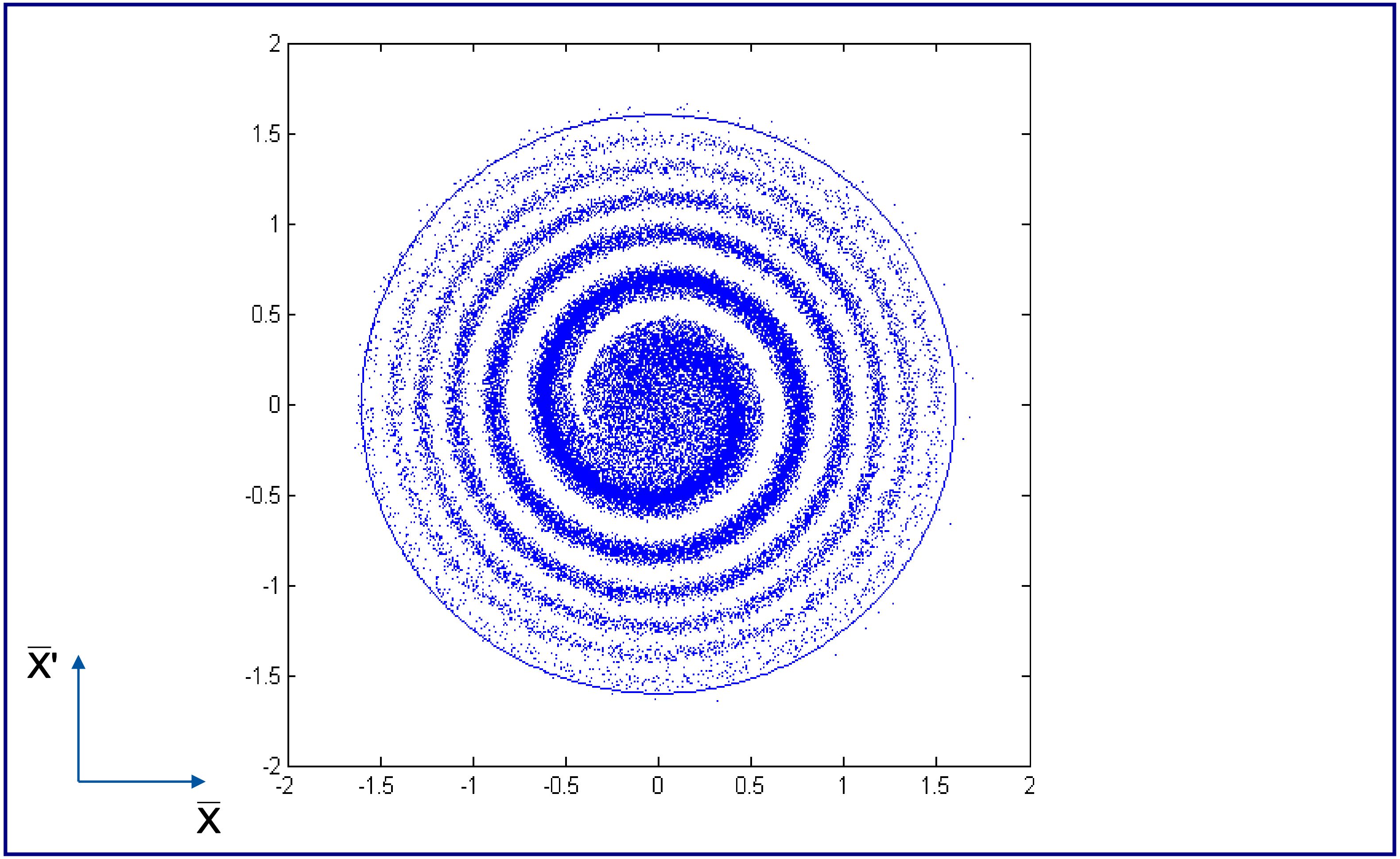

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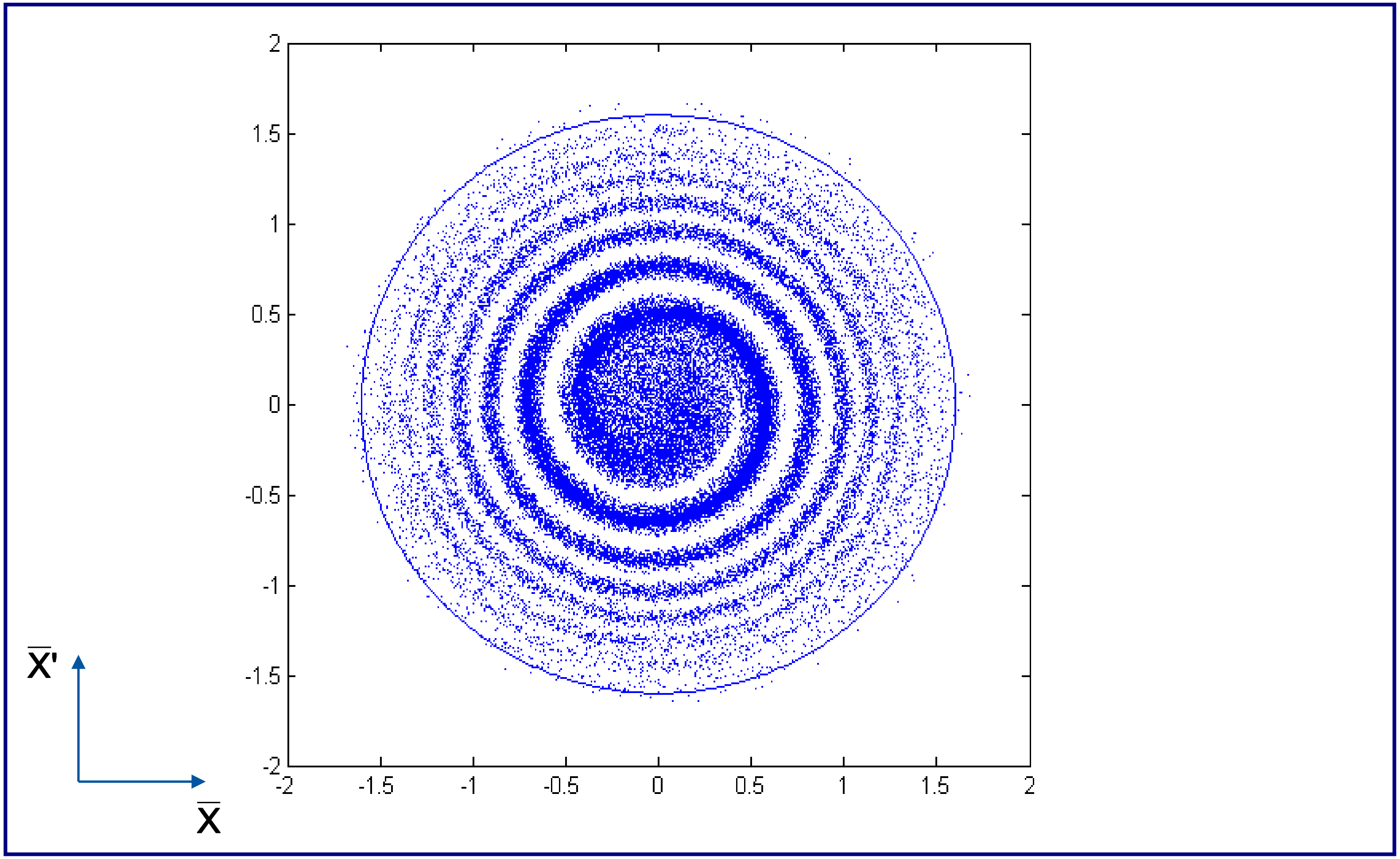

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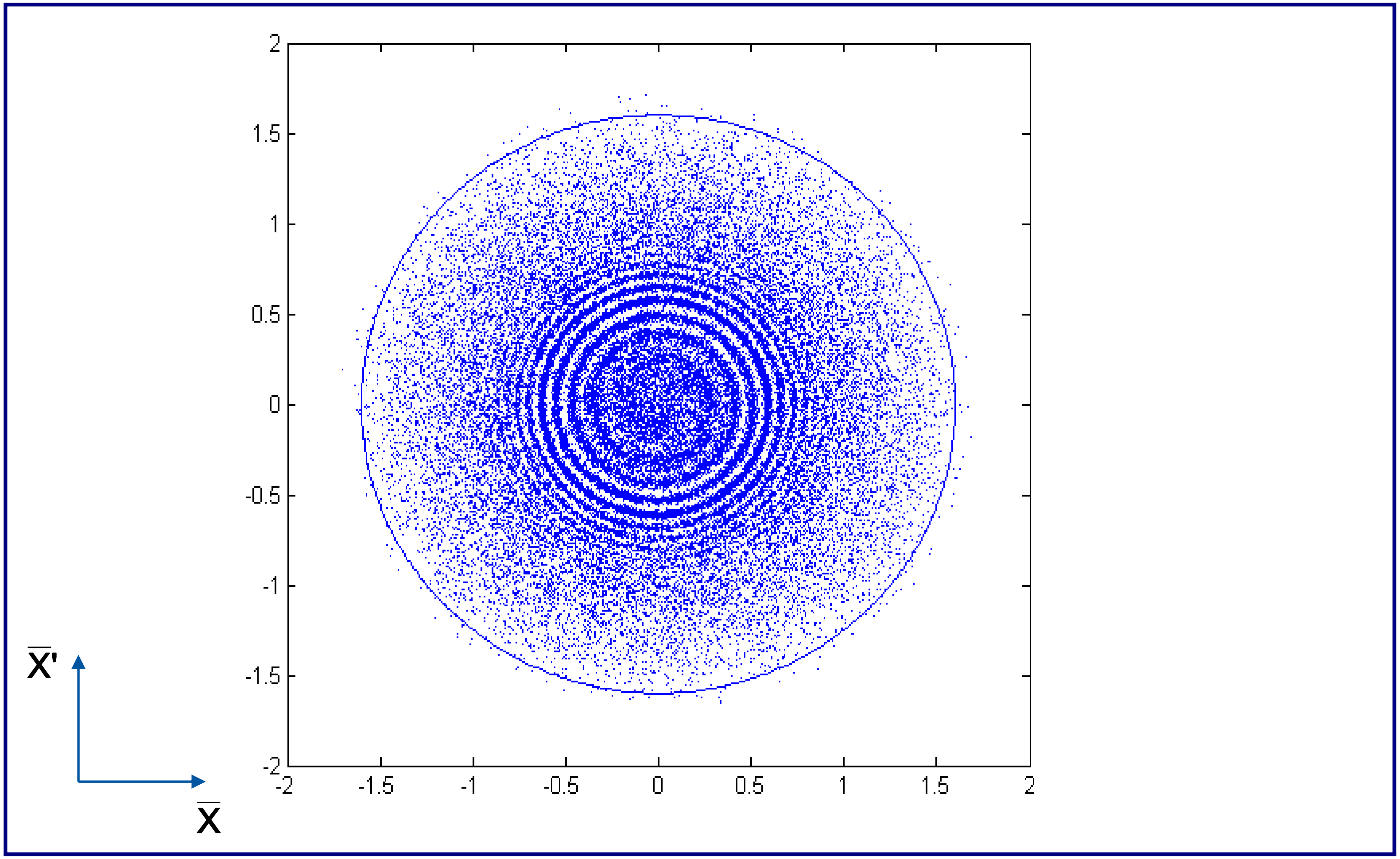

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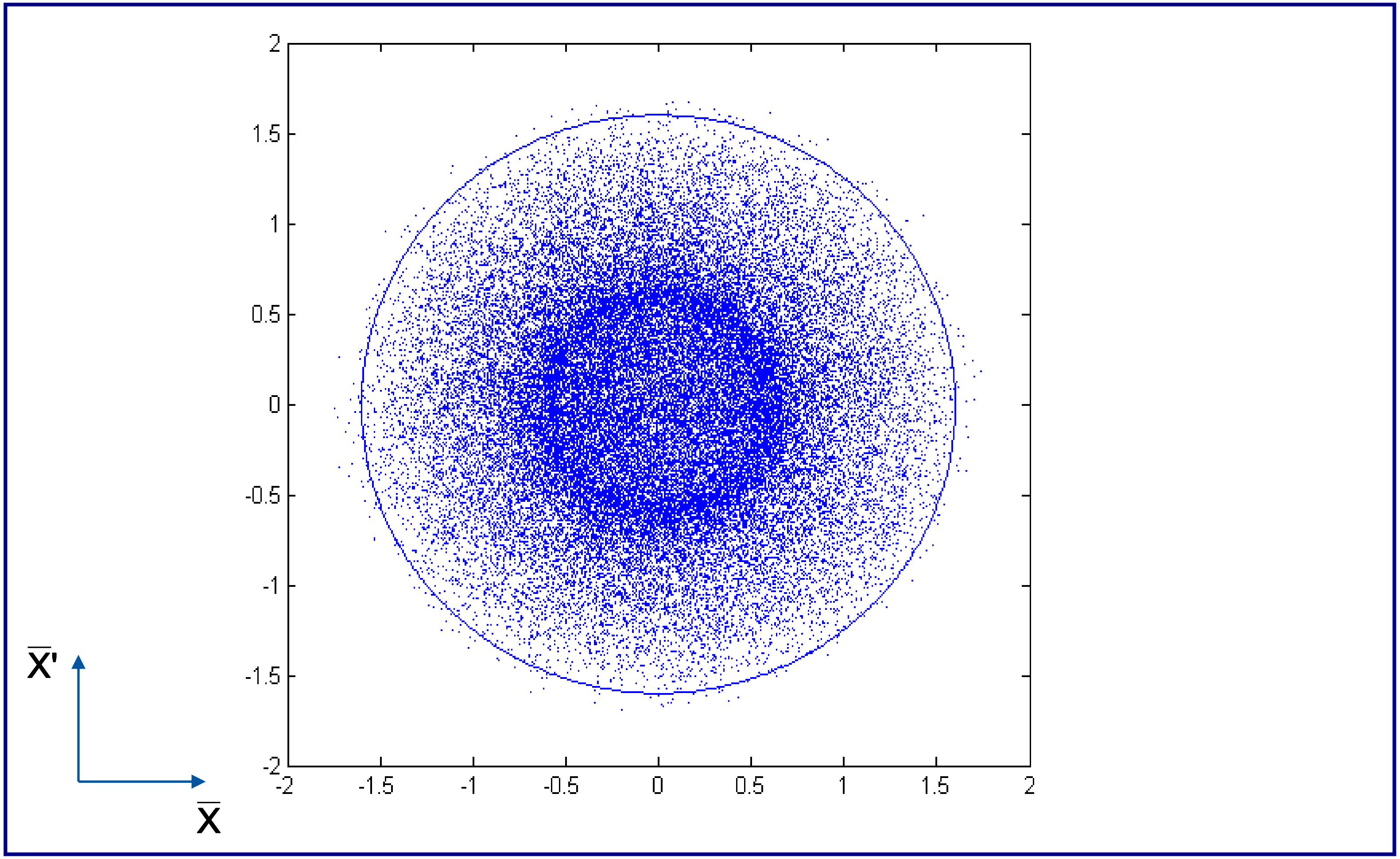


Blow-up from steering error

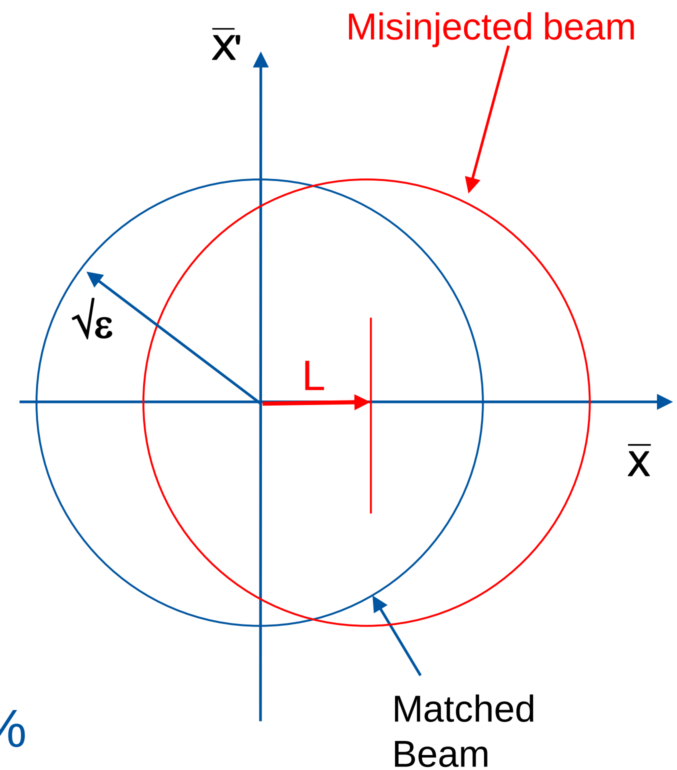
A numerical example….

Consider an offset Δa = 0.5σ for injected beam: $L = \Delta a \sqrt{\epsilon_{matched}}$

$$\epsilon_{diluted} = \epsilon_{matched} + \frac{L^2}{2}$$

$$= \epsilon_{matched}\left[1 + \frac{\Delta a^2}{2}\right]$$

$$= \epsilon_{matched}\left[1.125\right]$$

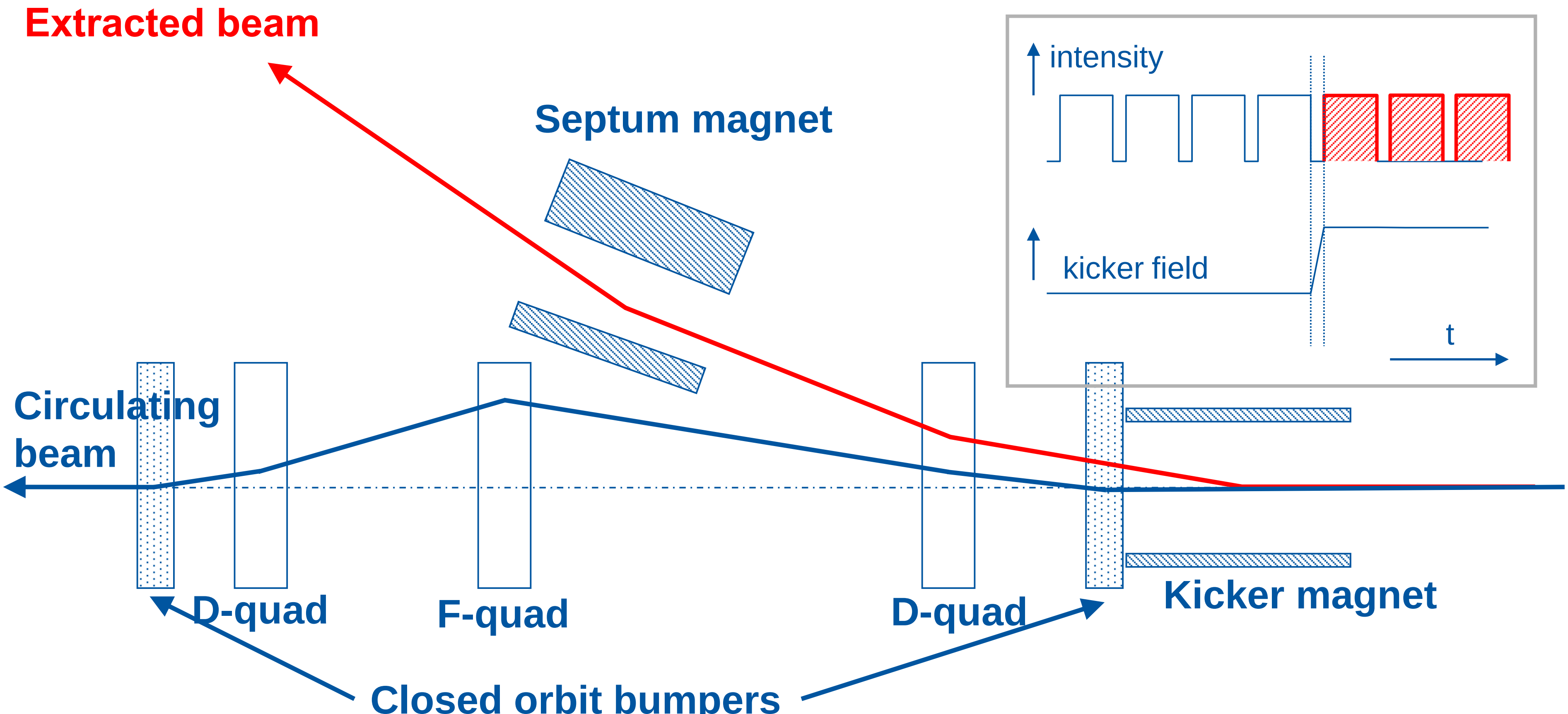


For nominal LHC beam:

…allowed growth through LHC cycle ~10 %

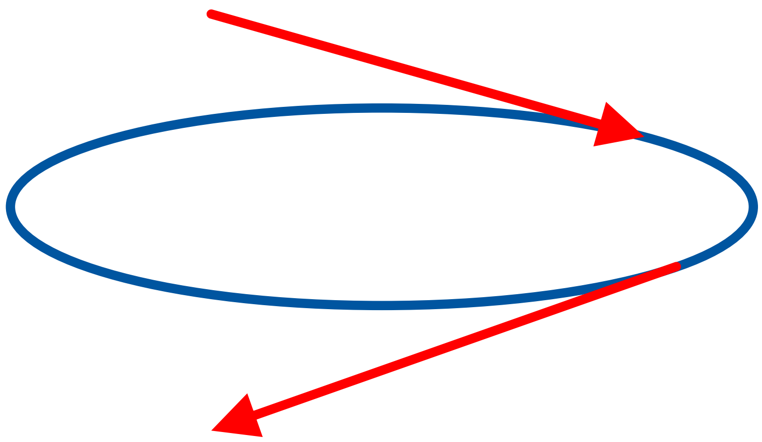
Fast single turn extraction

Entire beam kicked into septum gap and extracted over a single turn



- Bumpers move circulating beam close to septum to reduce kicker strength
- Kicker deflects the entire beam into the septum in a single turn
- Most efficient (lowest deflection angles required) for $\pi/2$ phase advance between kicker and septum

Overview



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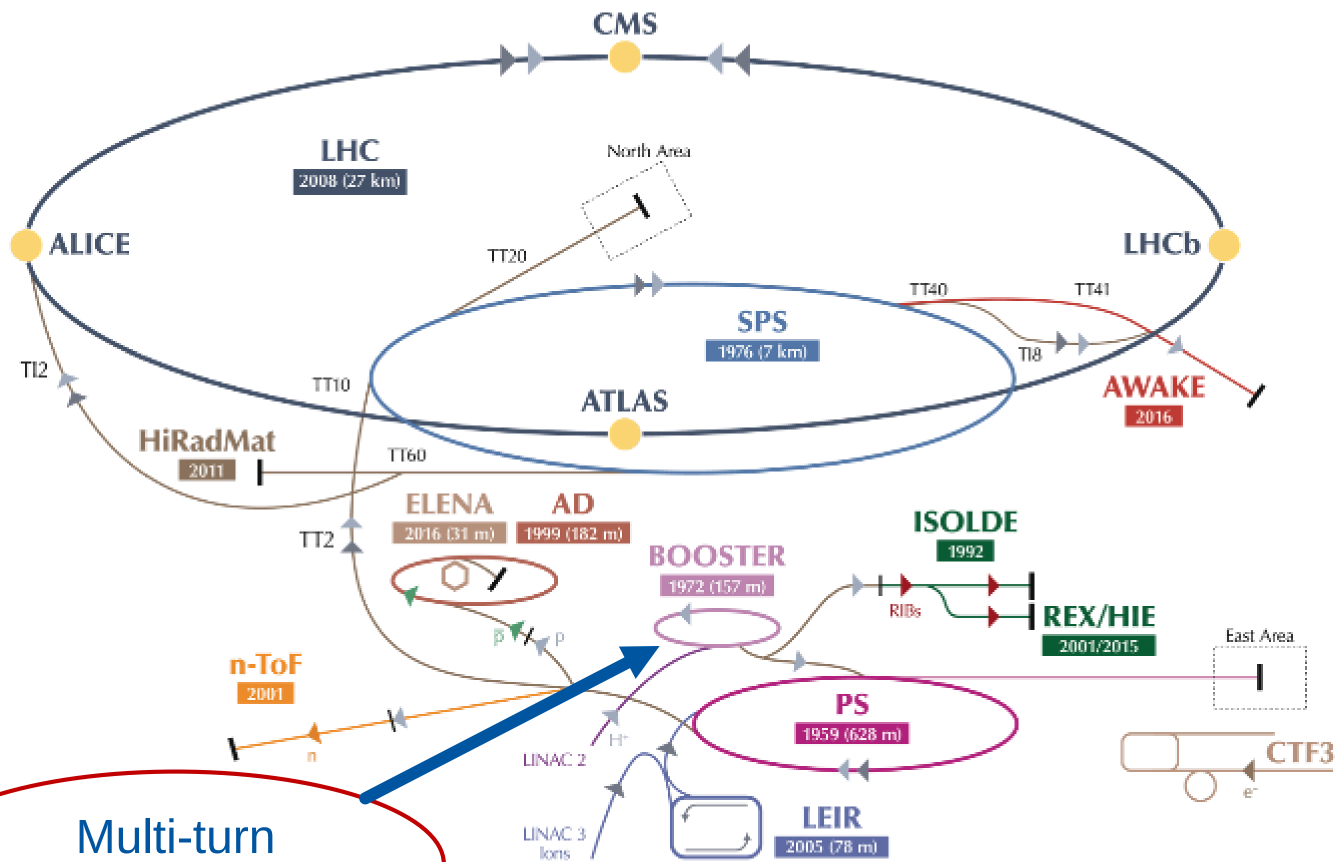
Multi-turn methods

CERN Accelerator Complex



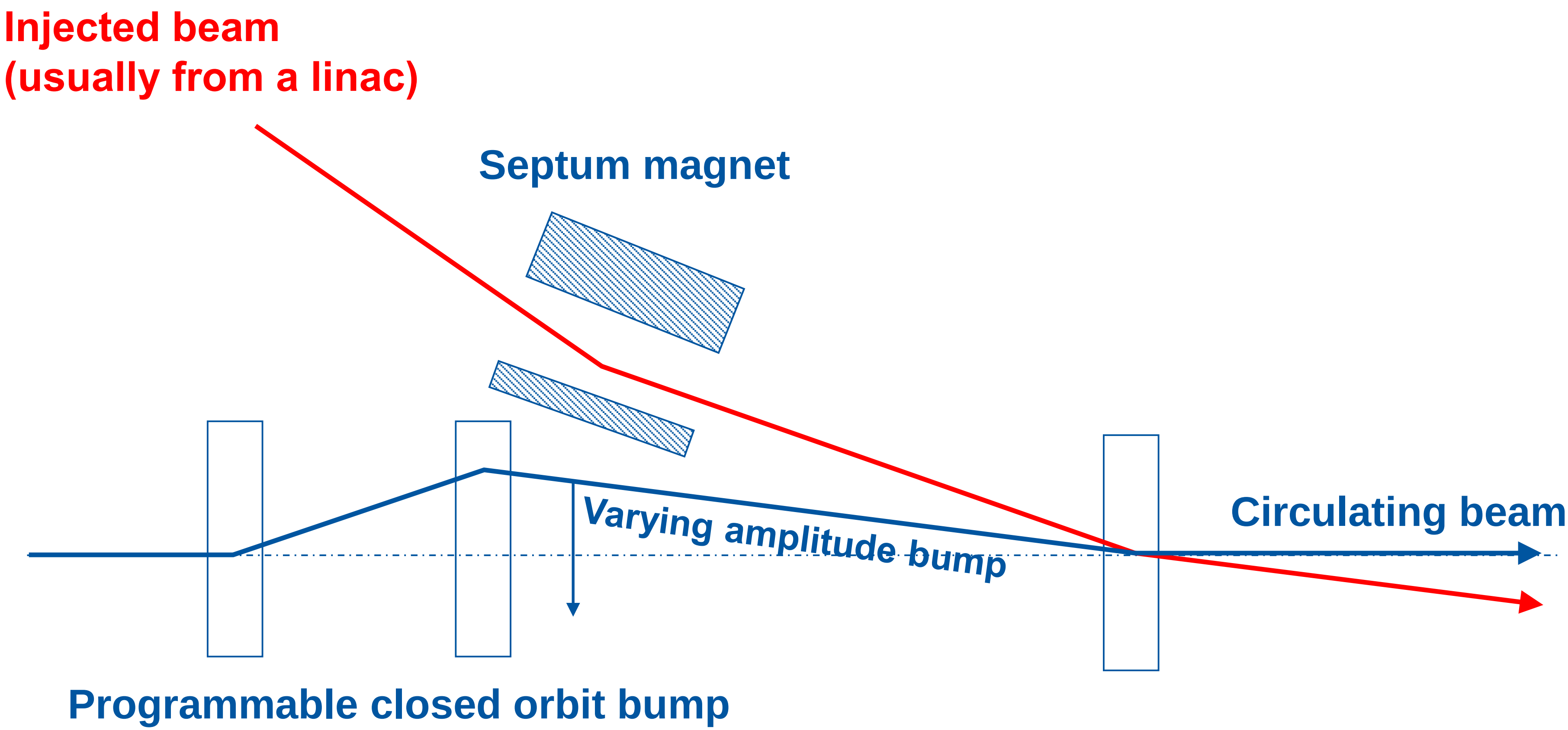
Multi-turn injection

Filling of acceptance of receiving machine in multiple turns

Multi-turn injection

- Limitation of beam density at injection (for hadrons) by:
  - space charge effects
  - the injector capacity
- Increase overall injected intensity : fill the horizontal phase space
  - Requires large acceptance of receiving machine compared to beam emittance from injector
  - ➔ no increase of beam density!

Multi-turn injection for hadrons



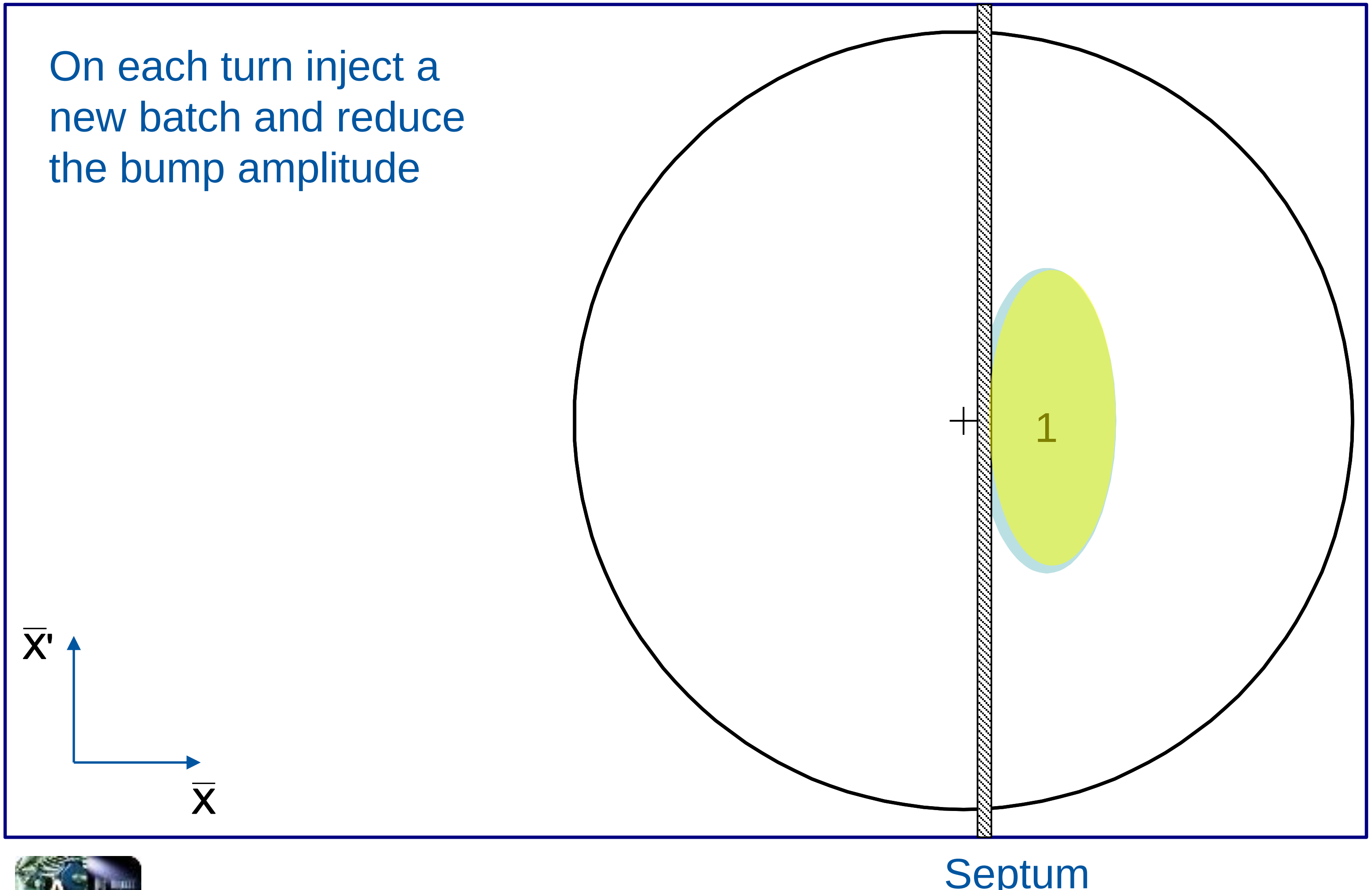
- No kicker but fast programmable bumpers
- Bump amplitude decreases and a new batch injected turn-by-turn
- Phase-space "painting"

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
fractional tune $Q_h \approx 0.25$ ➔ beam rotates π/2 per turn in phase space

Turn 1

On each turn inject a new batch and reduce the bump amplitude



Multi-turn injection for hadrons

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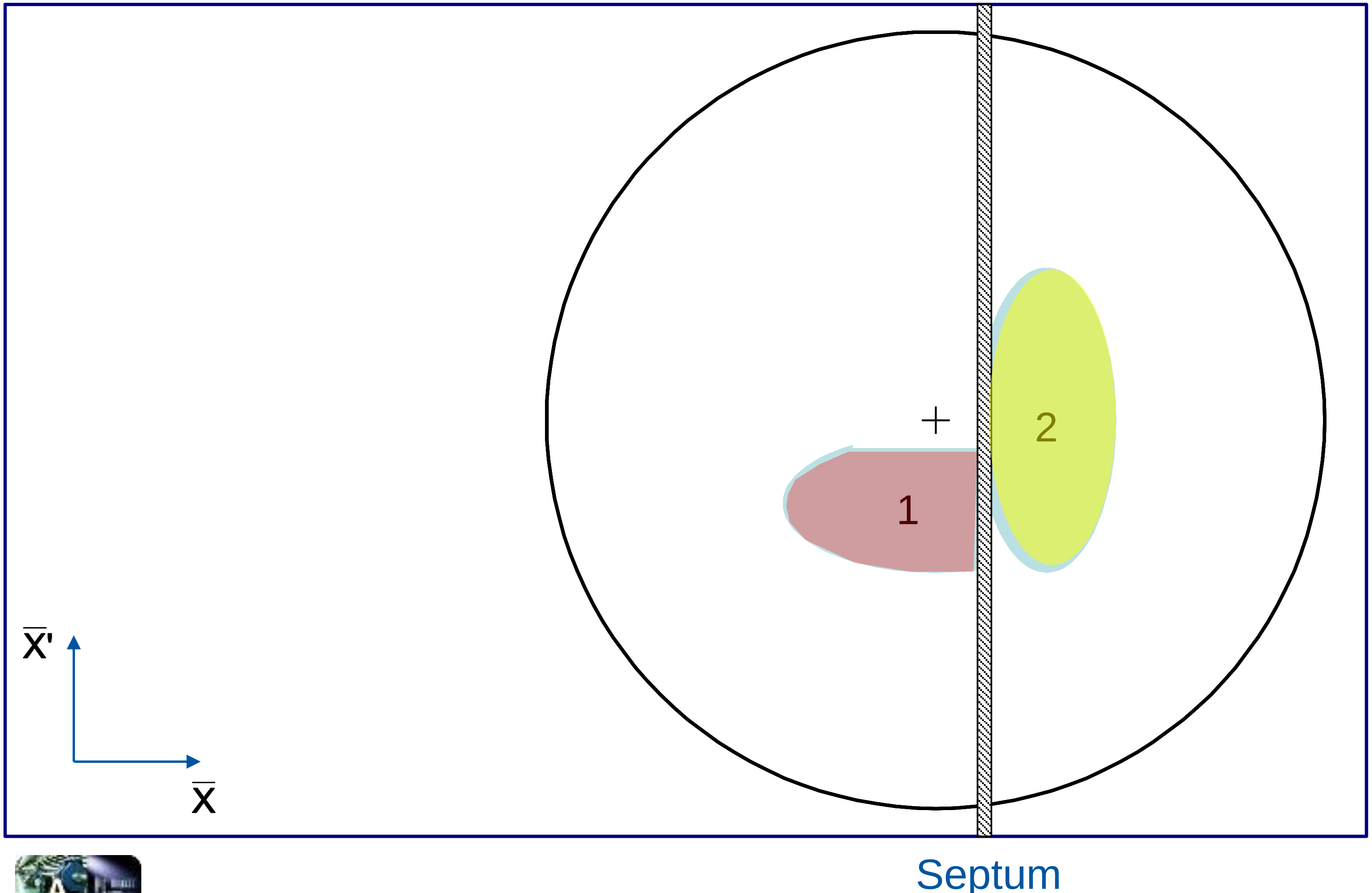
Turn 2



Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
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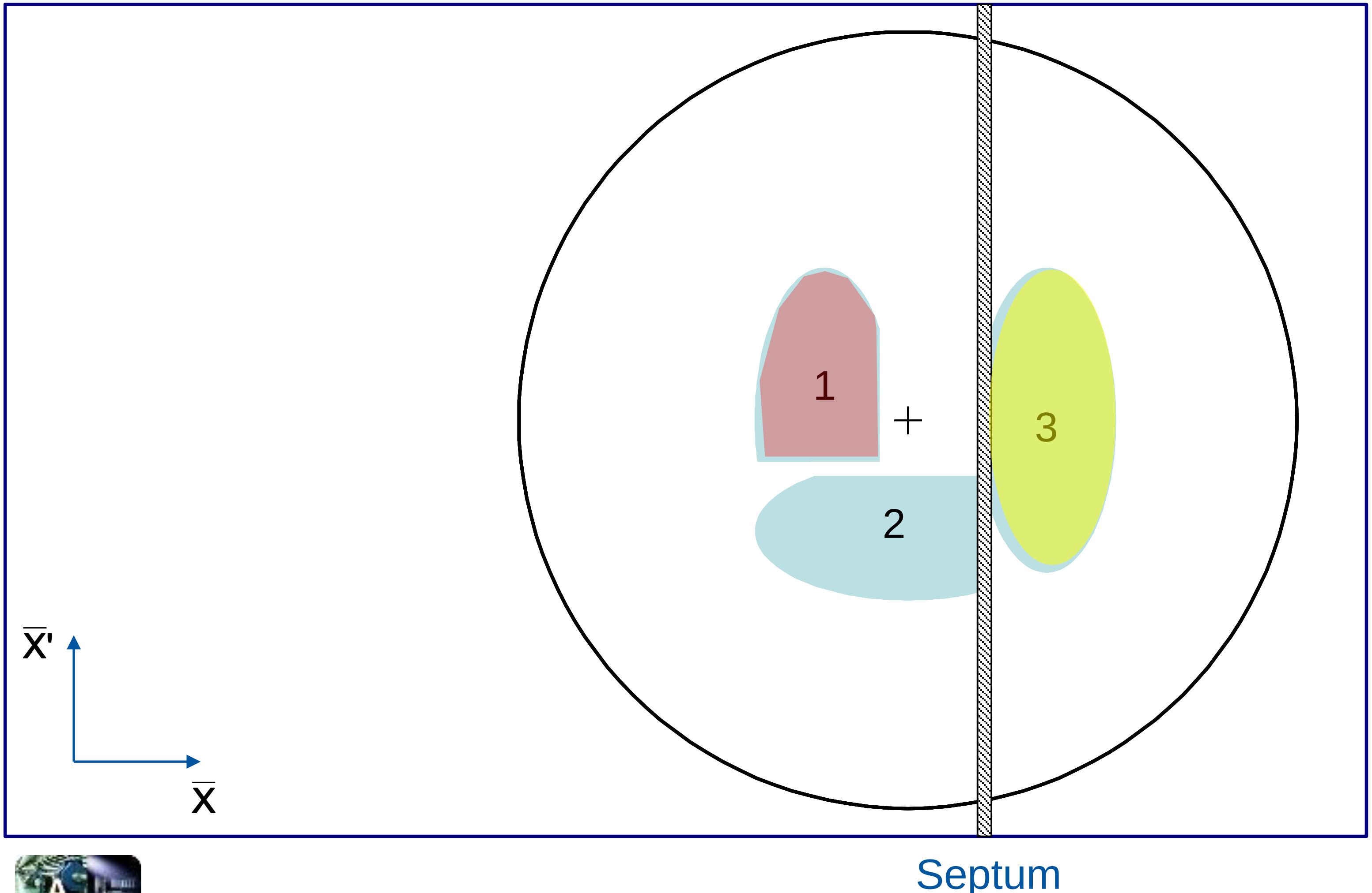
Turn 3



Multi-turn injection for hadrons

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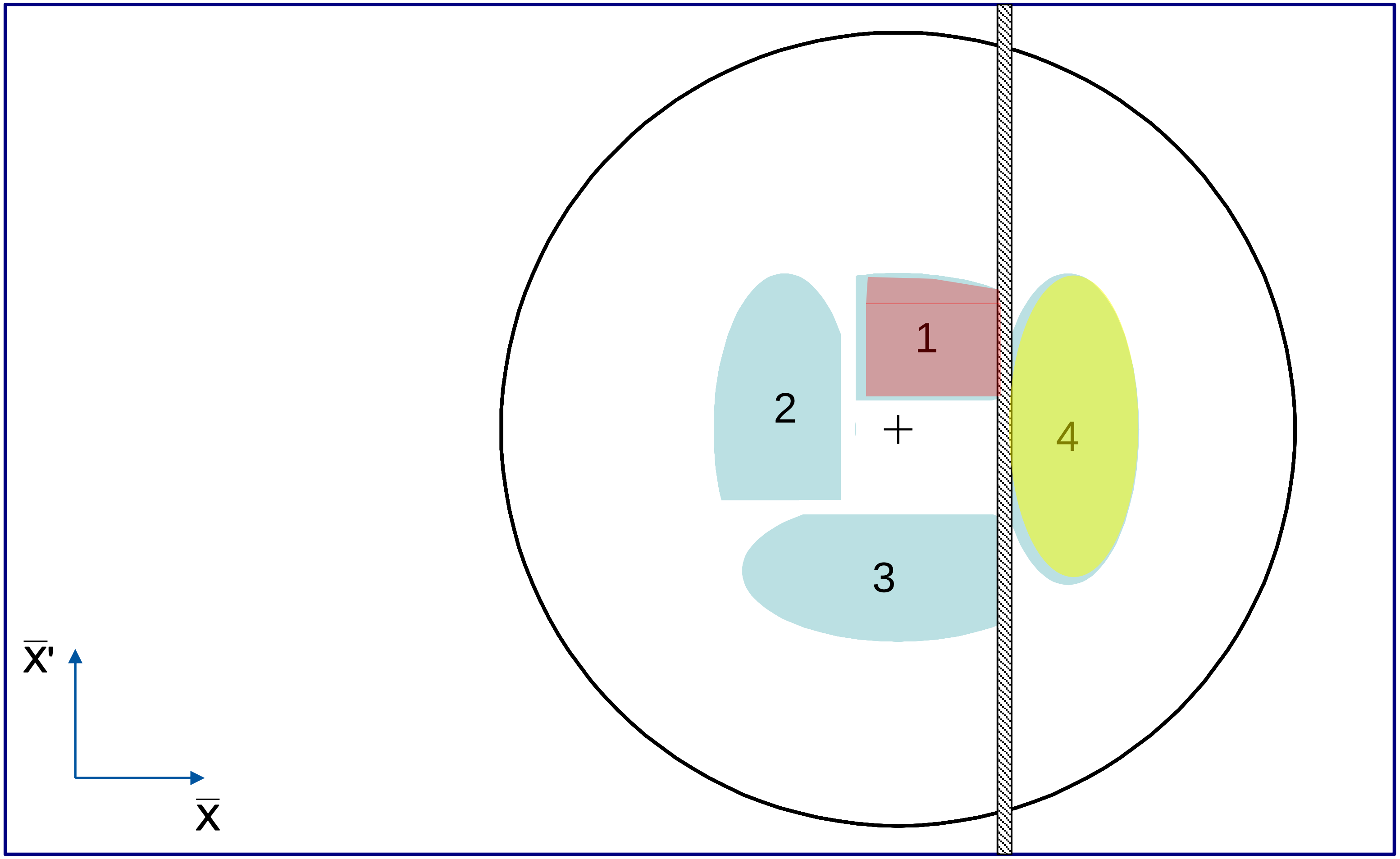
Turn 4



Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
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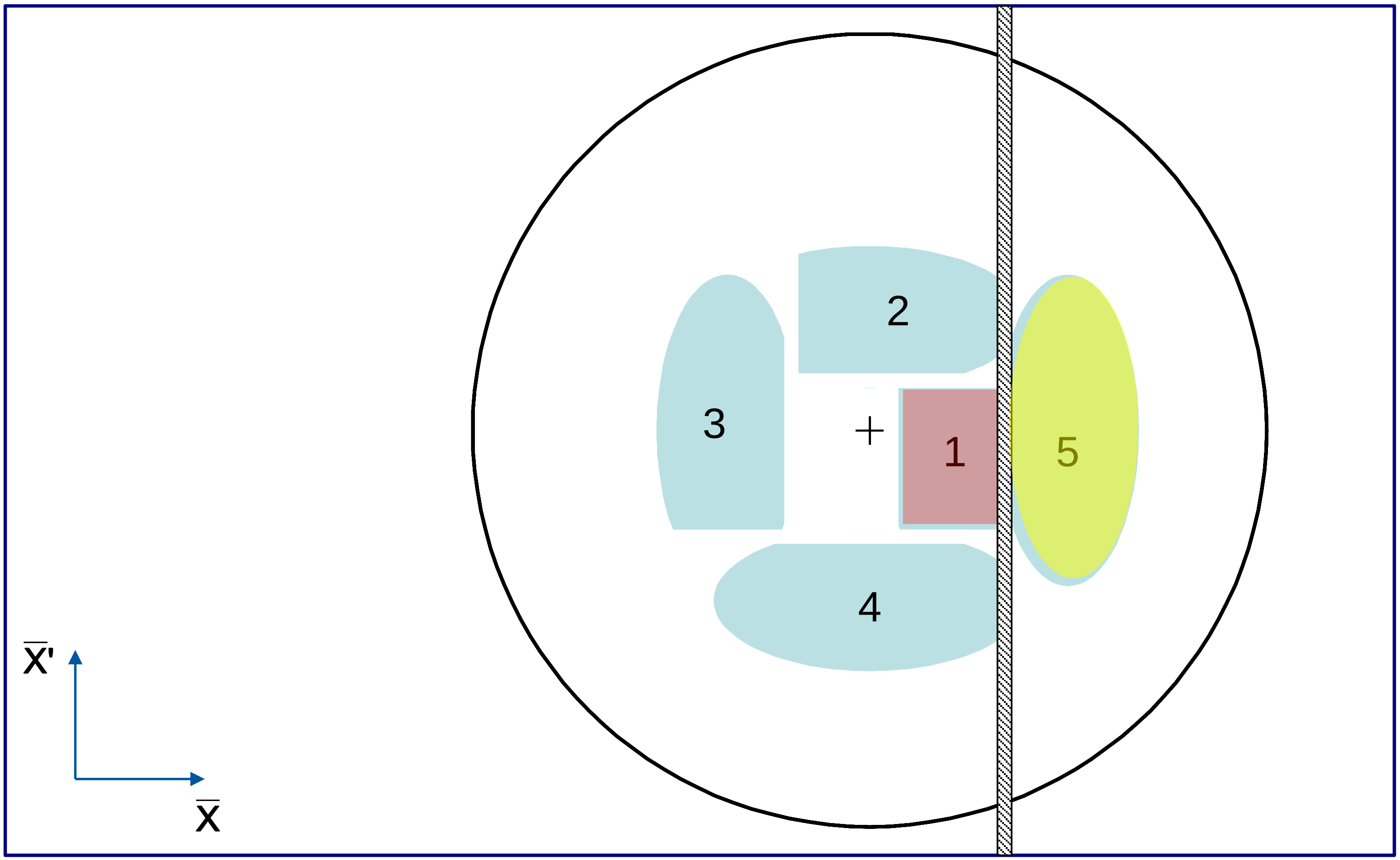
Turn 5



Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
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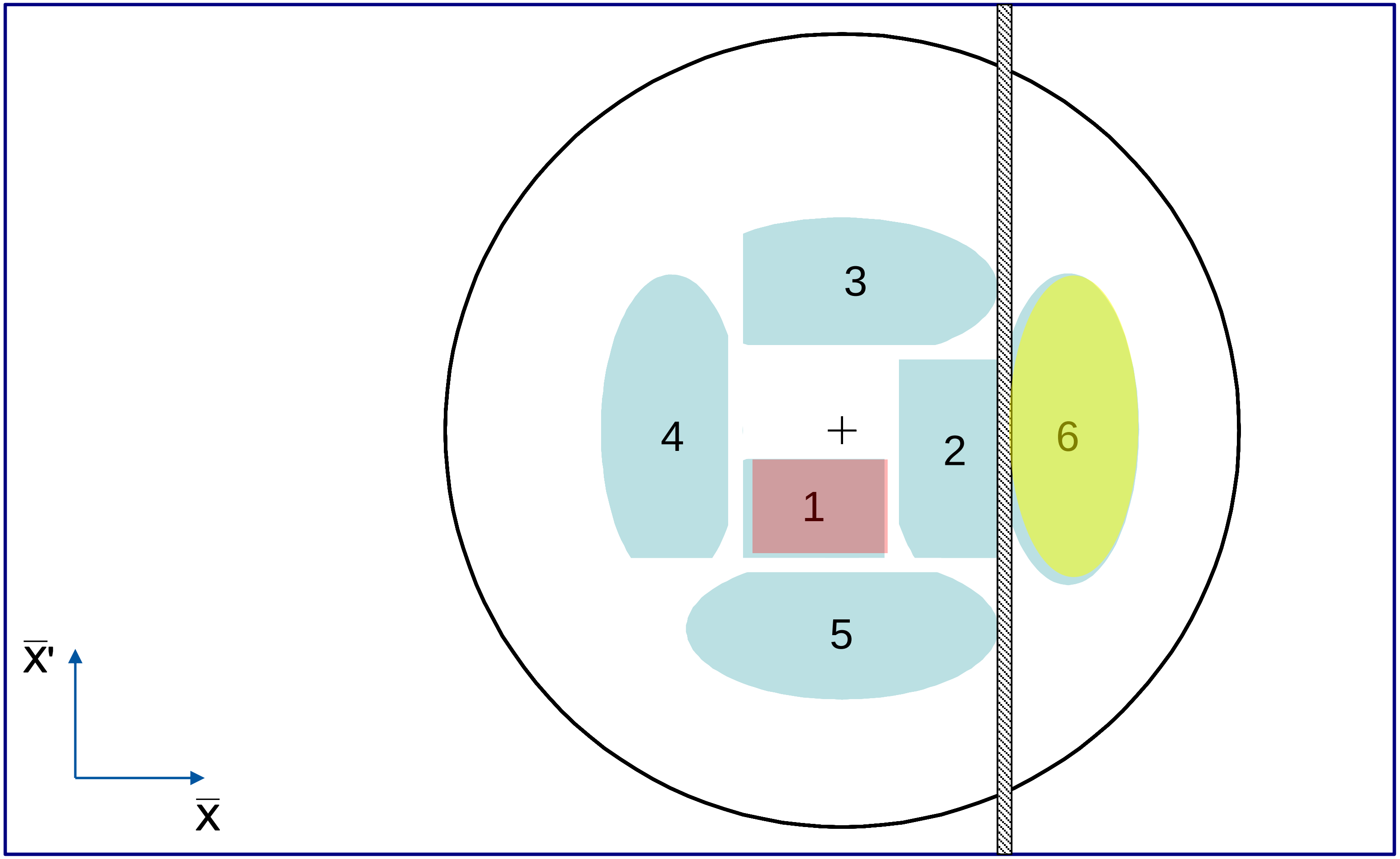
Turn 6



Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
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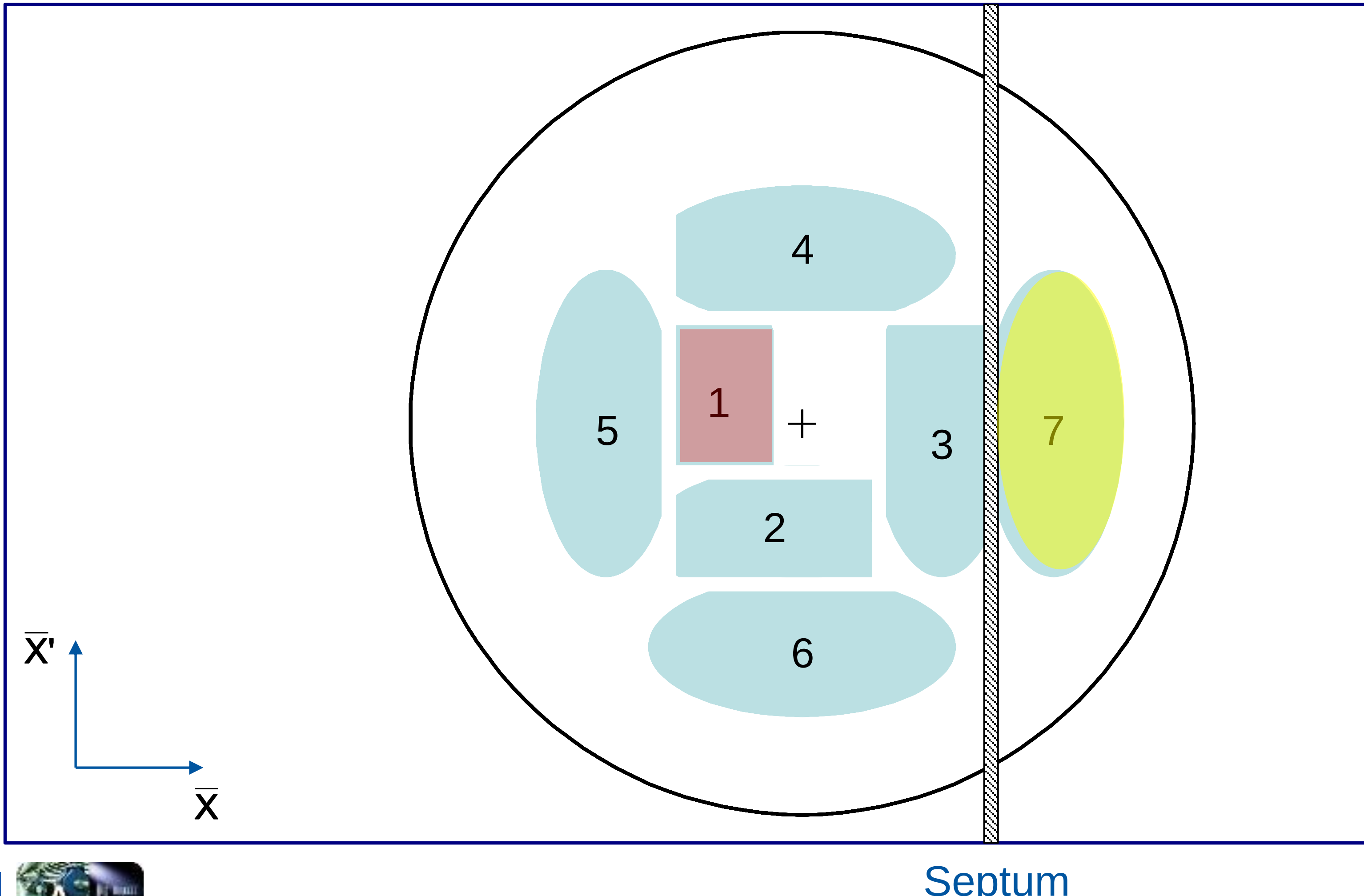
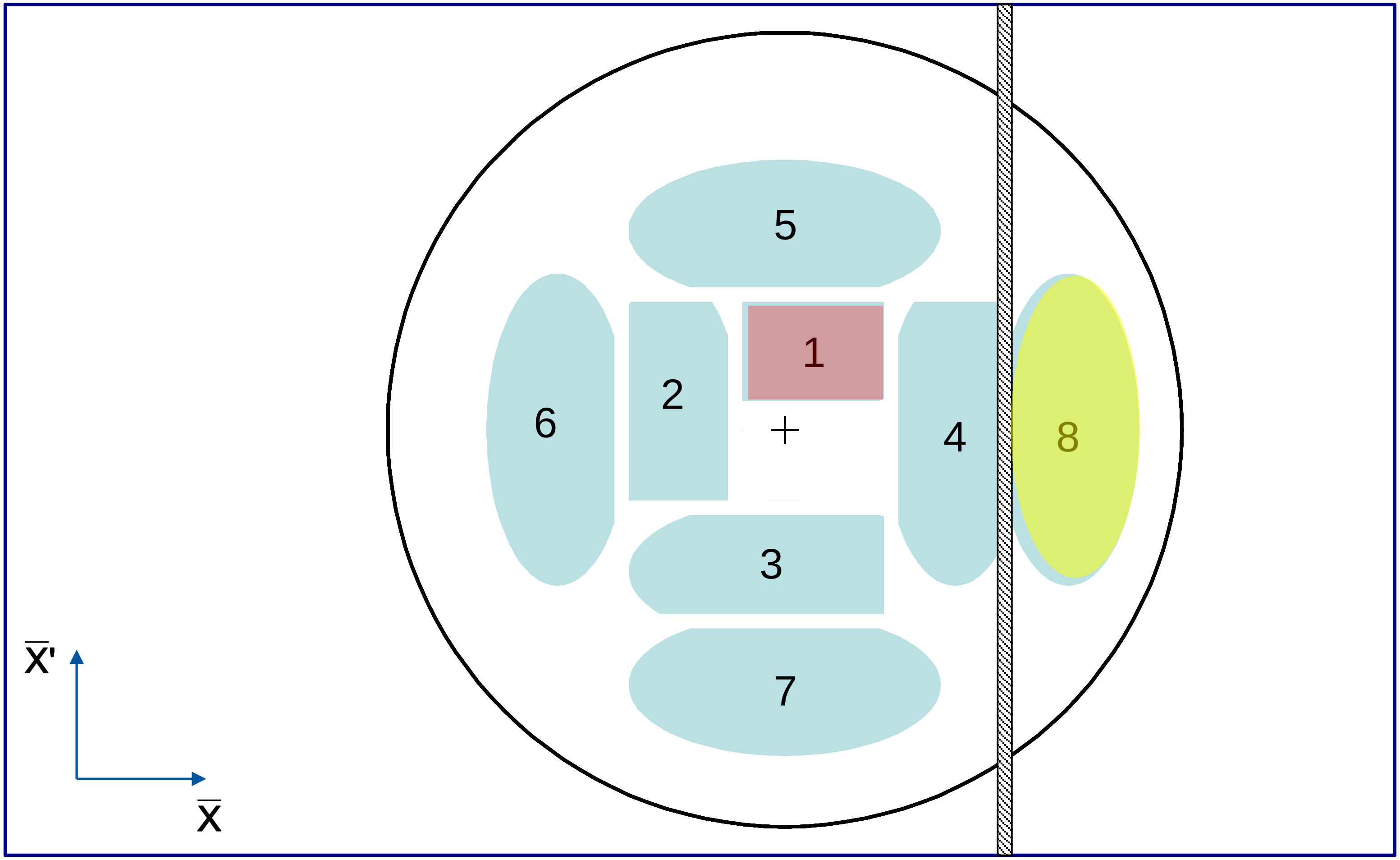
Turn 7



Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
fractional tune $Q_h \approx 0.25$ ➔ beam rotates $\pi/2$ per turn in phase space

Turn 8

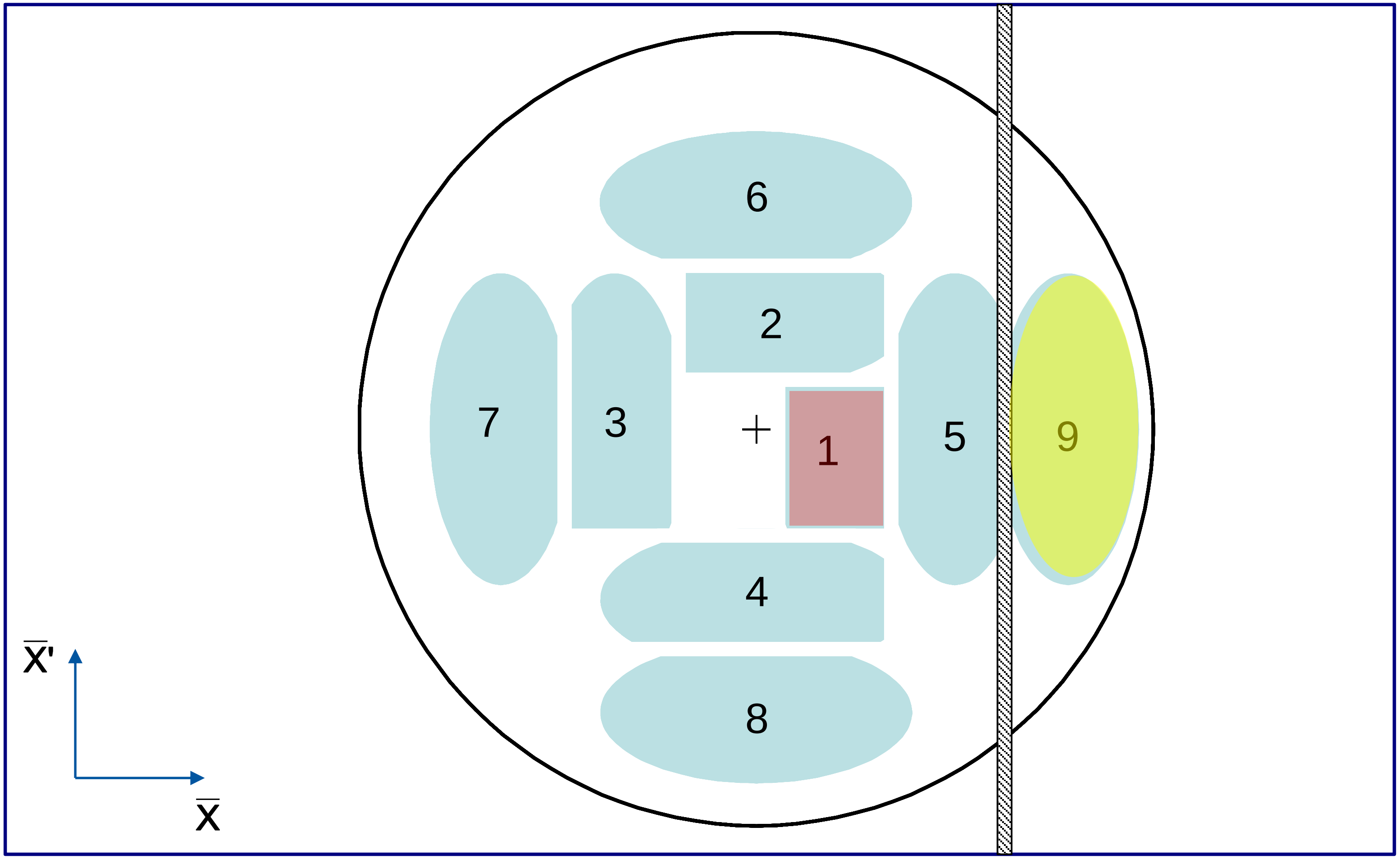


Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
fractional tune $Q_h \approx 0.25$ ➔ beam rotates π/2 per turn in phase space

Turn 9

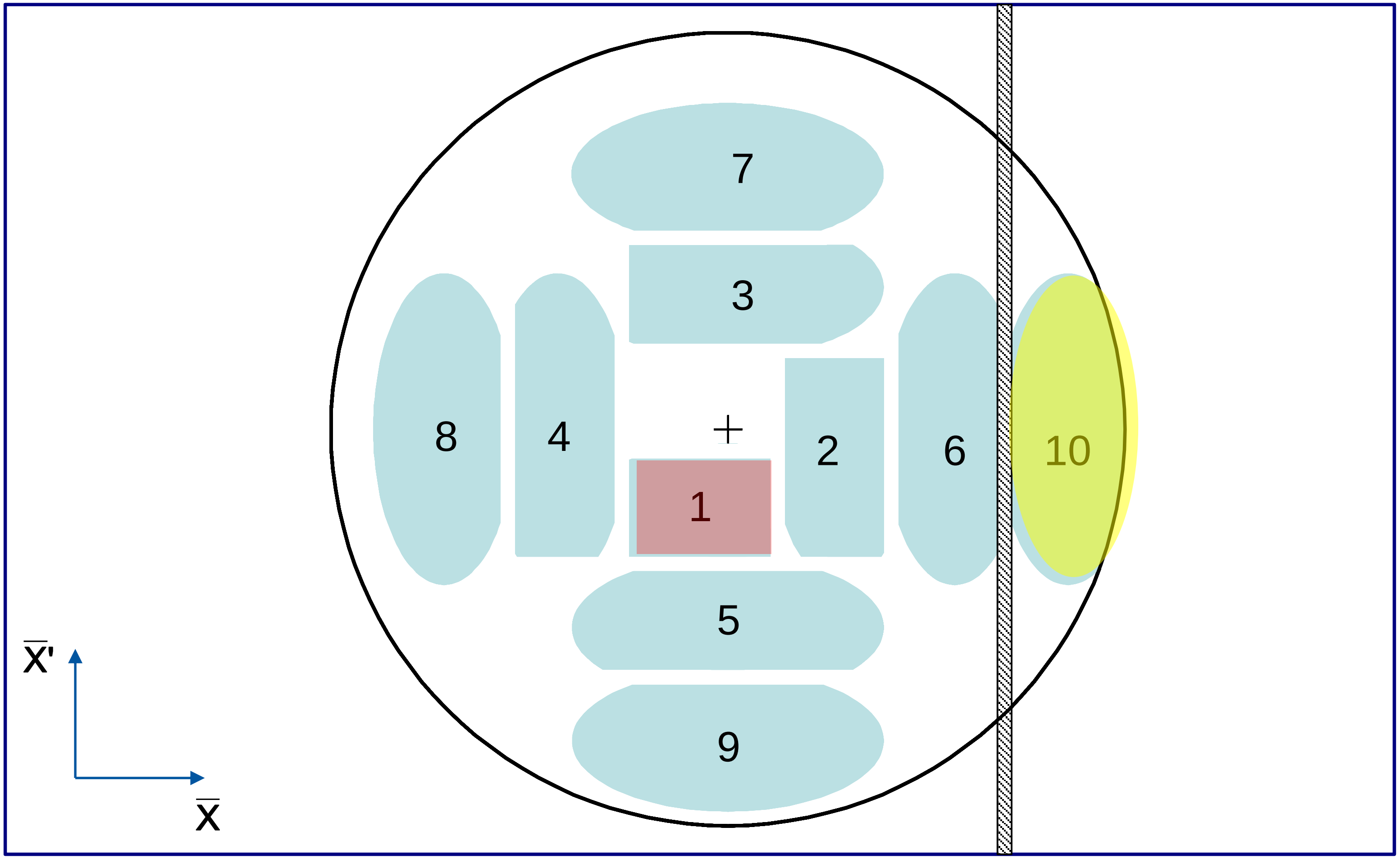


Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
fractional tune $Q_h \approx 0.25$ ➔ beam rotates $\pi/2$ per turn in phase space

Turn 10

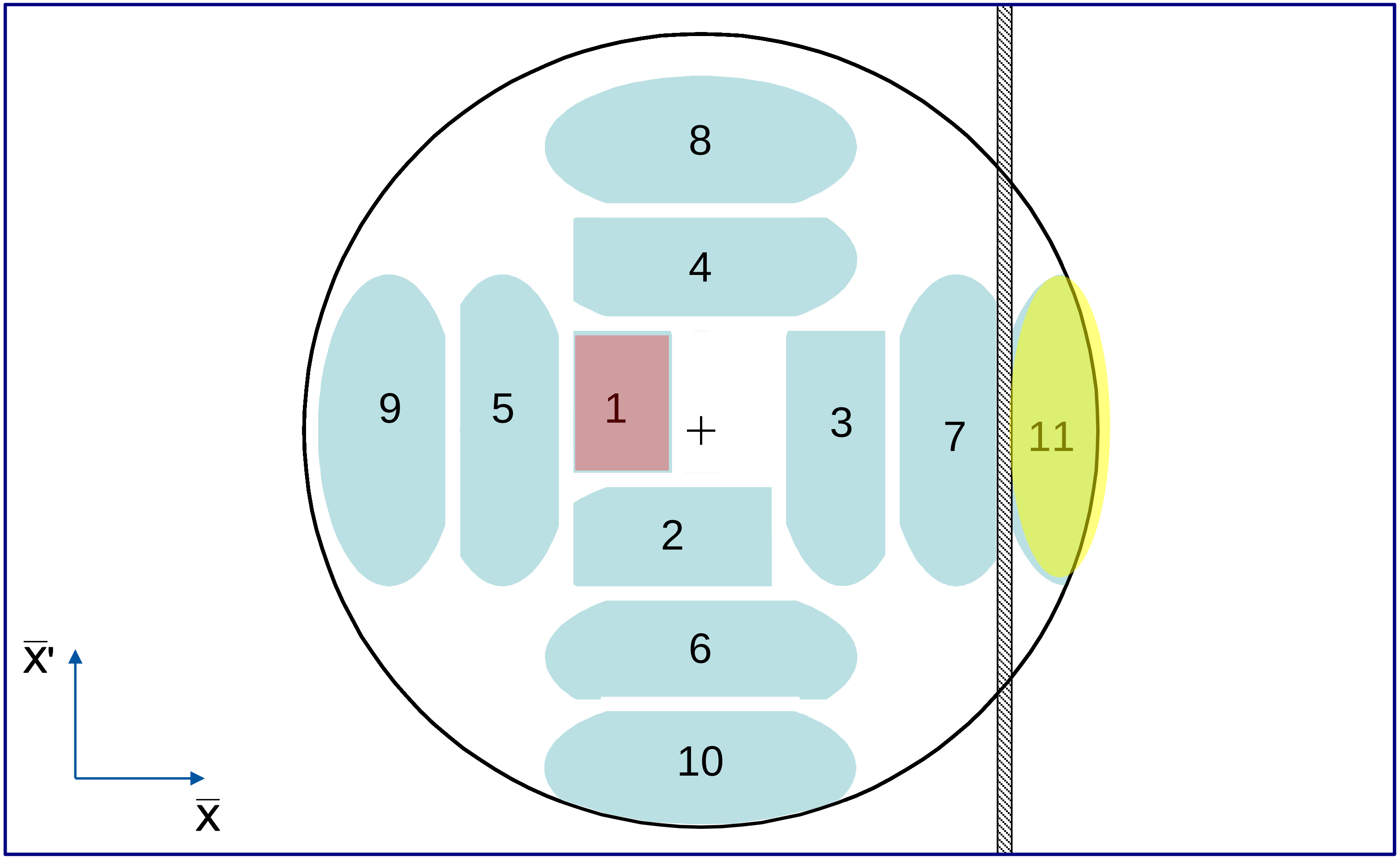


Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
fractional tune $Q_h \approx 0.25$ ➔ beam rotates π/2 per turn in phase space

Turn 11

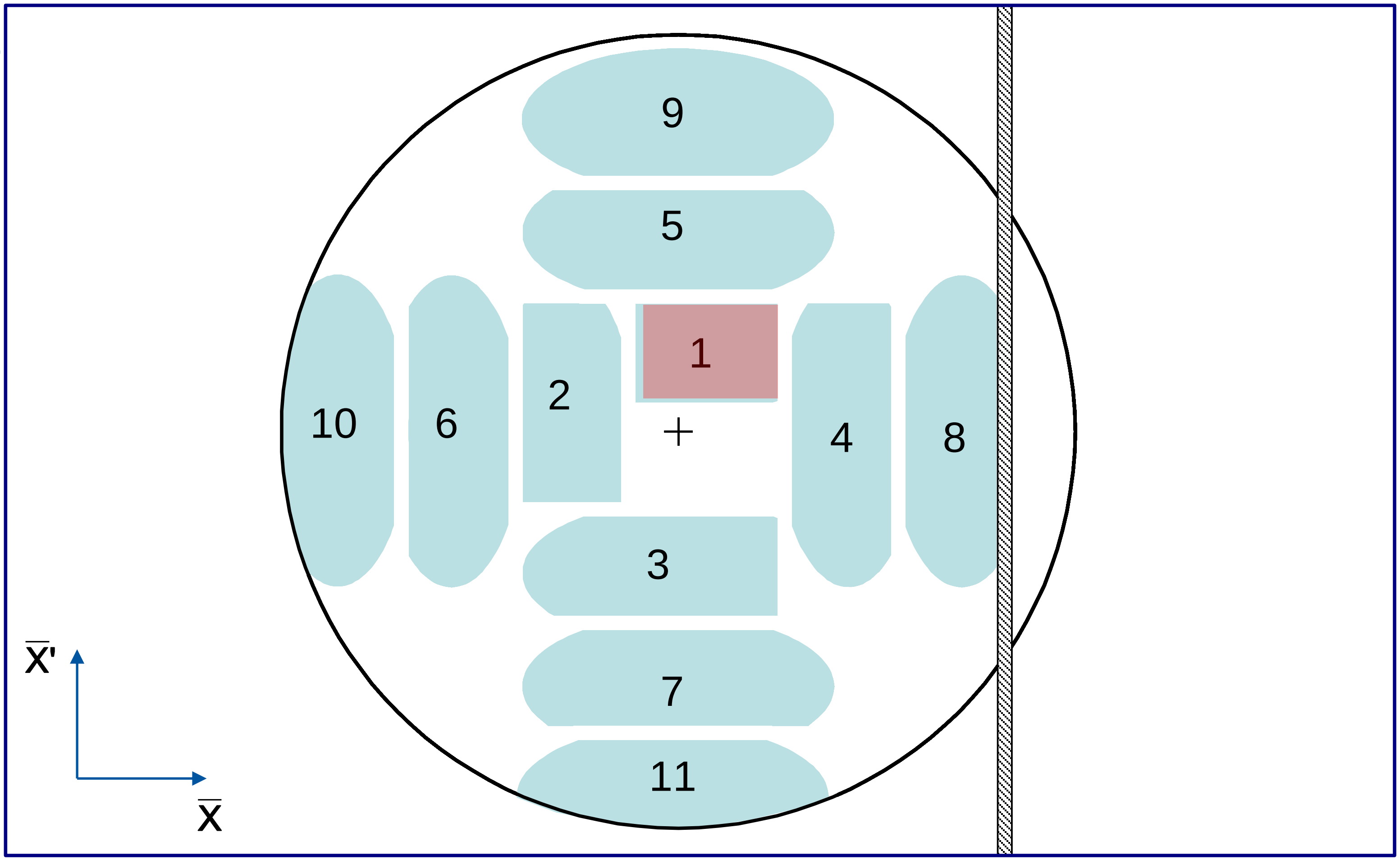


Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
fractional tune $Q_h \approx 0.25$ ➔ beam rotates π/2 per turn in phase space

Turn 12

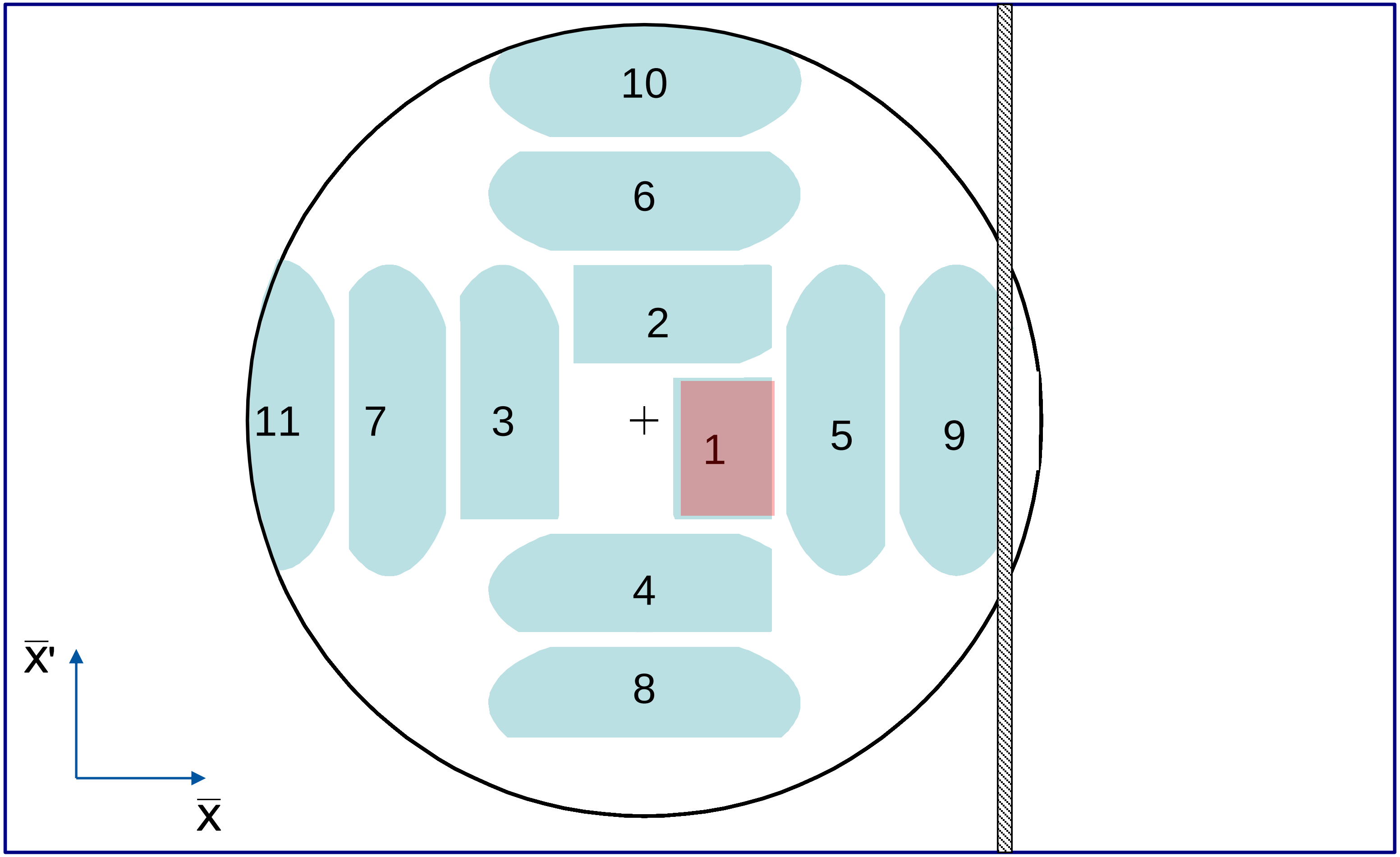


Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
fractional tune $Q_h \approx 0.25$ ➔ beam rotates π/2 per turn in phase space

Turn 13

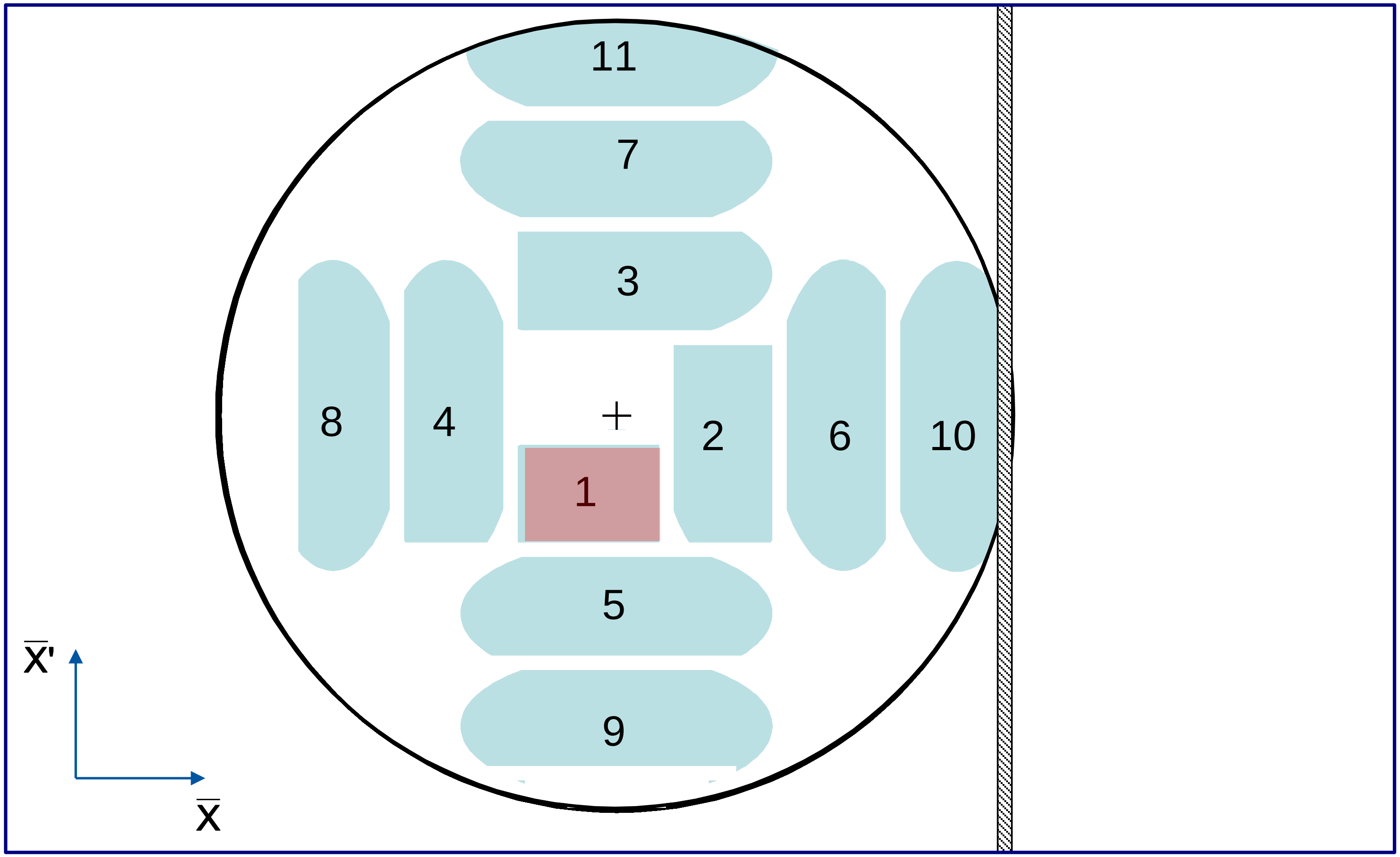


Septum

Multi-turn injection for hadrons

Example: CERN PSB injection, high intensity beams,
fractional tune $Q_h \approx 0.25$ ➔ beam rotates $\pi/2$ per turn in phase space

Turn 14

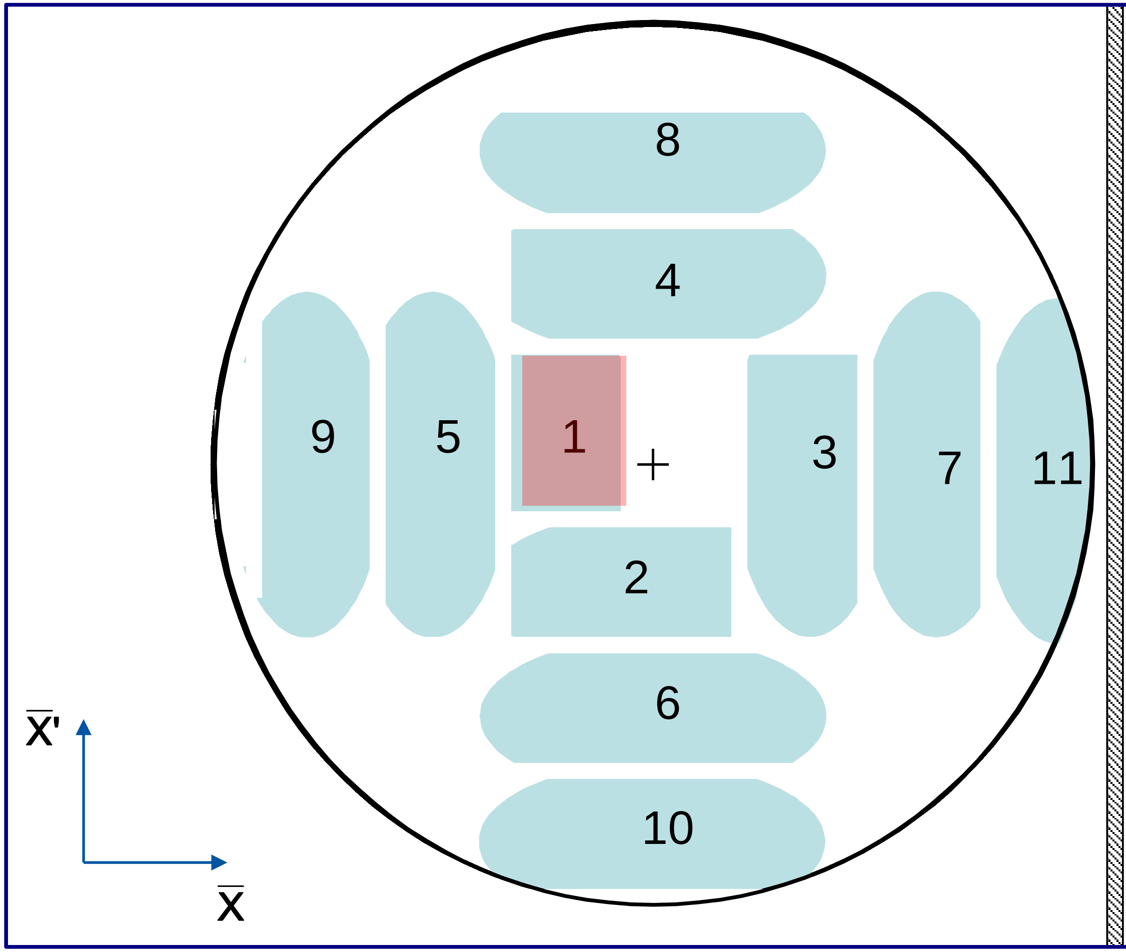


Septum

Multi-turn injection for hadrons

Phase space has been “**painted**”

Turn 15



In reality, filamentation (often space-charge driven) occurs to produce a quasi-uniform beam

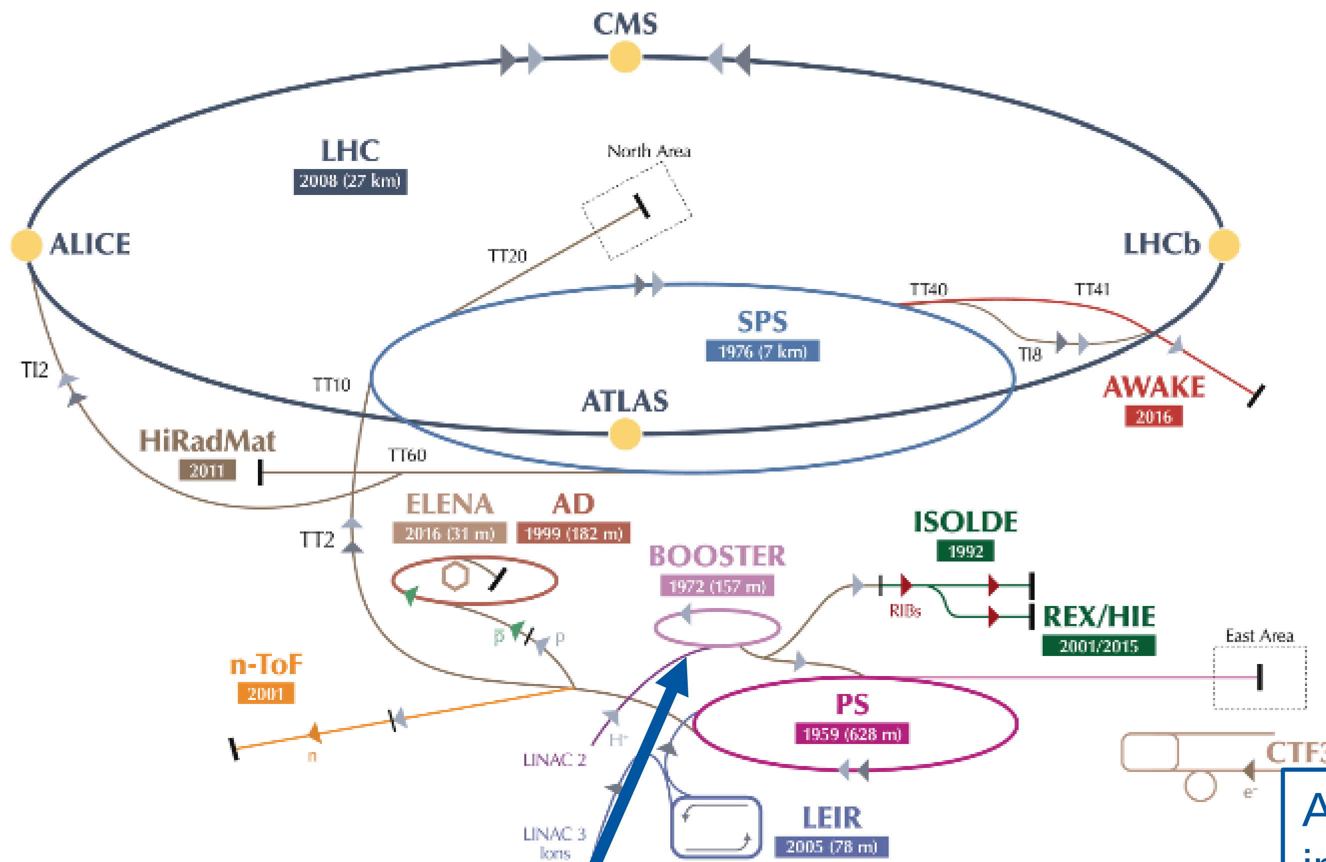
Charge exchange H^- injection

- Multi-turn injection is essential to accumulate high intensity
- Disadvantages inherent in using an injection septum:
  - Width of several mm reduces aperture
  - Beam losses from circulating beam hitting septum:
    - typically up to 50% for the CERN PSB injection at 50 MeV
  - Limits number of injected turns to 10 – 20
- **Charge-exchange injection** provides elegant alternative
  - Convert H^- to H^+ using a thin stripping foil, allowing **injection into the same phase space area**

➔ **increase of beam density**

Multi-turn methods

CERN Accelerator Complex

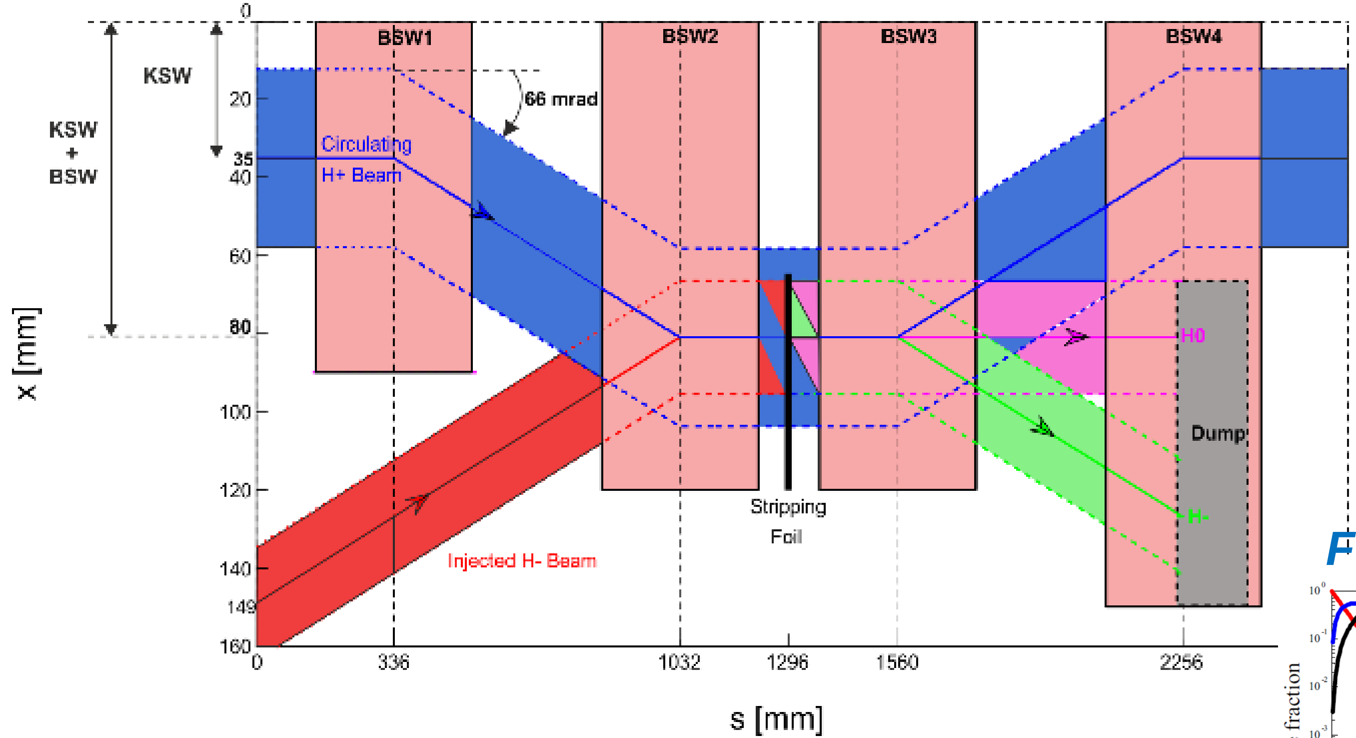


With LINAC4:
Charge exchange H^-
injection

Accumulate high intensity and density by “cheating” Liouville

H⁻ charge exchange injection (Example: PSB future injection)

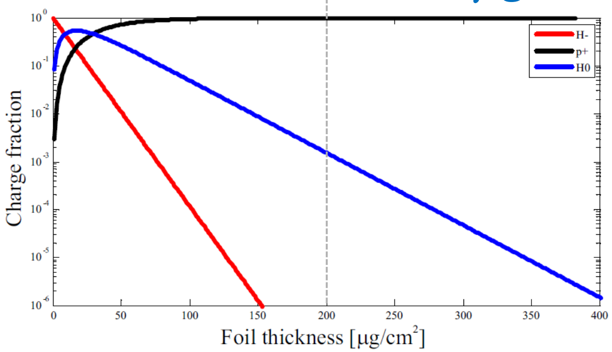
- The Linac4 will generate H^- beams to the PSB injection at 160 MeV.
- Protons are stripped (>98% efficiency) from H^- ions through a stripping carbon foil.



The stripping foil

Measured efficiency 98-99%

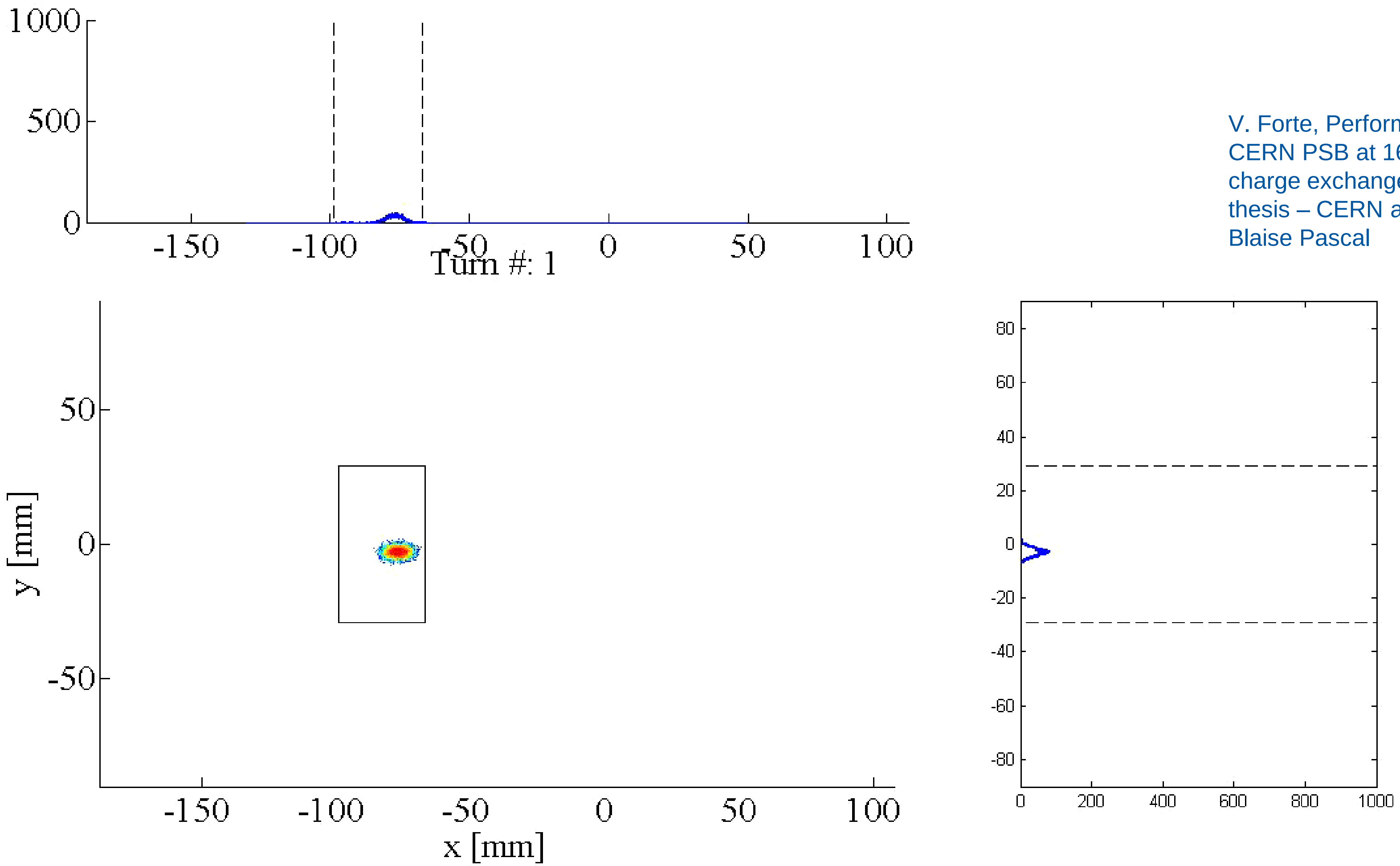
Foil thickness 200 $\mu g/cm^2$



Accumulation process on foil

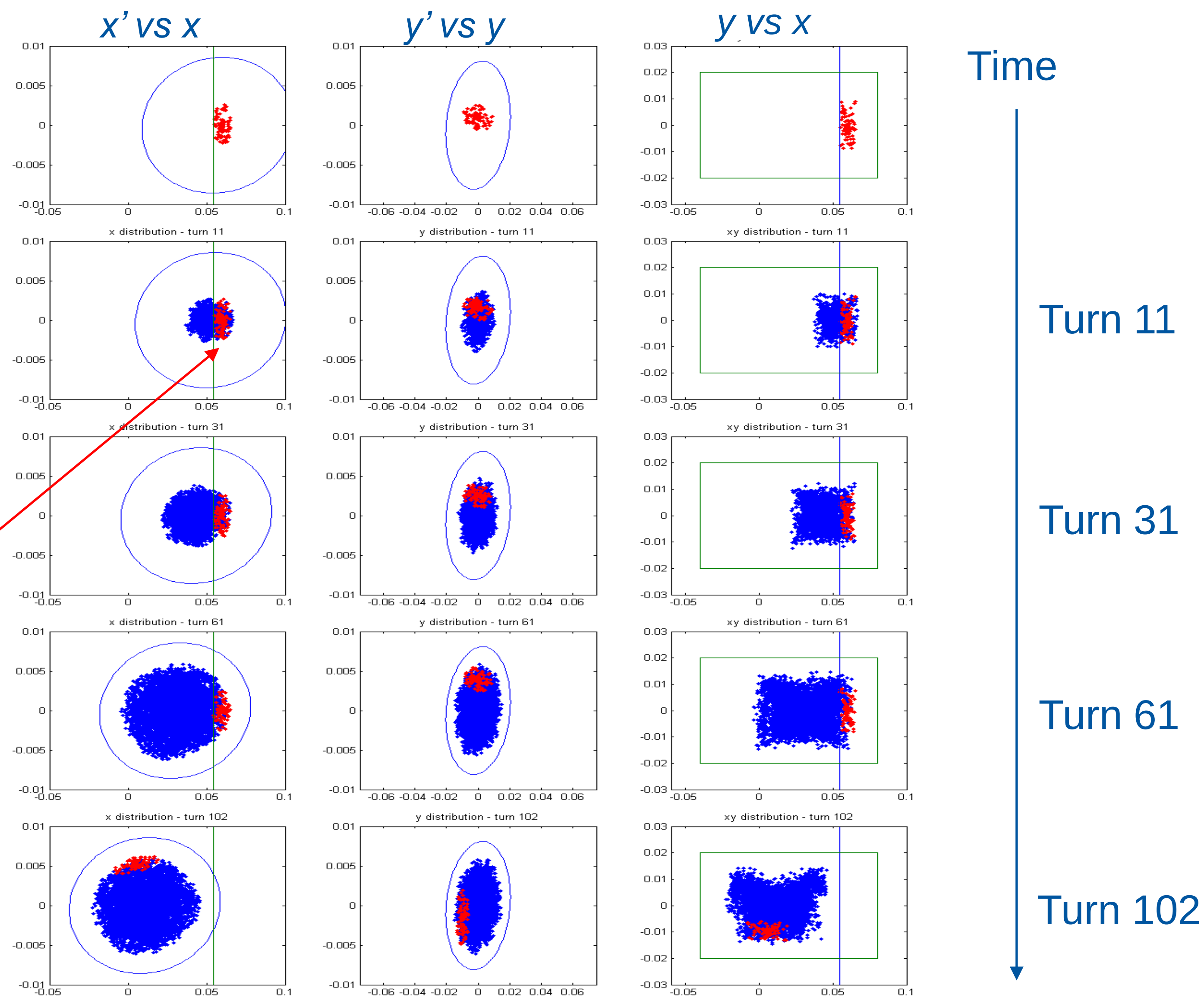
Linac4 connection to the PS Booster at 160 MeV:

H^- stripped to H^+ with Carbon foil 200 μg.cm^{-2}



V. Forte, Performance of the CERN PSB at 160 MeV with H- charge exchange injection, PhD thesis – CERN and Université Blaise Pascal

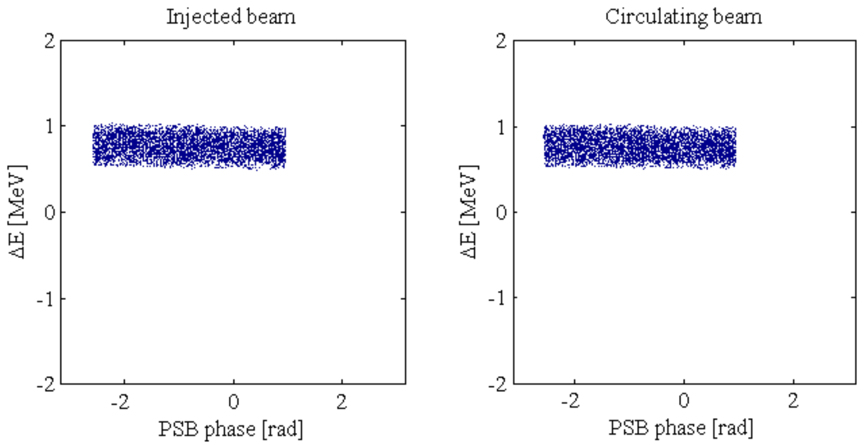
H⁻ injection - painting



Note injection into same phase space area as circulating beam

Charge exchange H^- injection

- Paint uniform transverse phase space density by modifying closed orbit bump and steering injected beam
- Injection chicane reduced or switched off after injection, to avoid excessive foil heating and beam blow-up
- Longitudinal phase space can also be painted turn-by-turn:
  - Variation of the injected beam energy turn-by-turn
  - Chopper system in linac to match length of injected batch to bucket

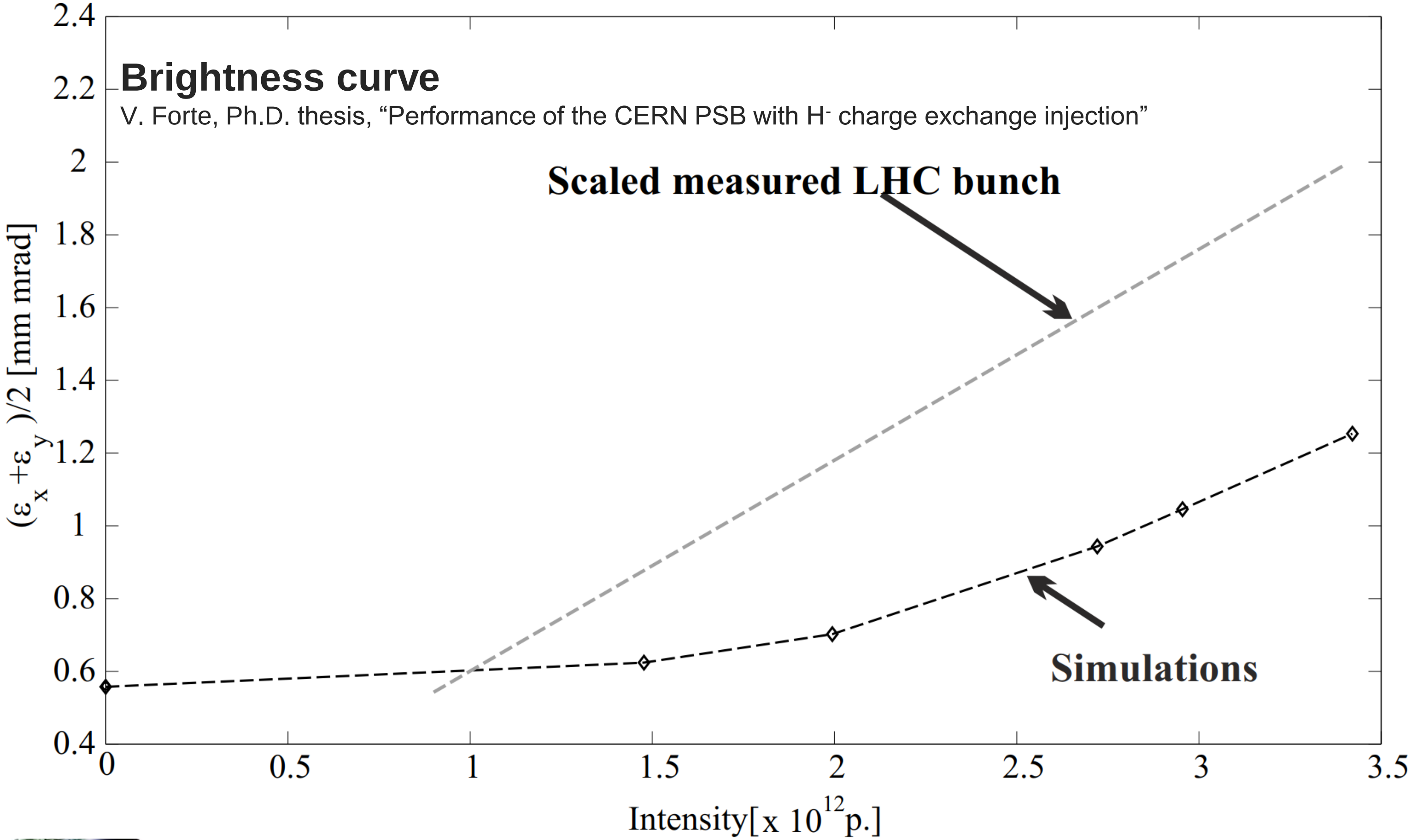


V. Forte

Effect on beam emittance

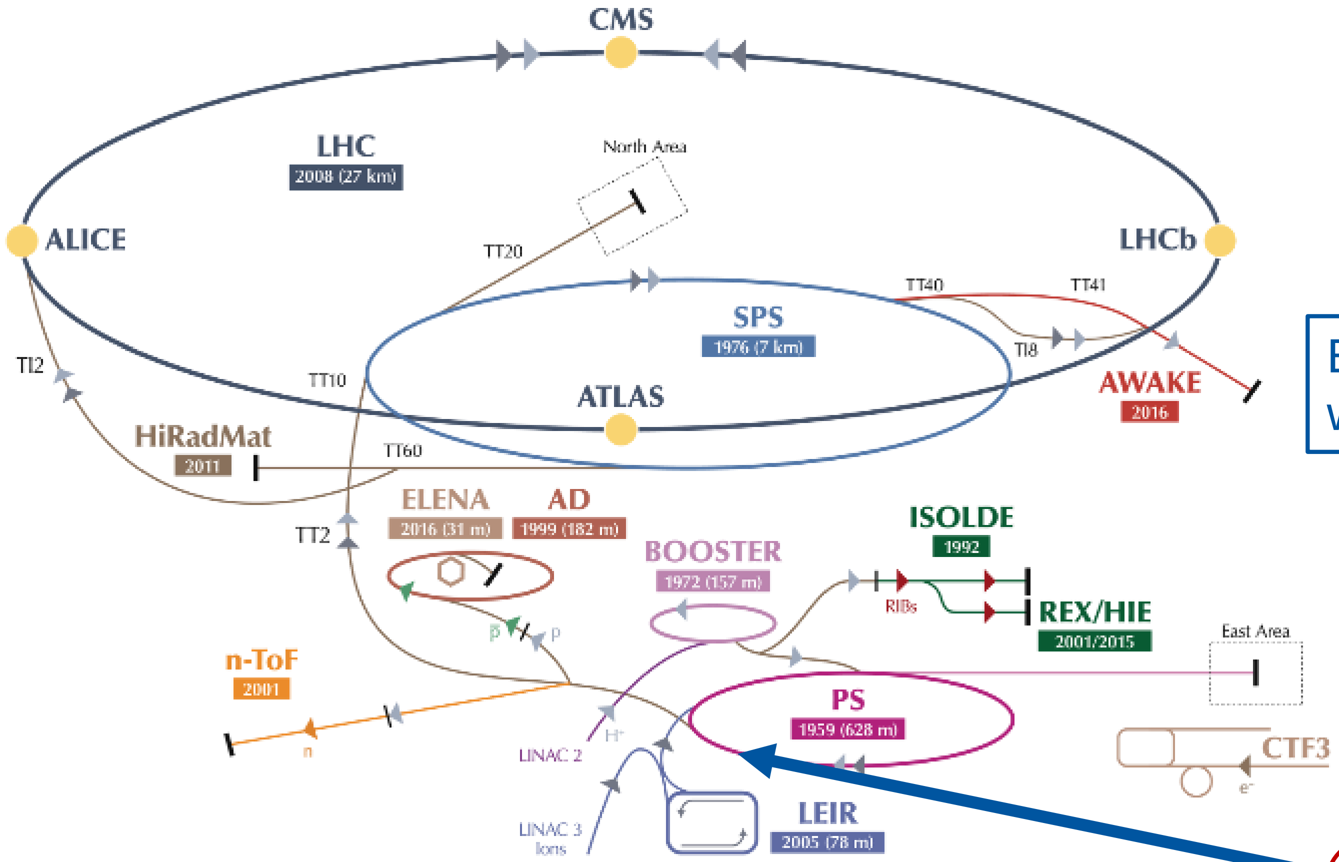
Simulations of future PSB emittance evolution with Linac4 160 MeV injection

- Increased brightness (smaller slope of the curve) for CERN LIU and HL-LHC project



Multi-turn methods

CERN Accelerator Complex

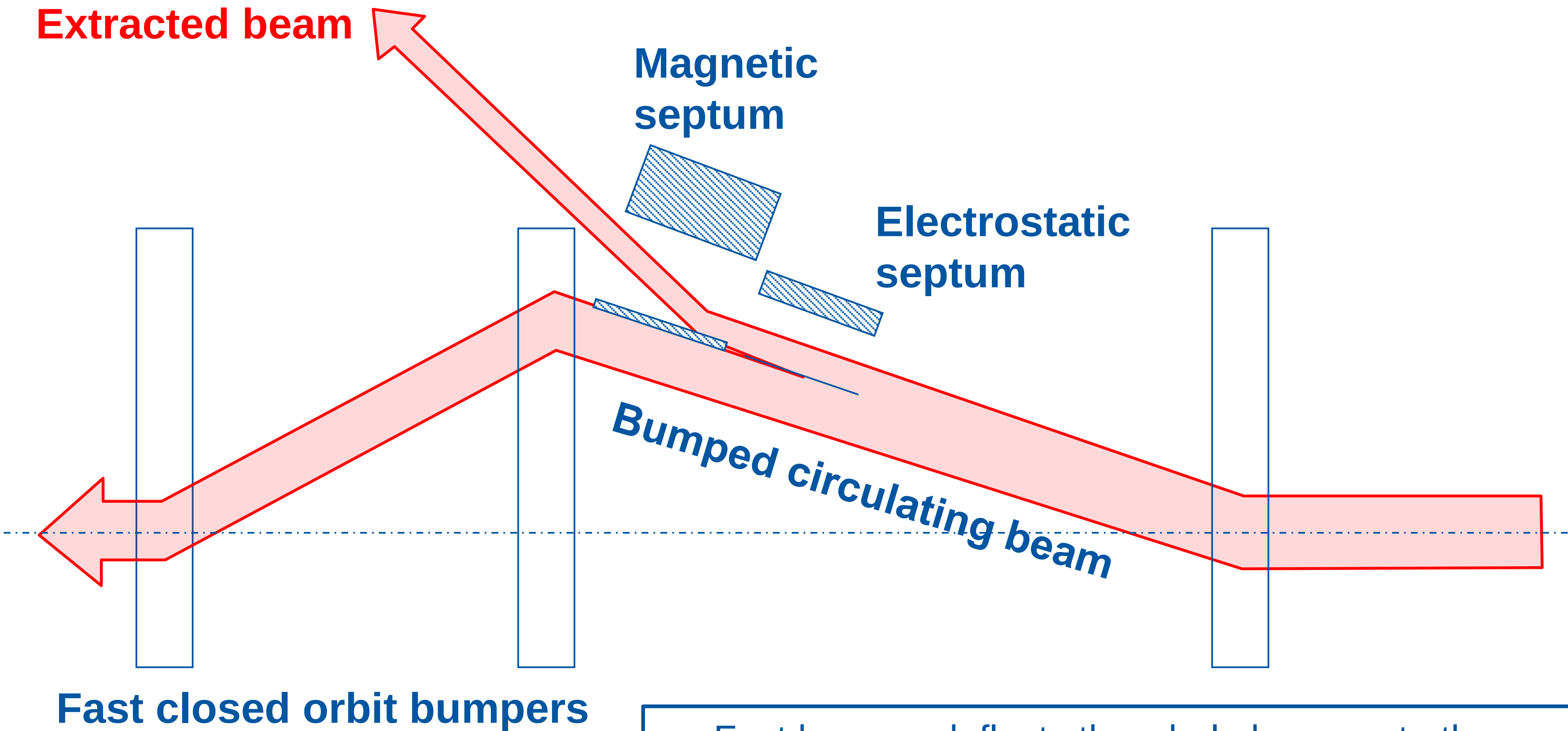


Extraction of beams within several turns

Multi-turn extraction

Non-resonant multi-turn extraction

Beam bumped to septum; part of beam ‘shaved’ off each turn



- Fast bumper deflects the whole beam onto the septum
- Beam extracted in a few turns, with the machine tune rotating the beam
- Intrinsically a high-loss process: thin septum essential
- Often combine thin electrostatic septa with magnetic septa

Magnetic vs. electrostatic septum

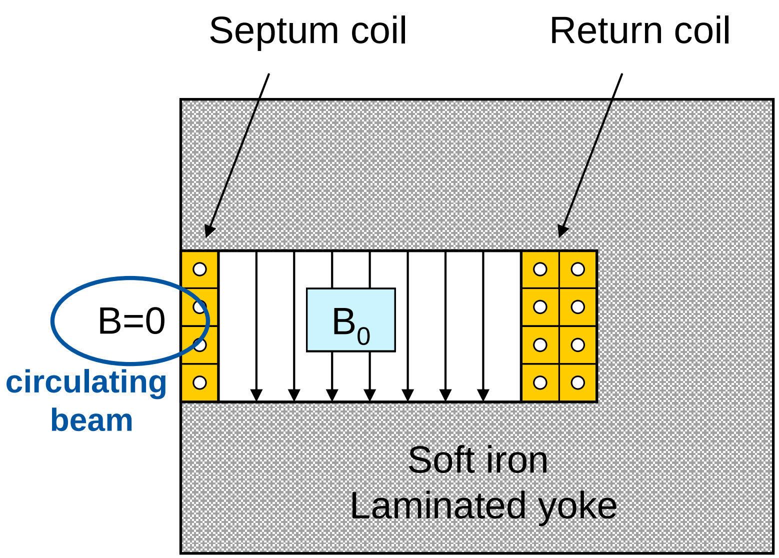
More details in T. Kramer's talk later

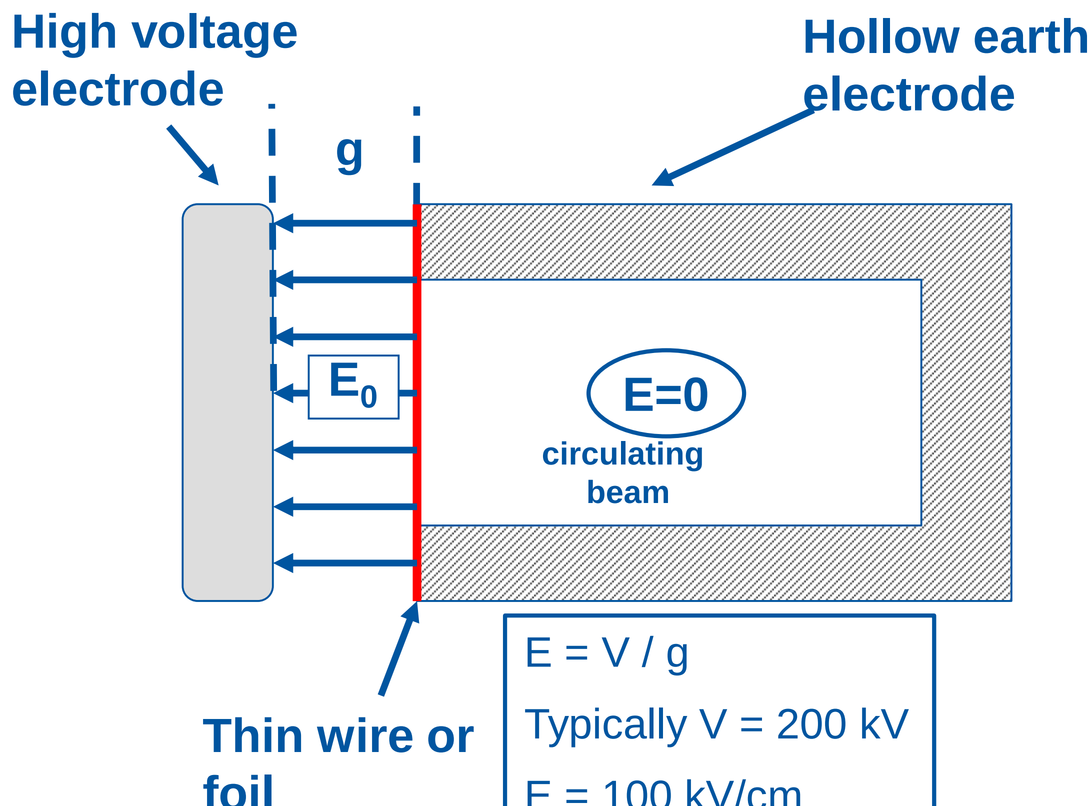
Magnetic

Septum coil: 2 – 20 mm

Electrostatic

Thin wire or coil: ~0.1 mm





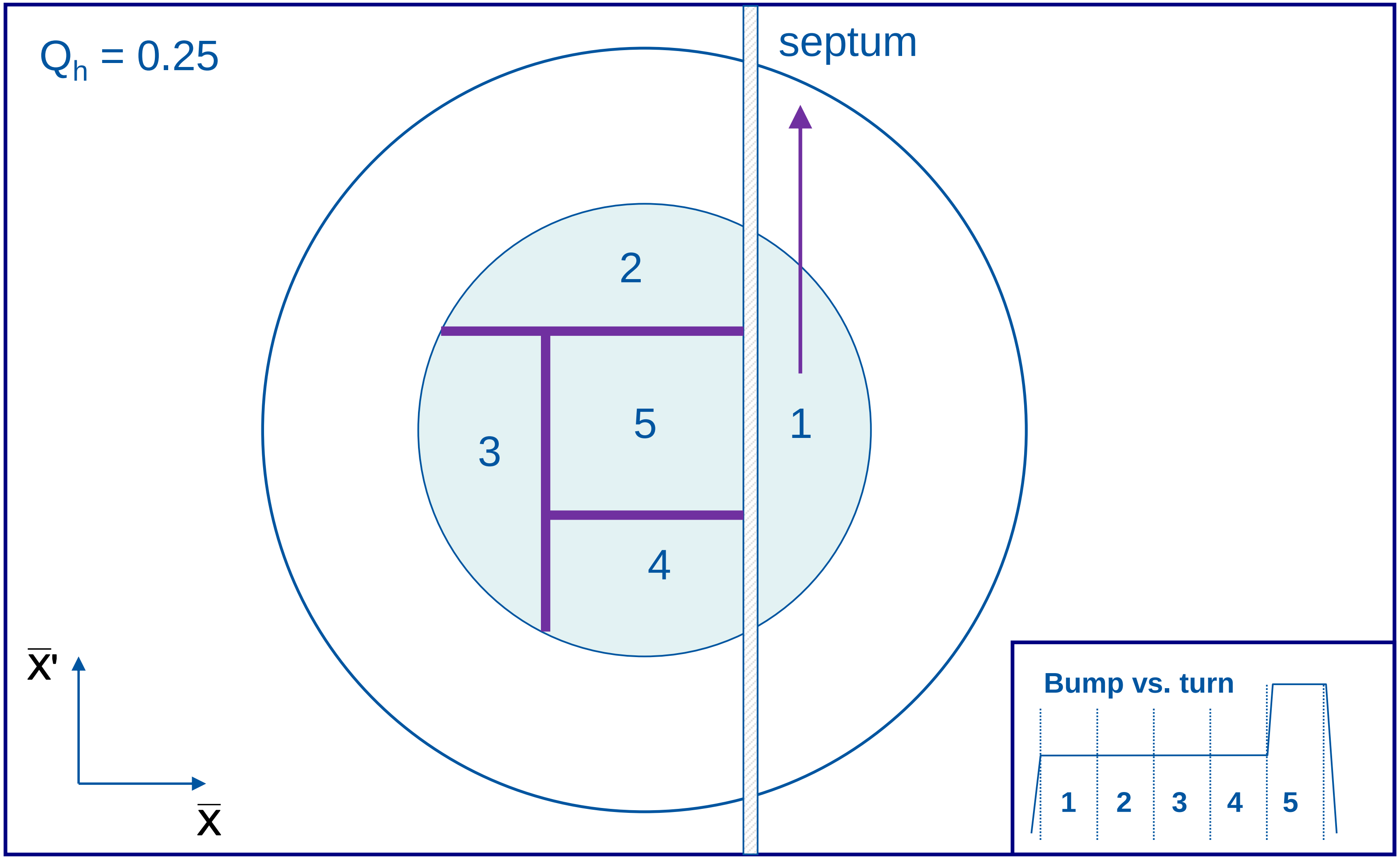
E = V / g

Typically V = 200 kV

E = 100 kV/cm

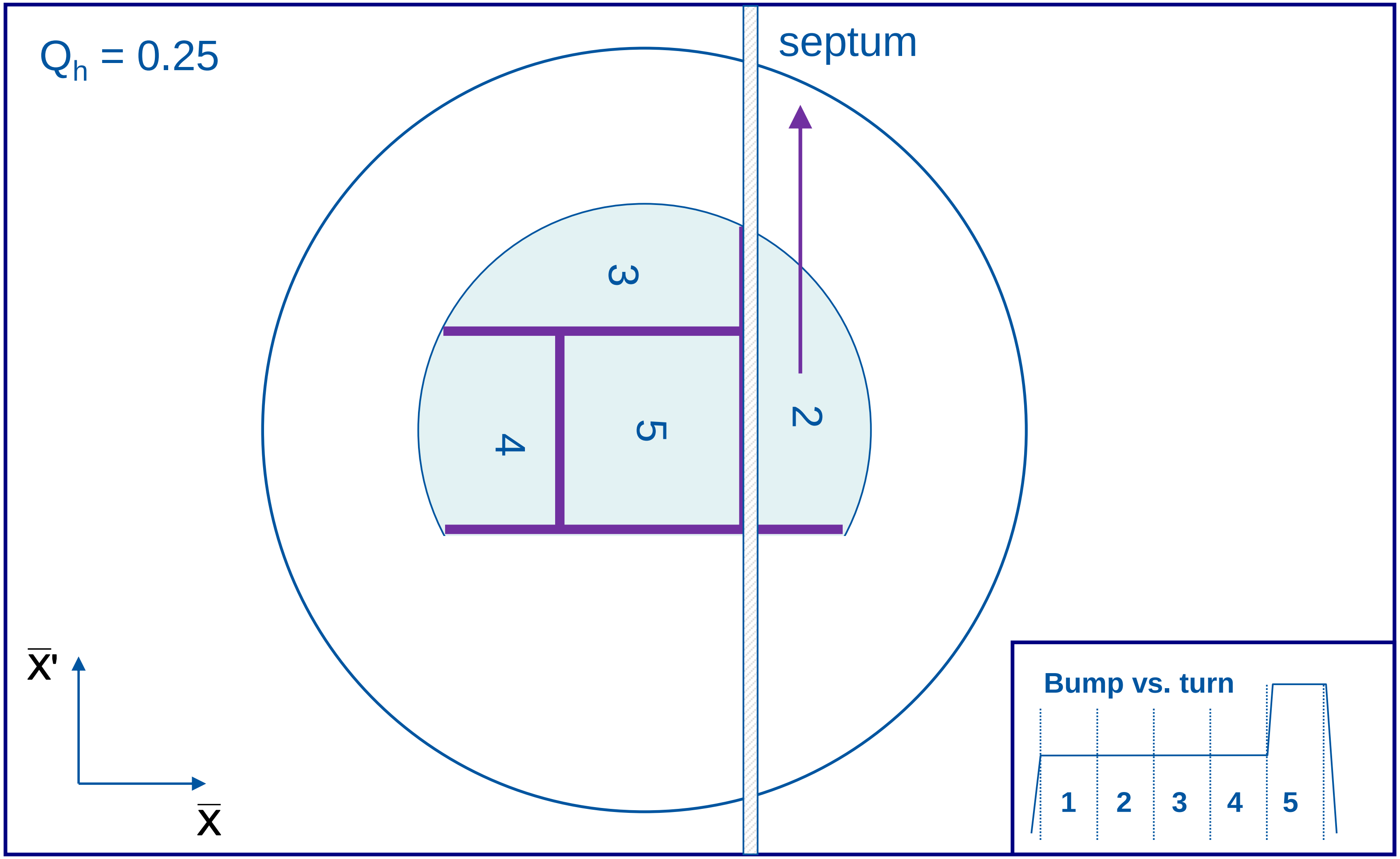
Non-resonant multi-turn extraction

CERN PS to SPS: 5-turn continuous transfer – 1st turn



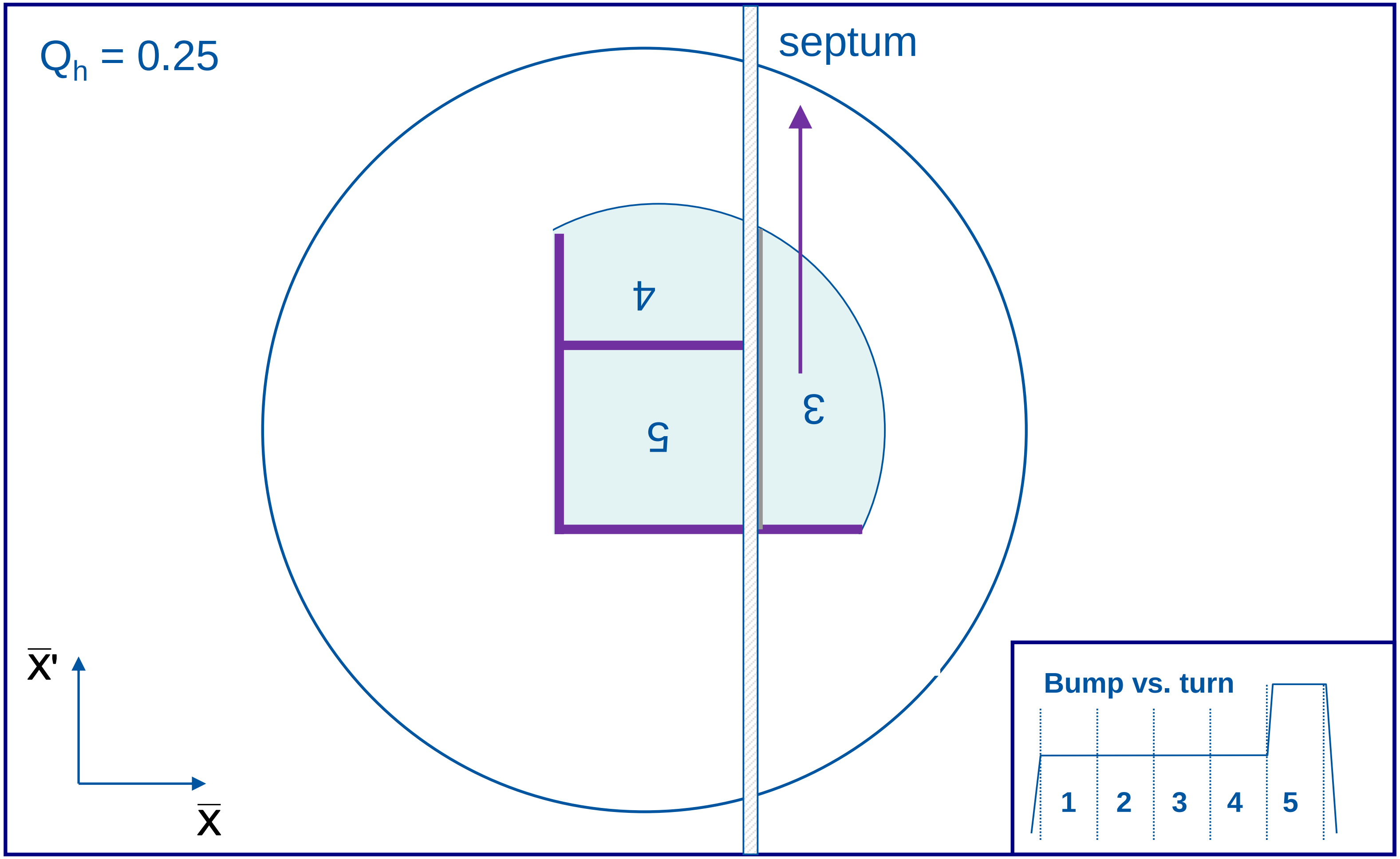
Non-resonant multi-turn extraction

CERN PS to SPS: 5-turn continuous transfer – 2nd turn



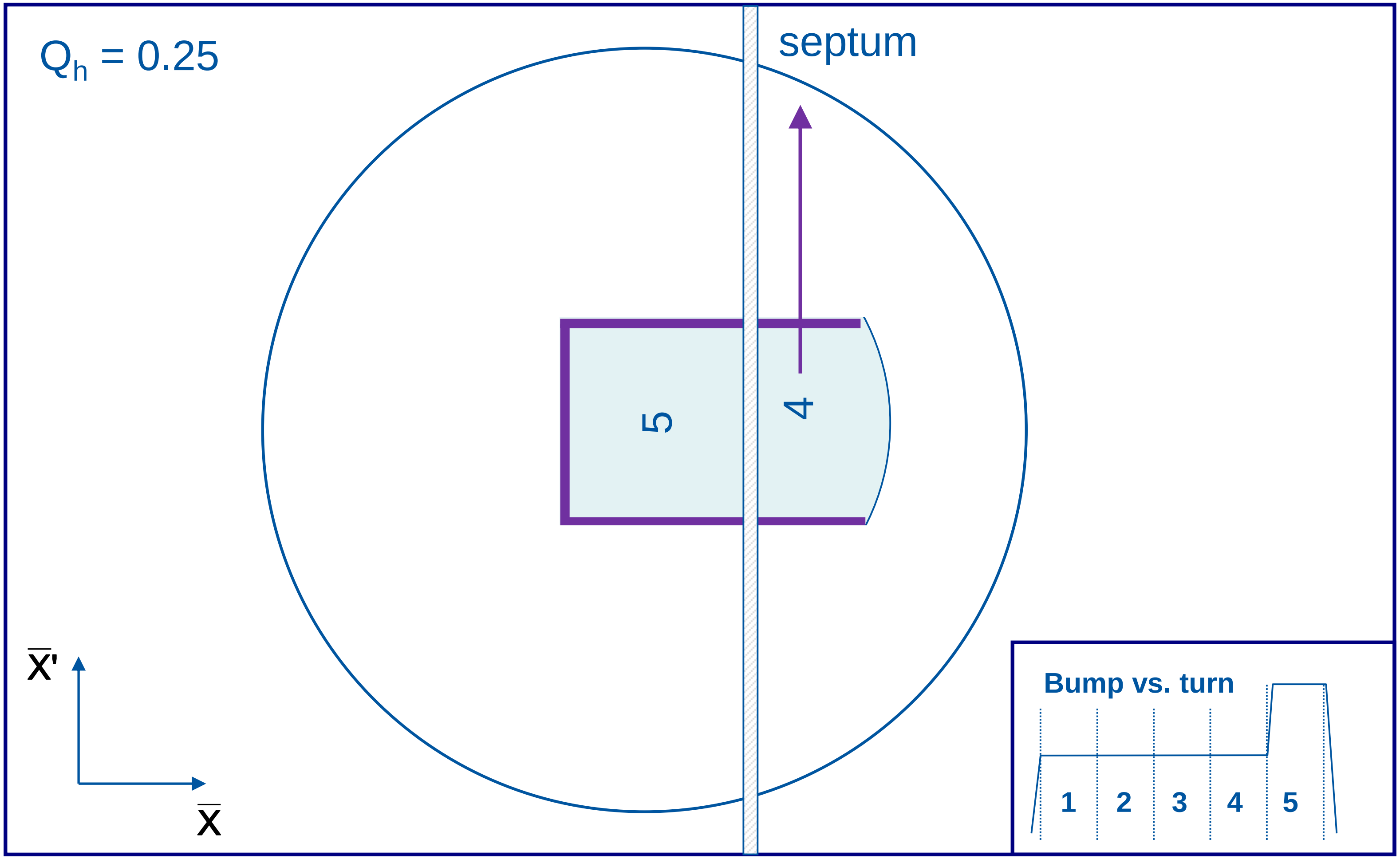
Non-resonant multi-turn extraction

CERN PS to SPS: 5-turn continuous transfer – 3rd turn



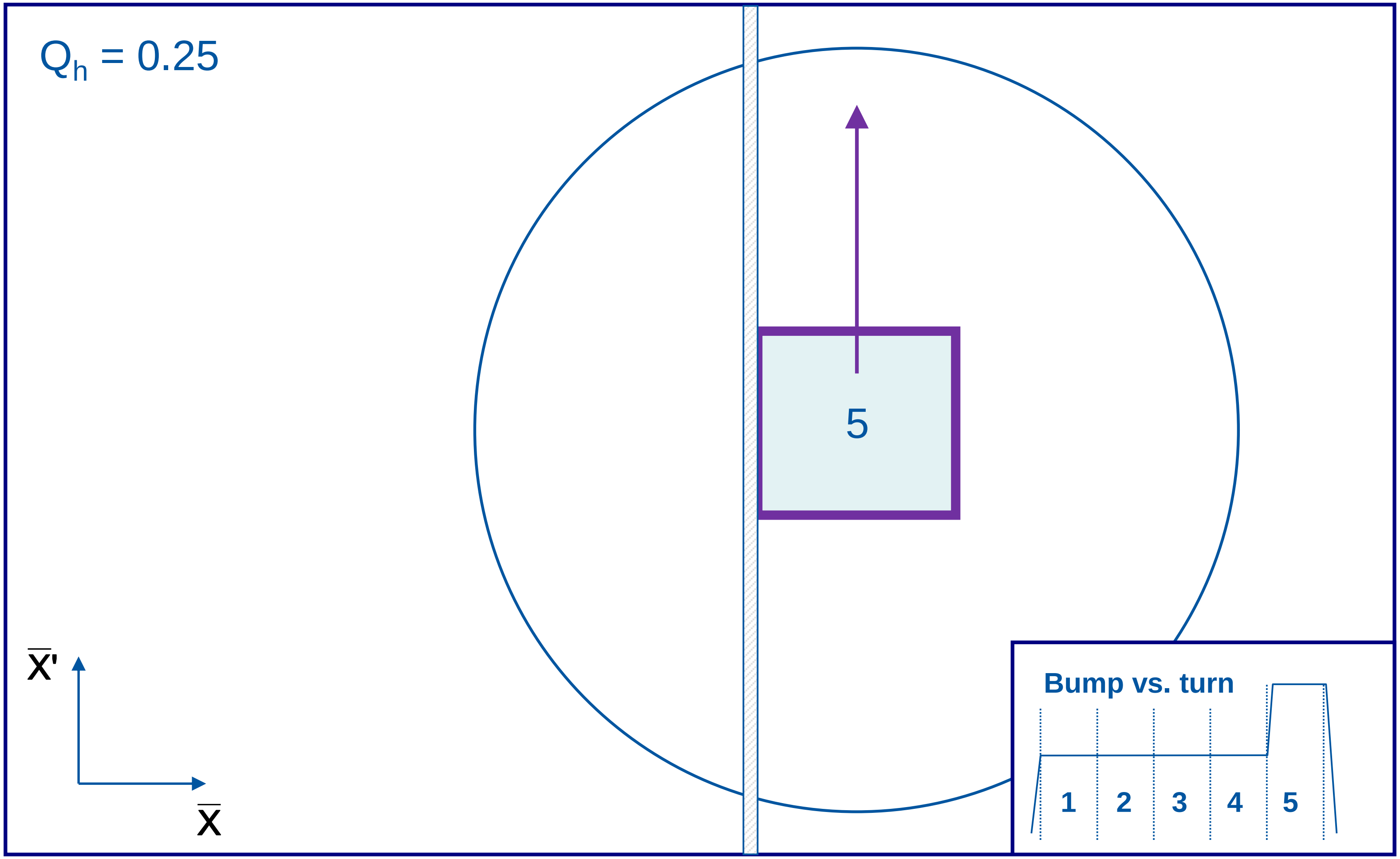
Non-resonant multi-turn extraction

CERN PS to SPS: 5-turn continuous transfer – 4th turn



Non-resonant multi-turn extraction

CERN PS to SPS: 5-turn continuous transfer – 5th turn

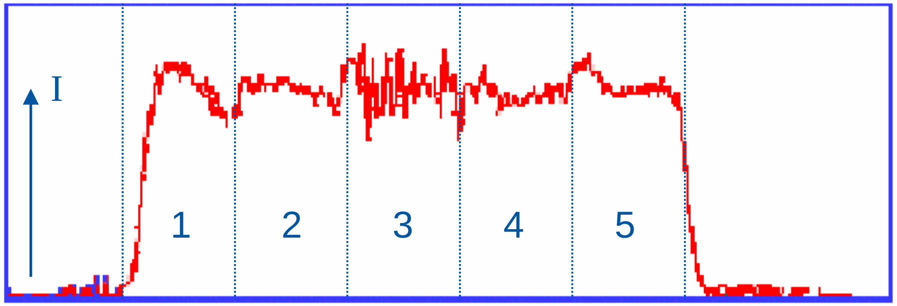


Non-resonant multi-turn extraction

- CERN PS to SPS: 5-turn continuous transfer

Fill SPS with 2 times 5-turn extractions (and 2 x 1 μs gap)

Total intensity in SPS 5×10^{13} p+

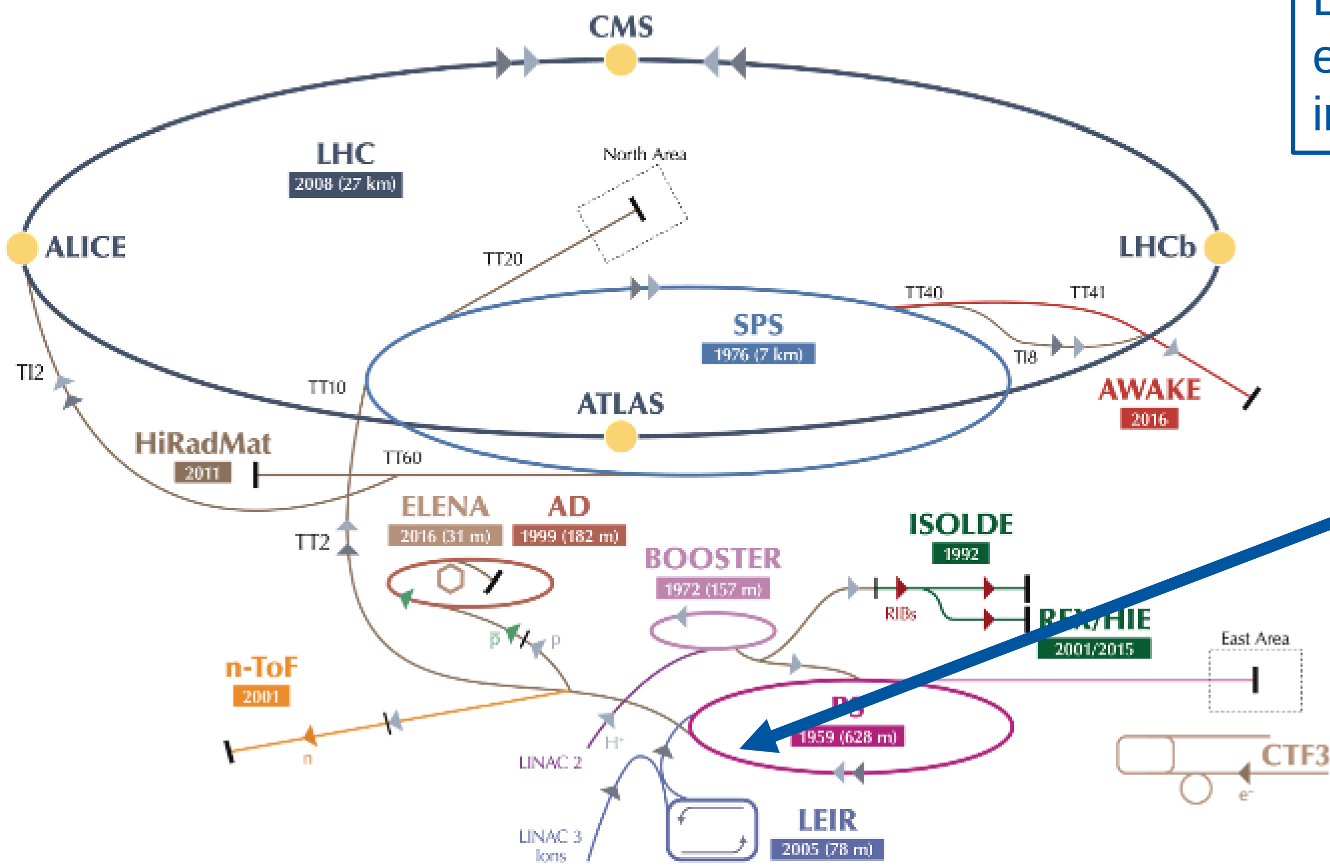


- Beamlets can have slightly different emittance
- Still about 15 % of beam lost in PS-SPS CT
  - Issue for maintenance of equipment due to radioprotection

Different method needed to extract very high intensity beams ➔ **use resonance**

Multi-turn methods

CERN Accelerator Complex



Low-loss multi-turn extraction for high intensity beams

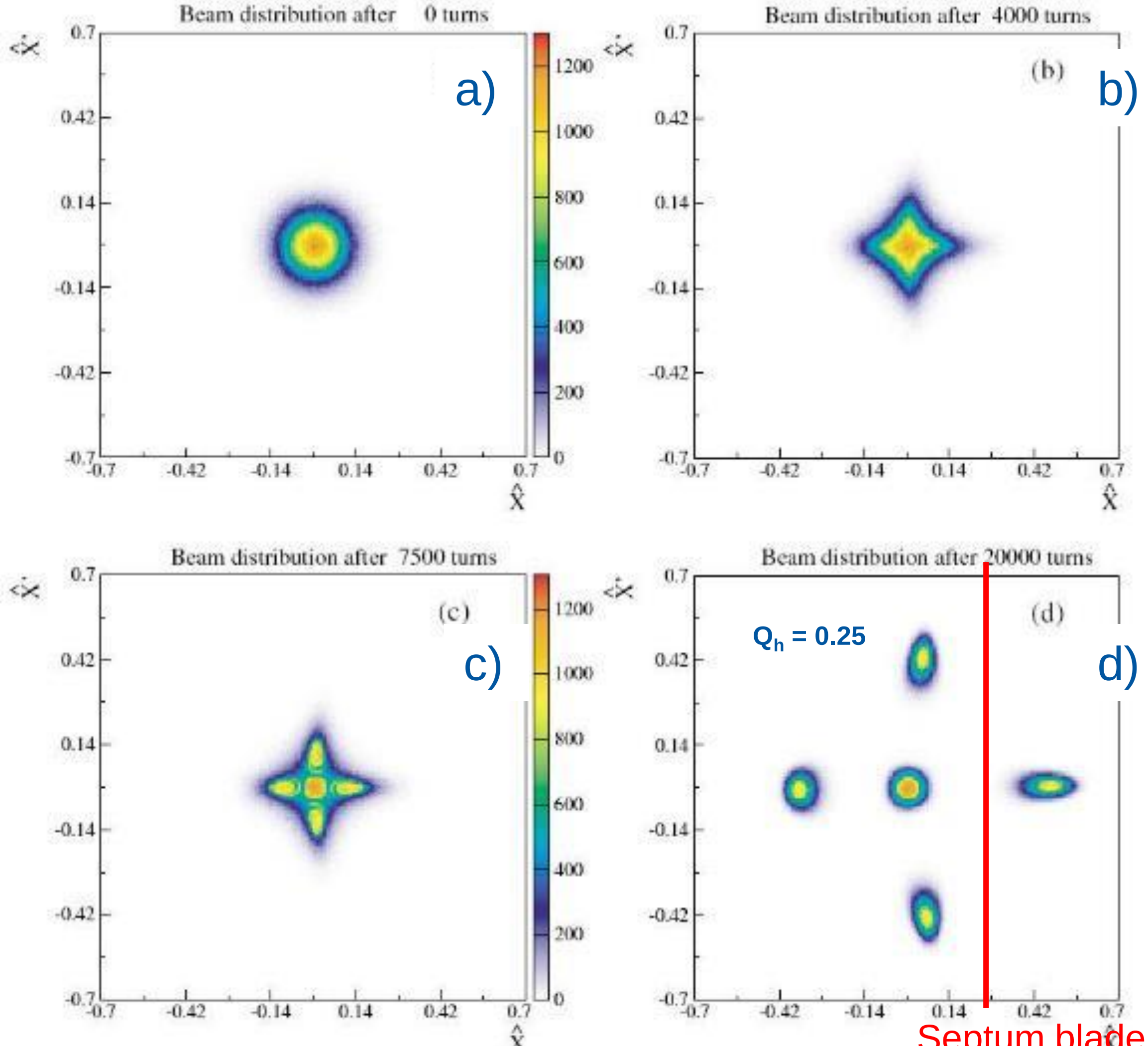
Resonant multi-turn extraction

Resonant low-loss multi-turn extraction

- **Adiabatic capture** of beam in stable "islands"
  - **Use non-linear fields (sextupoles and octupoles) to create islands** of stability in phase space
  - A **slow** (adiabatic) **tune variation to cross a resonance** and to drive particles into the islands (capture) with the help of transverse excitation (using damper)
  - **Variation of field strengths** to separate the islands in phase space

- Several big advantages:
  - **Losses reduced significantly** (no particles at the septum in transverse plane)
  - **Phase space matching improved with respect to existing non-resonant multi-turn extraction** - 'beamlets' have similar emittance and optical parameters

Resonant low-loss multi-turn extraction

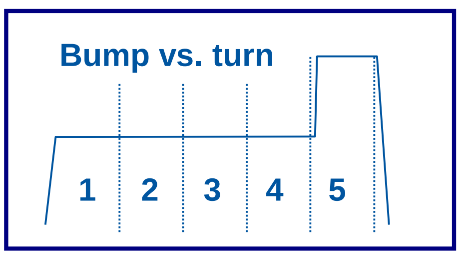


a) Unperturbed beam

b) Increasing non-linear fields

c) Beam captured in stable islands

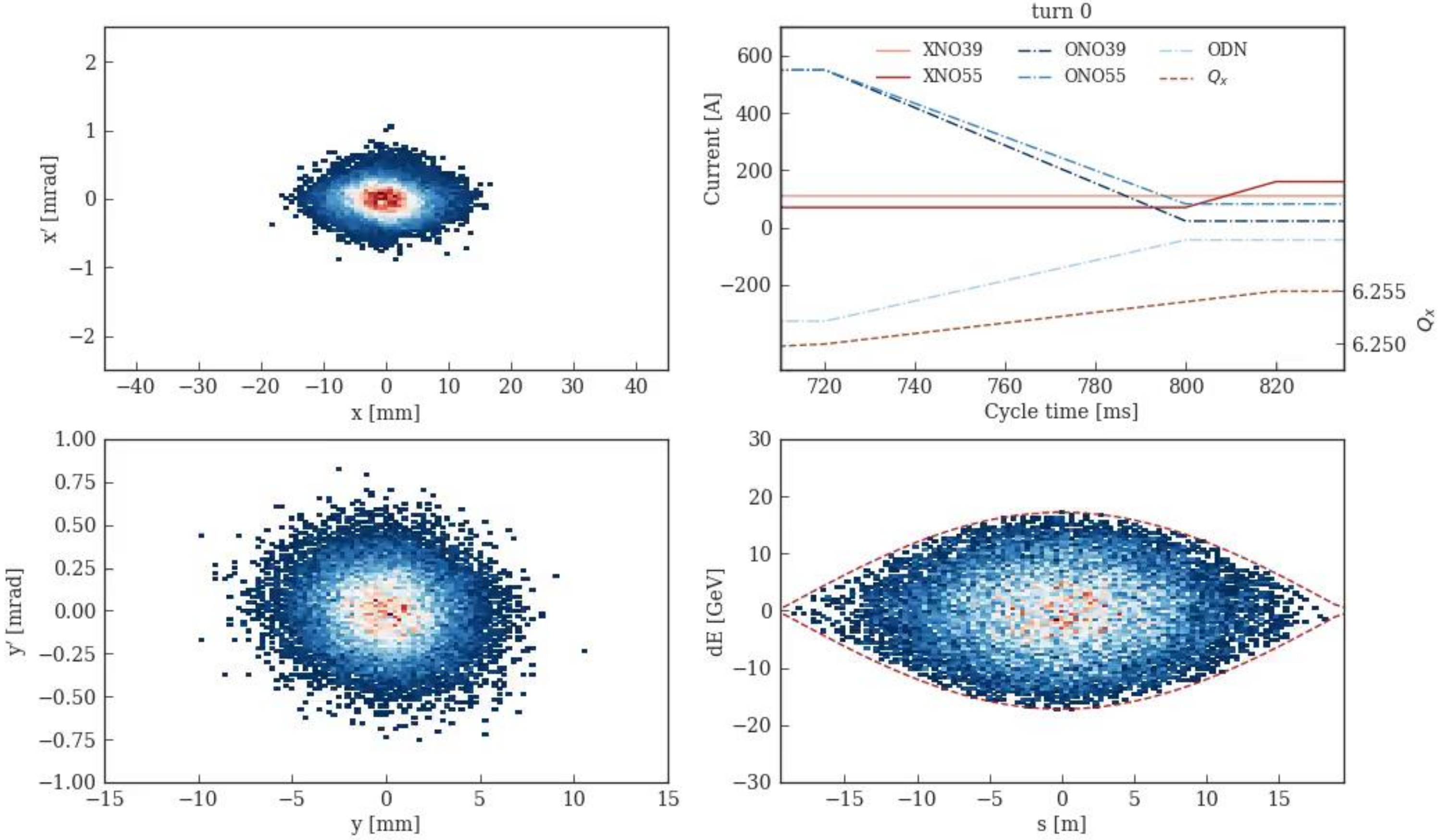
d) Islands separated and beam bumped across septum – extracted in 5 turns



Courtesy M. Giovannozzi: MTE Design Report, CERN-2006-011, 2006

Resonant low-loss multi-turn extraction

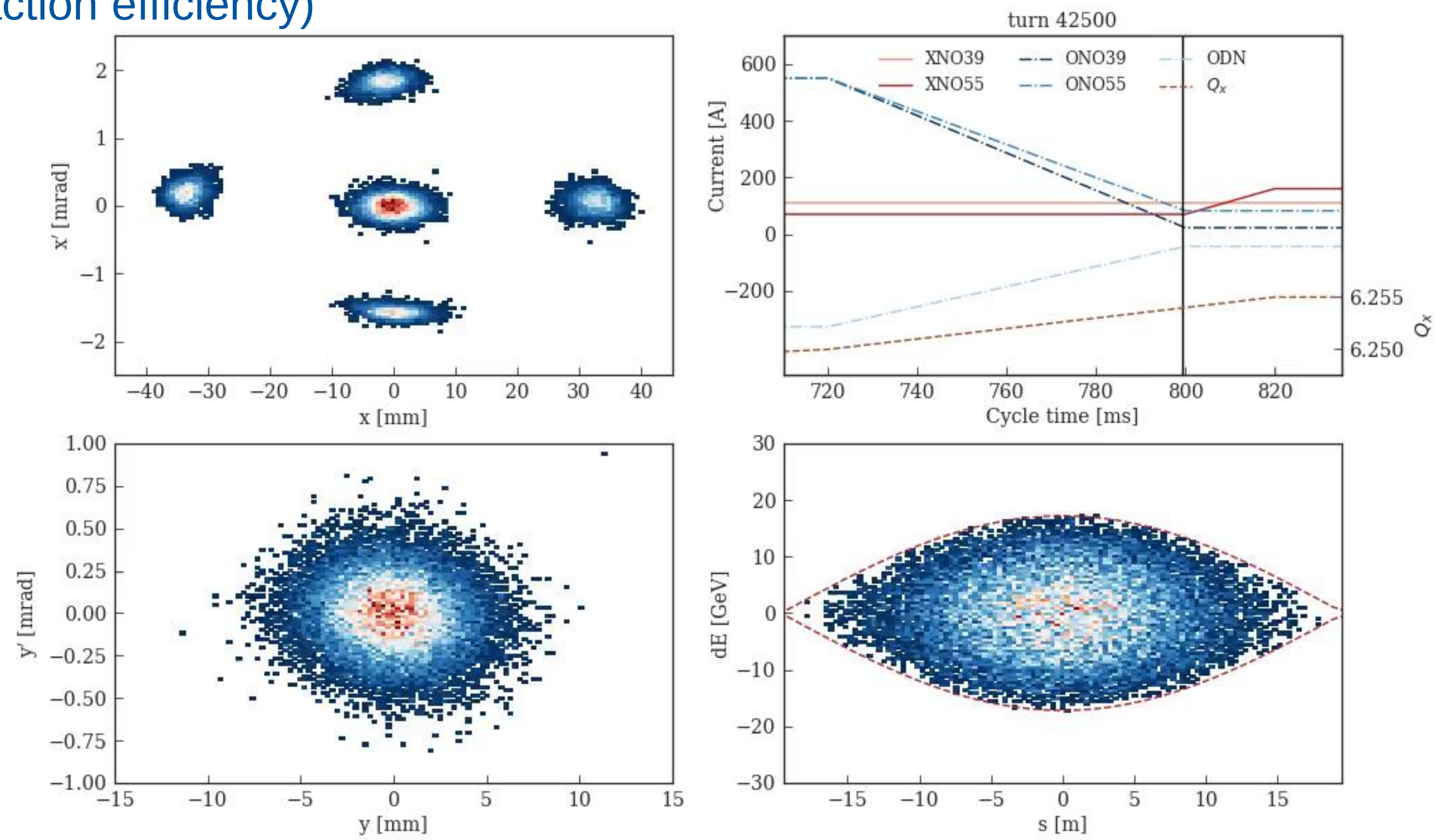
Time-dependent 6D simulations to investigate dynamics during splitting



Courtesy A. Huschauer

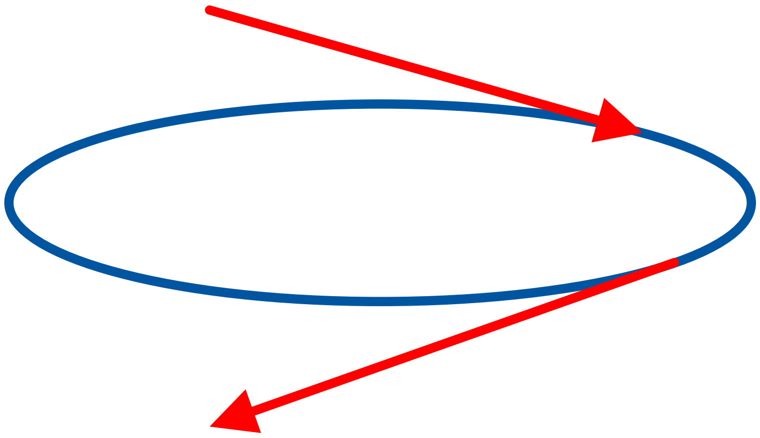
Resonant low-loss multi-turn extraction

Phase rotation and non-linear optics change prior to extraction (to optimise extraction efficiency)



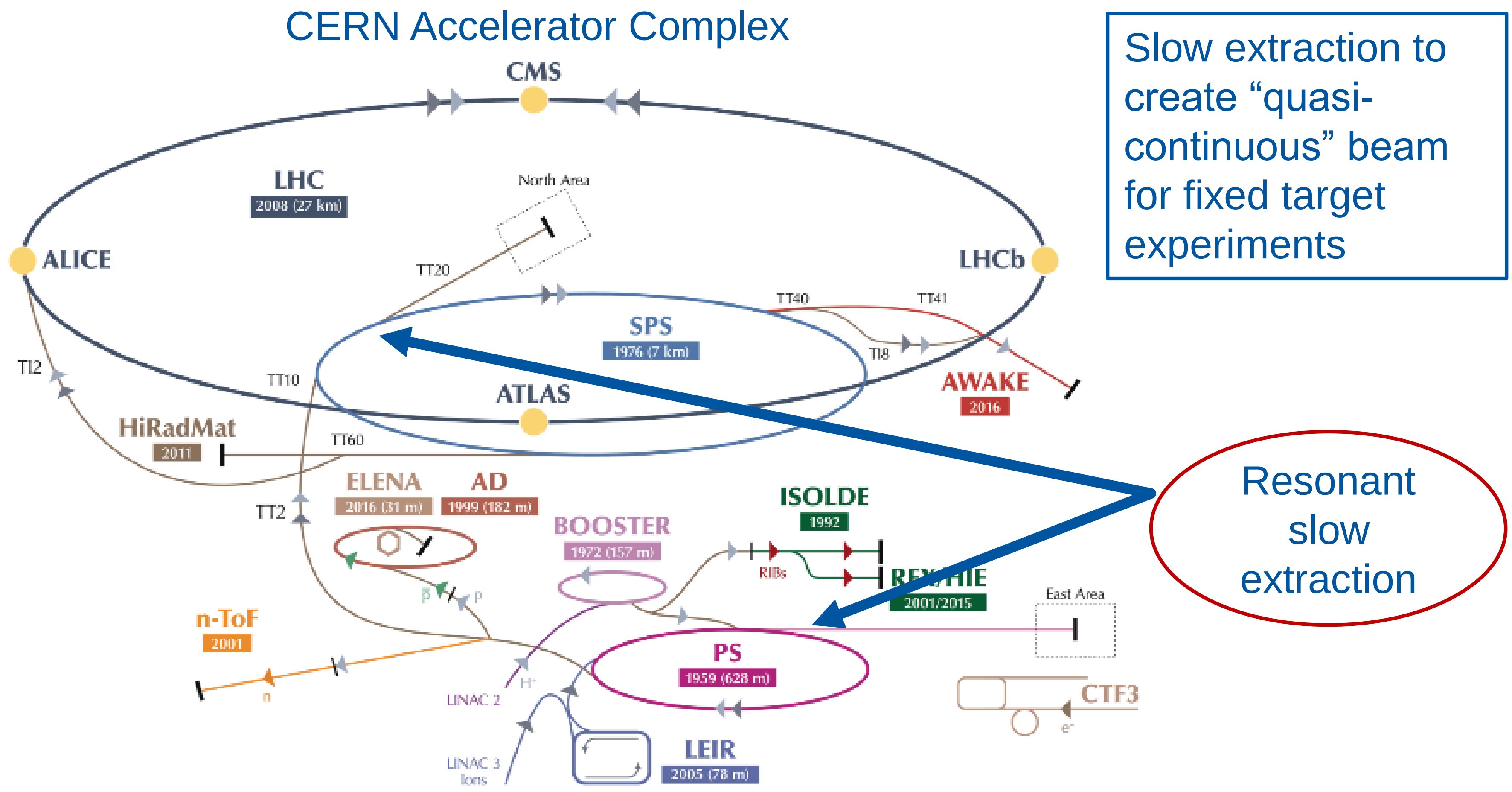
Courtesy A. Huschauer

Overview



- Introduction
- Single-turn methods
  - Injection
  - Fast extraction
- Multi-turn methods
  - Multi-turn hadron injection
  - Charge-exchange H- injection
  - Multi-turn extraction
- **Resonant slow extraction**
- Summary

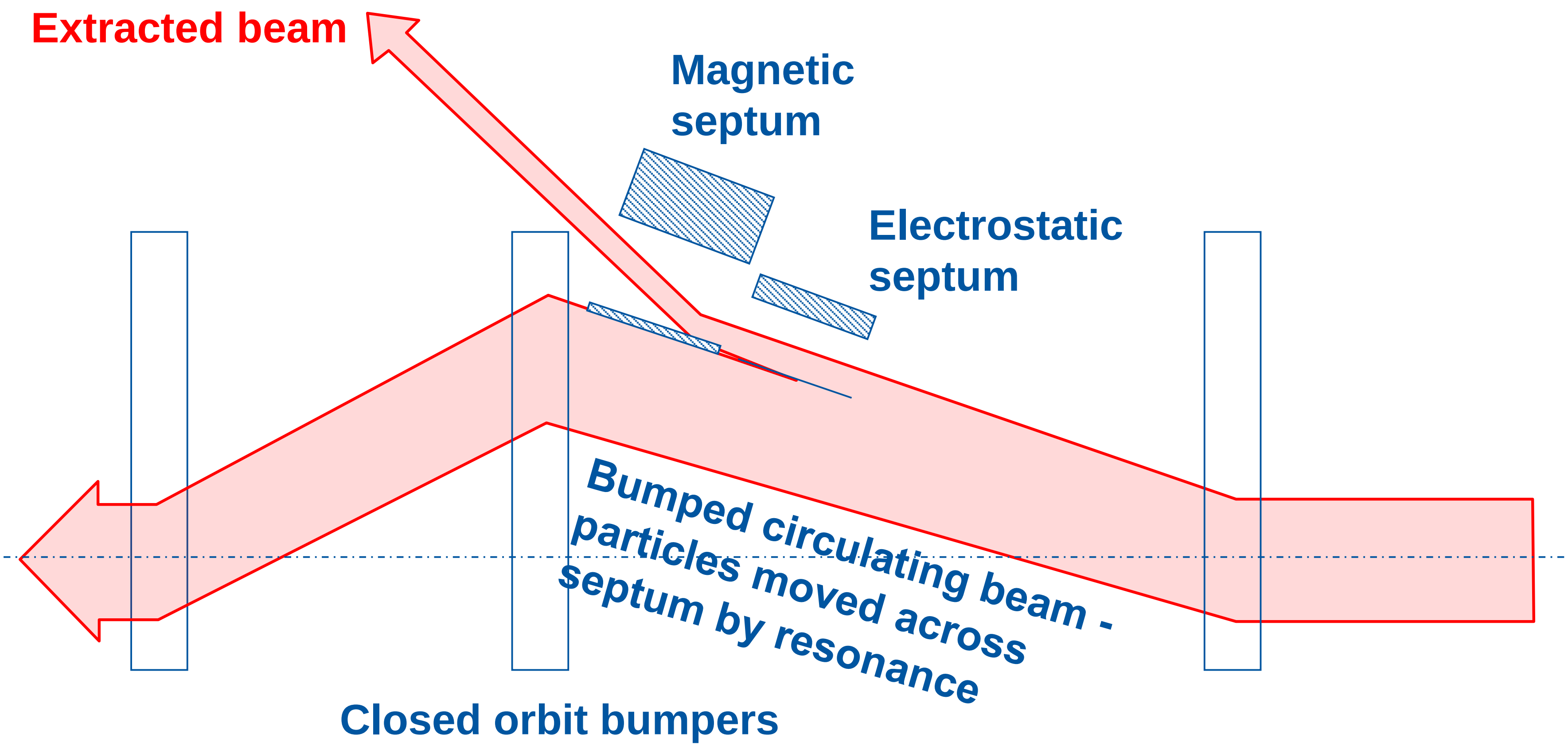
Resonant slow extraction



Slow extraction to create “quasi-continuous” beam for fixed target experiments

Resonant slow extraction

Non-linear fields excite resonances that drive the beam slowly across the septum



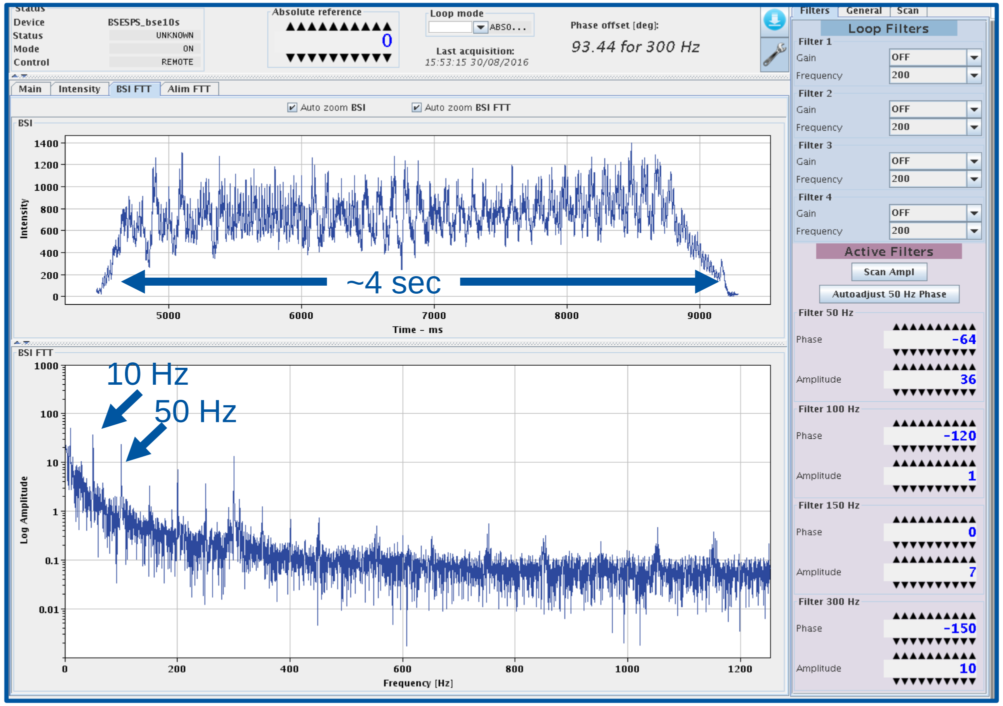
- Slow bumpers move the beam near the septum
- Tune adjusted close to nth order betatron resonance
- Multipole magnets excited to define stable area in phase space, size depends on $\Delta Q = Q - Q_r$

3rd order resonant slow extraction

- Sextupoles are used to excite a resonance for extraction

- This resonance slowly drives particles over septum for extraction (> 1000 turns)

- Results in long spills for experiments (milliseconds to hours)



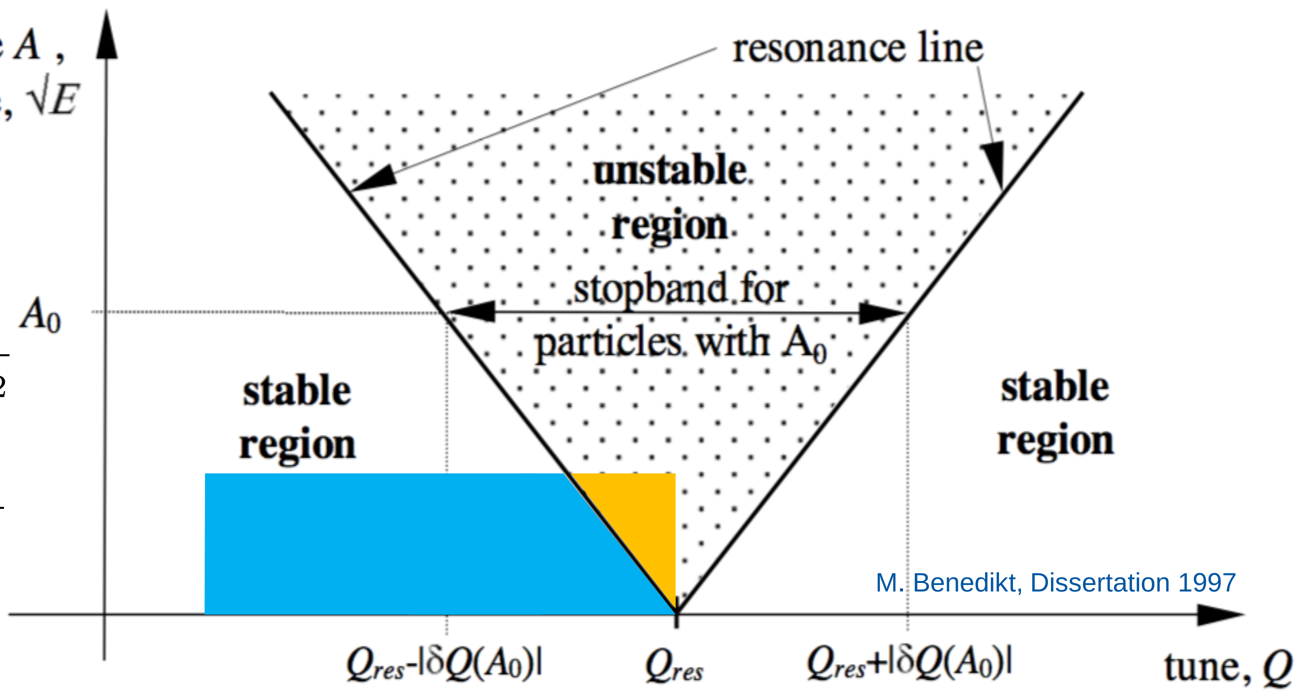
Example of a spill at SPS to the North Area with large n x 50 Hz components and another noise source at 10 Hz

Extraction process

- Increase the sextupole strength to excite resonance
- Large tune spread created with RF gymnastics (large momentum spread) and large chromaticity to reduce stable region in phase space
- Move beam into the resonance by changing the tune

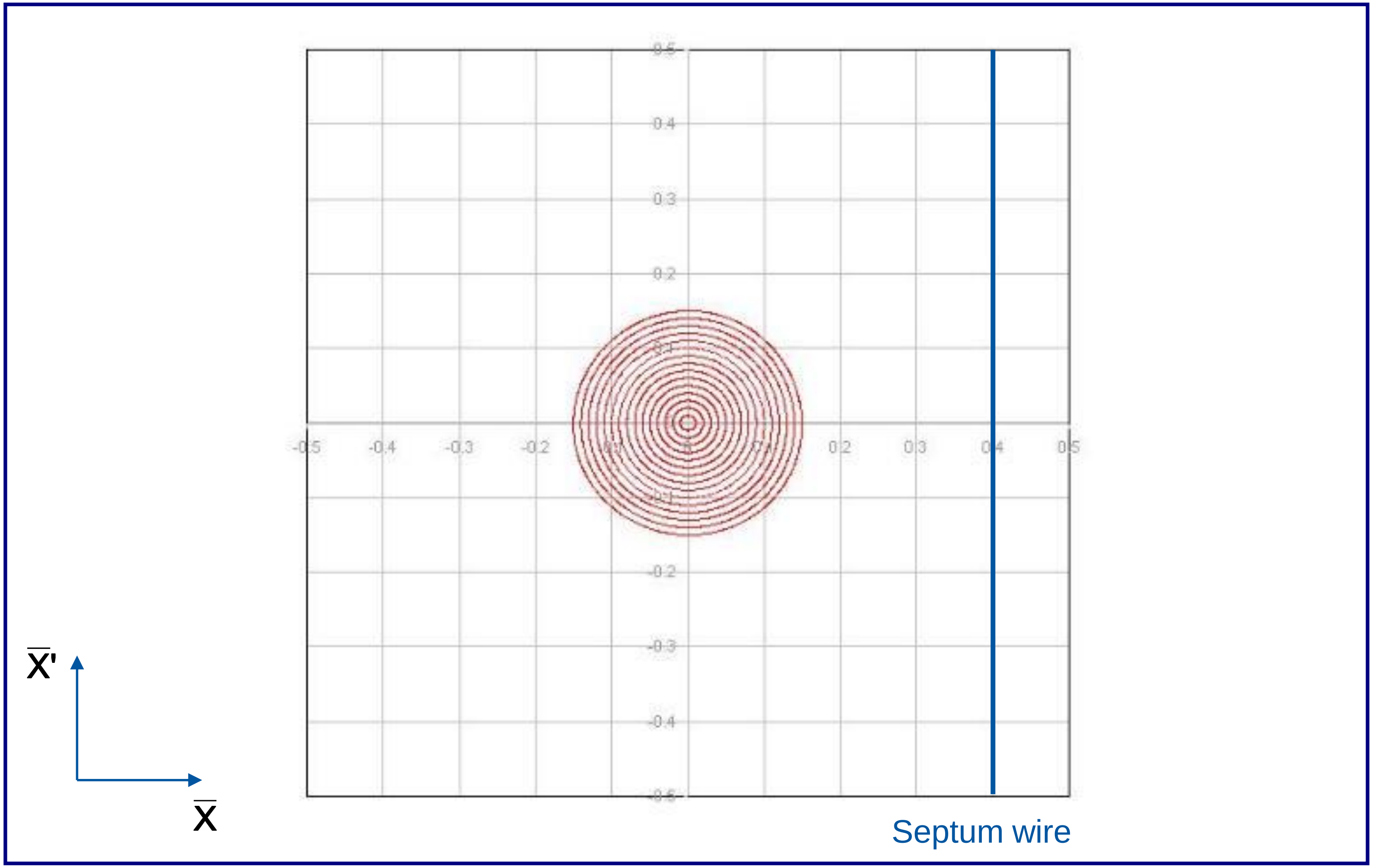
$$A = \sqrt{X_0^2 + X_0'^2}$$

$$A_{\text{resonance line}} \propto \frac{\delta Q}{k_{Sextupole}}$$



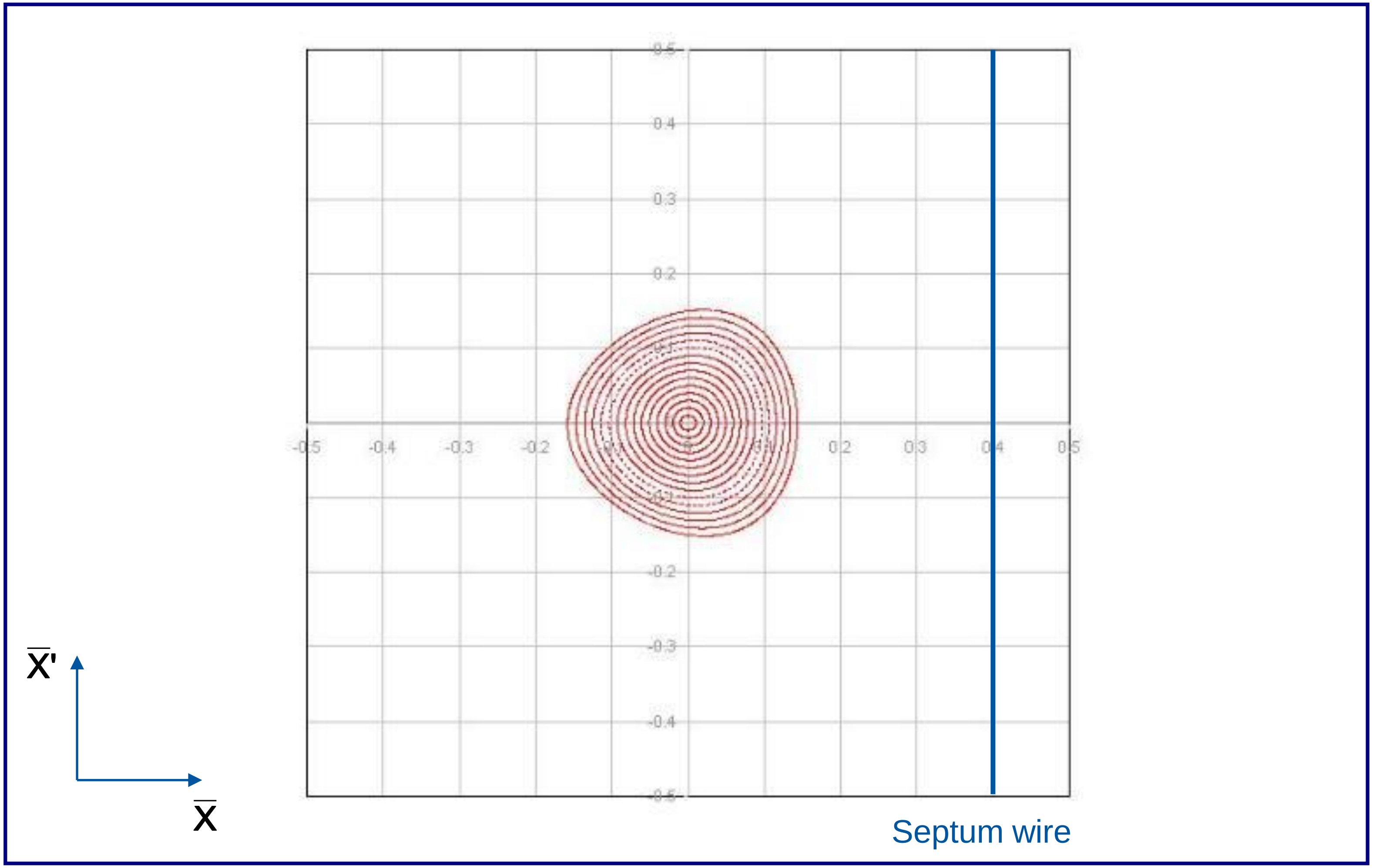
M. Benedikt, Dissertation 1997

3^{rd} order resonant slow extraction



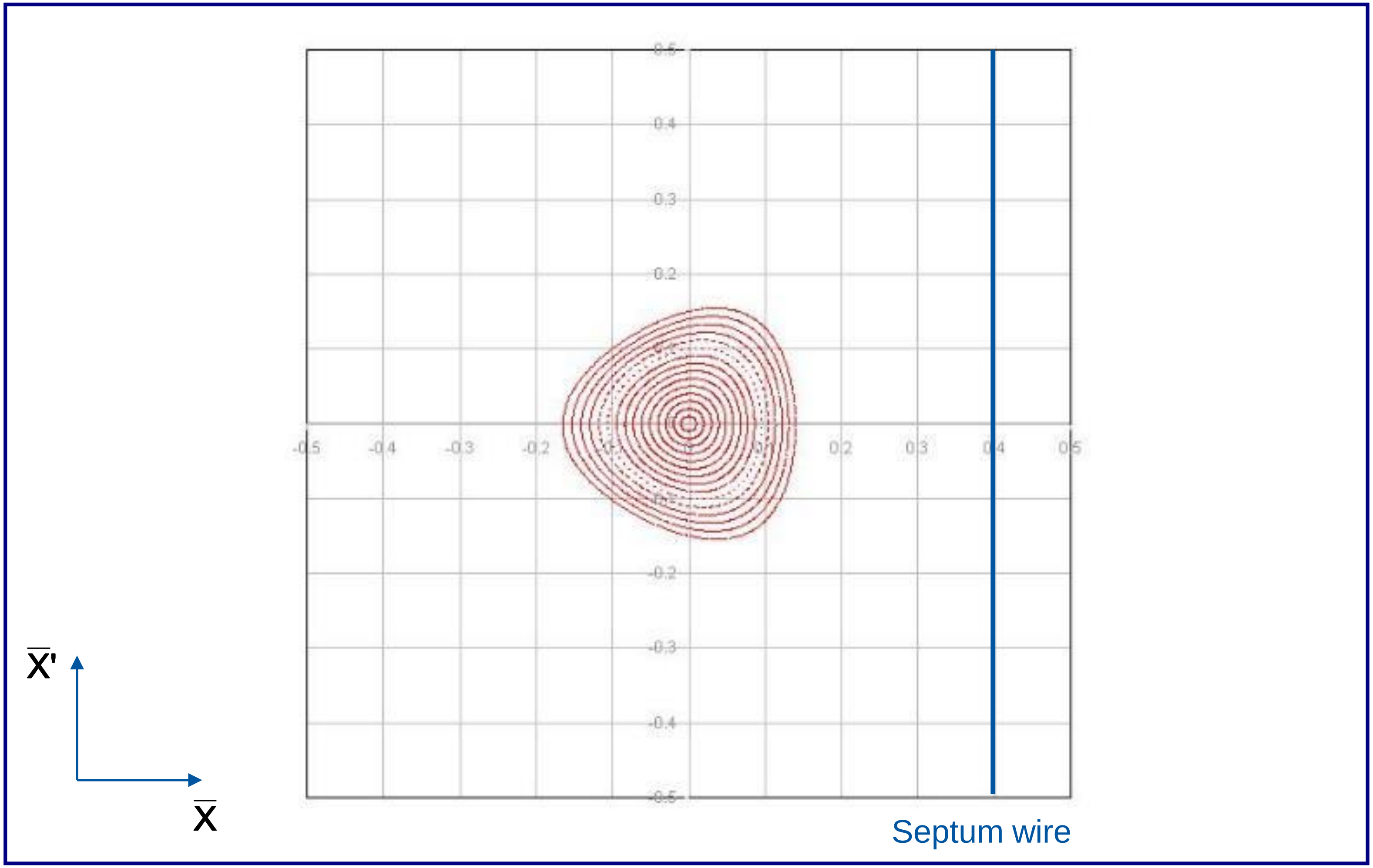
- Particles distributed on emittance contours
- ΔQ large – no phase space distortion

3rd order resonant slow extraction



- Sextupole magnets produce a triangular stable area in phase space
- ΔQ decreasing – phase space distortion for largest amplitudes

3rd order resonant slow extraction



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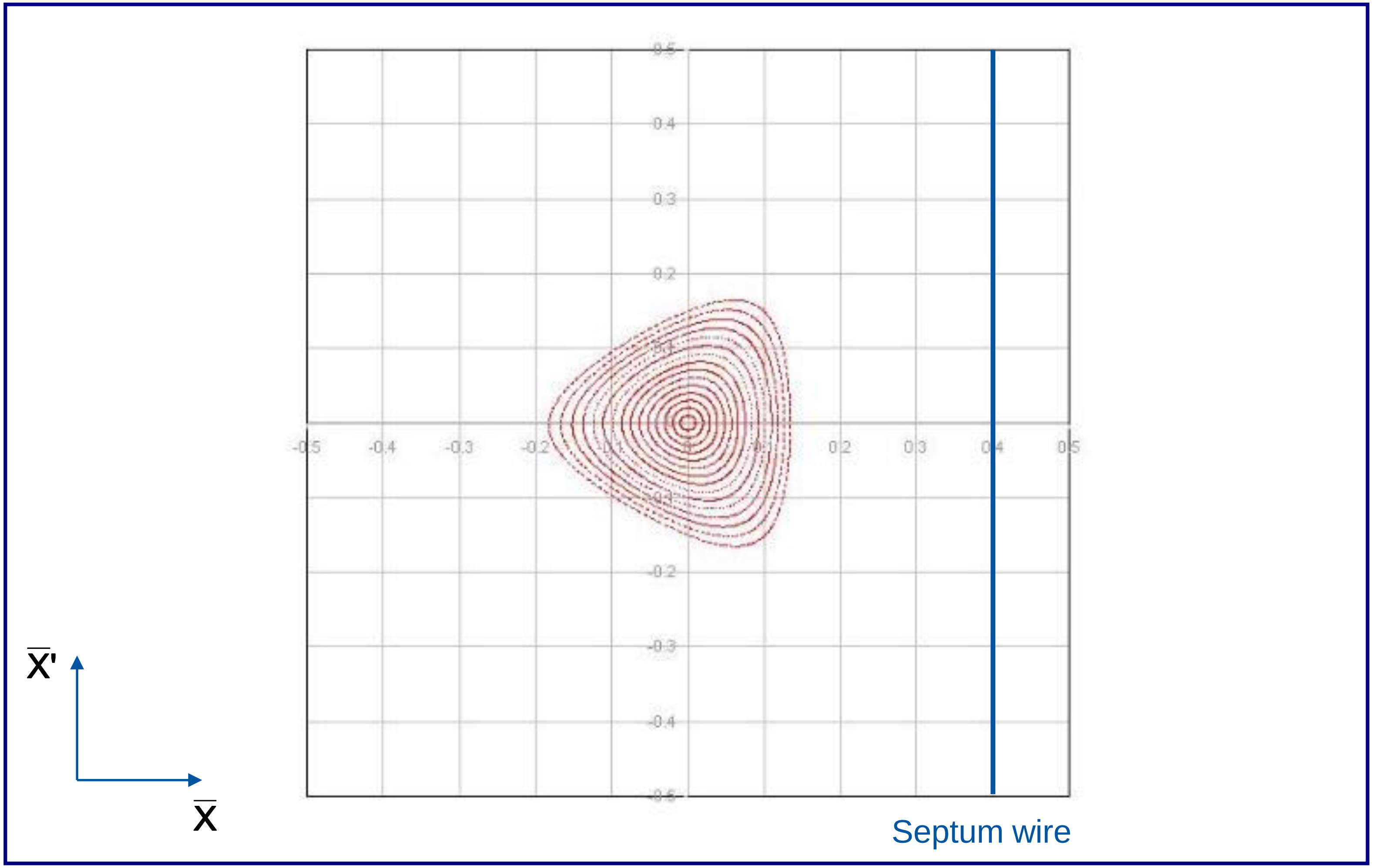
3rd order resonant slow extraction



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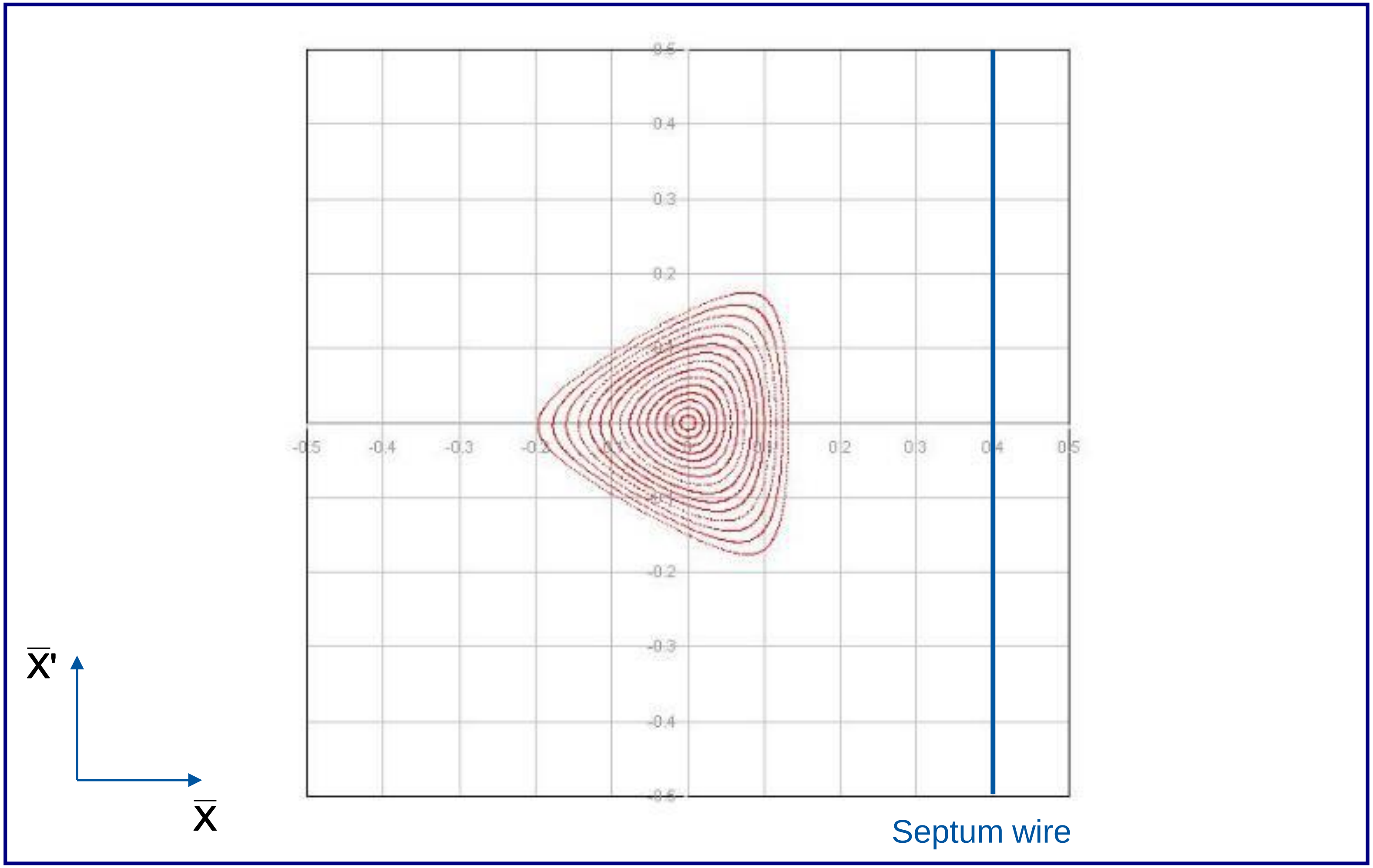

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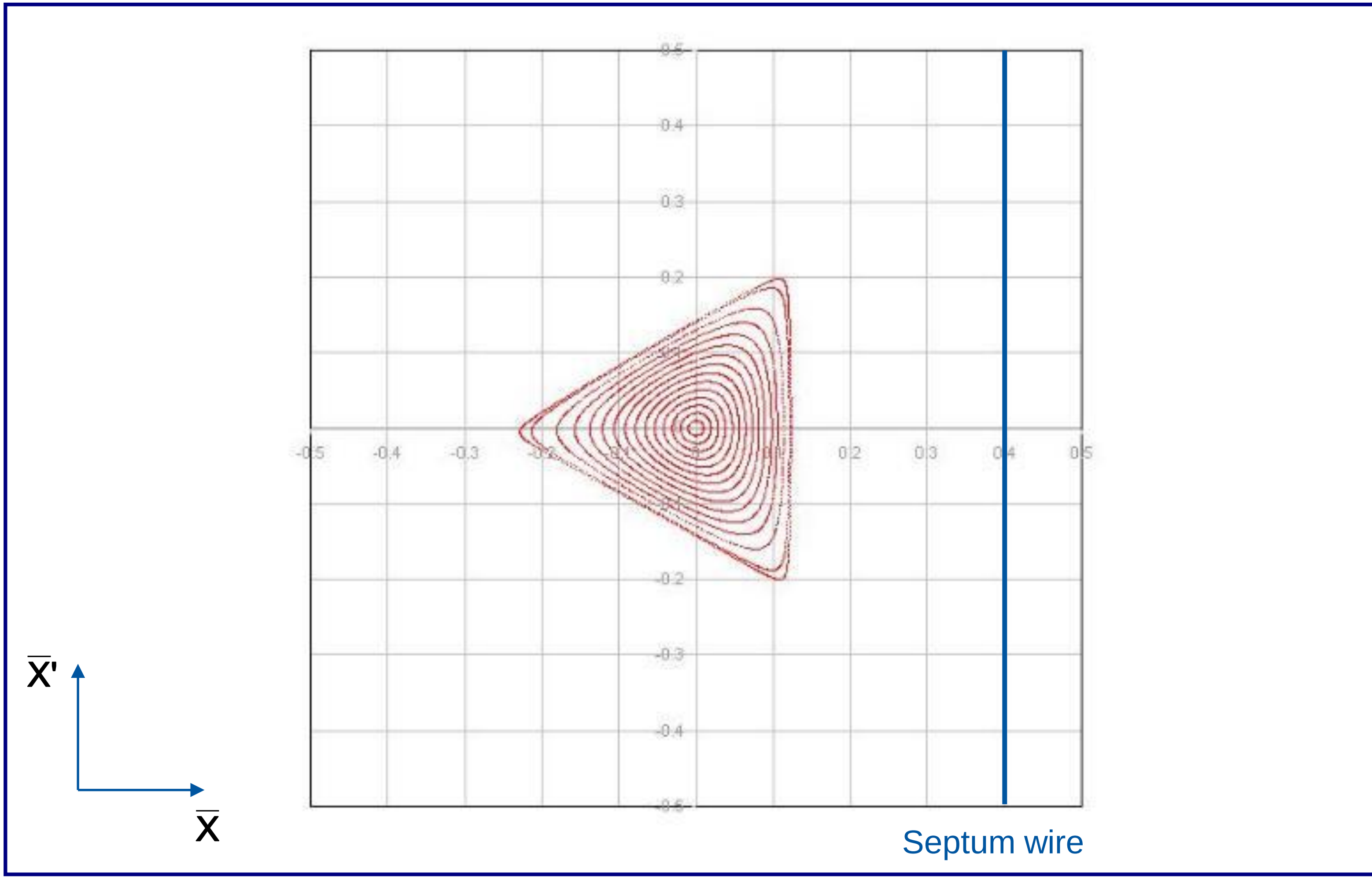
3rd-order resonant slow extraction



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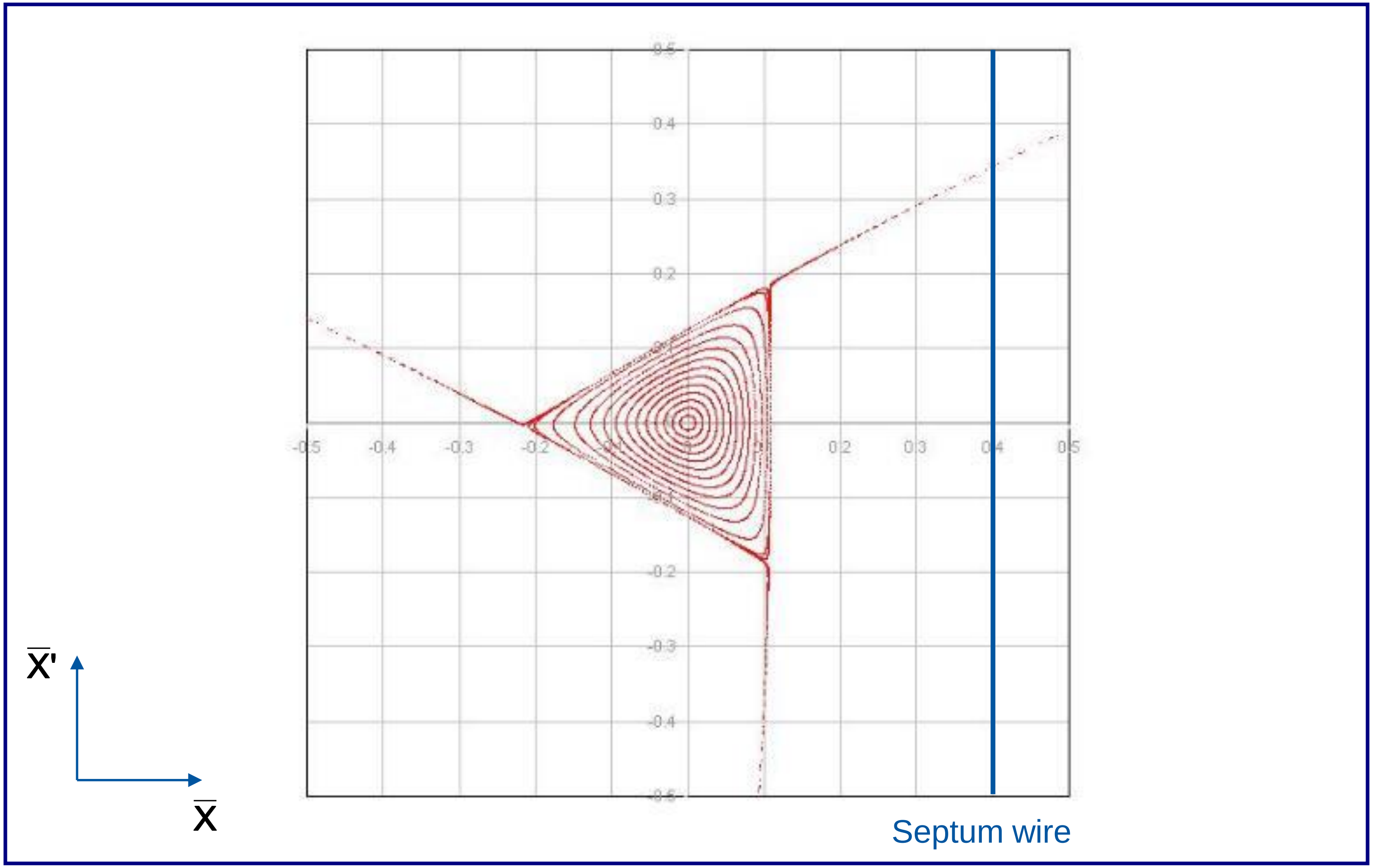
3rd-order resonant slow extraction



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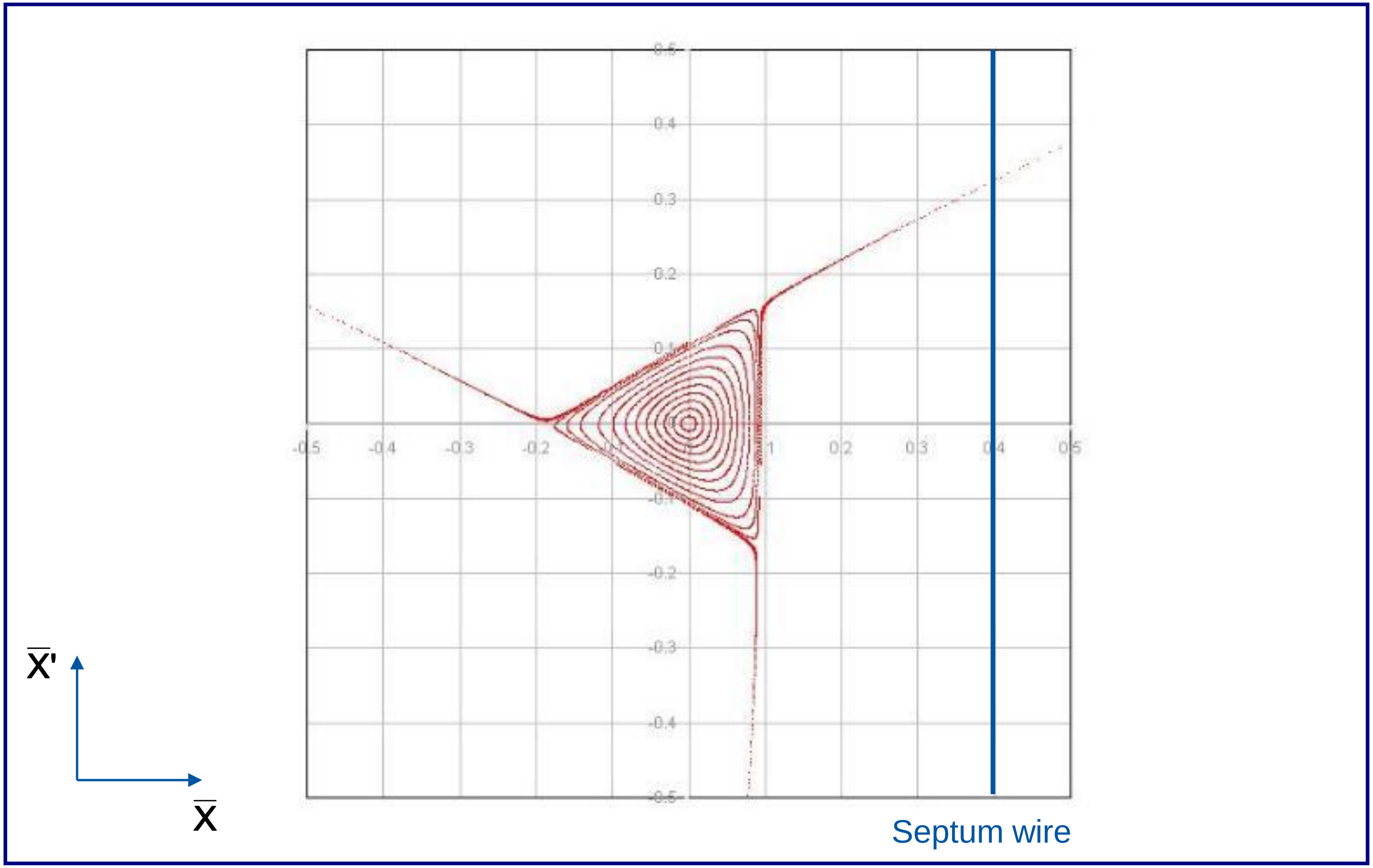
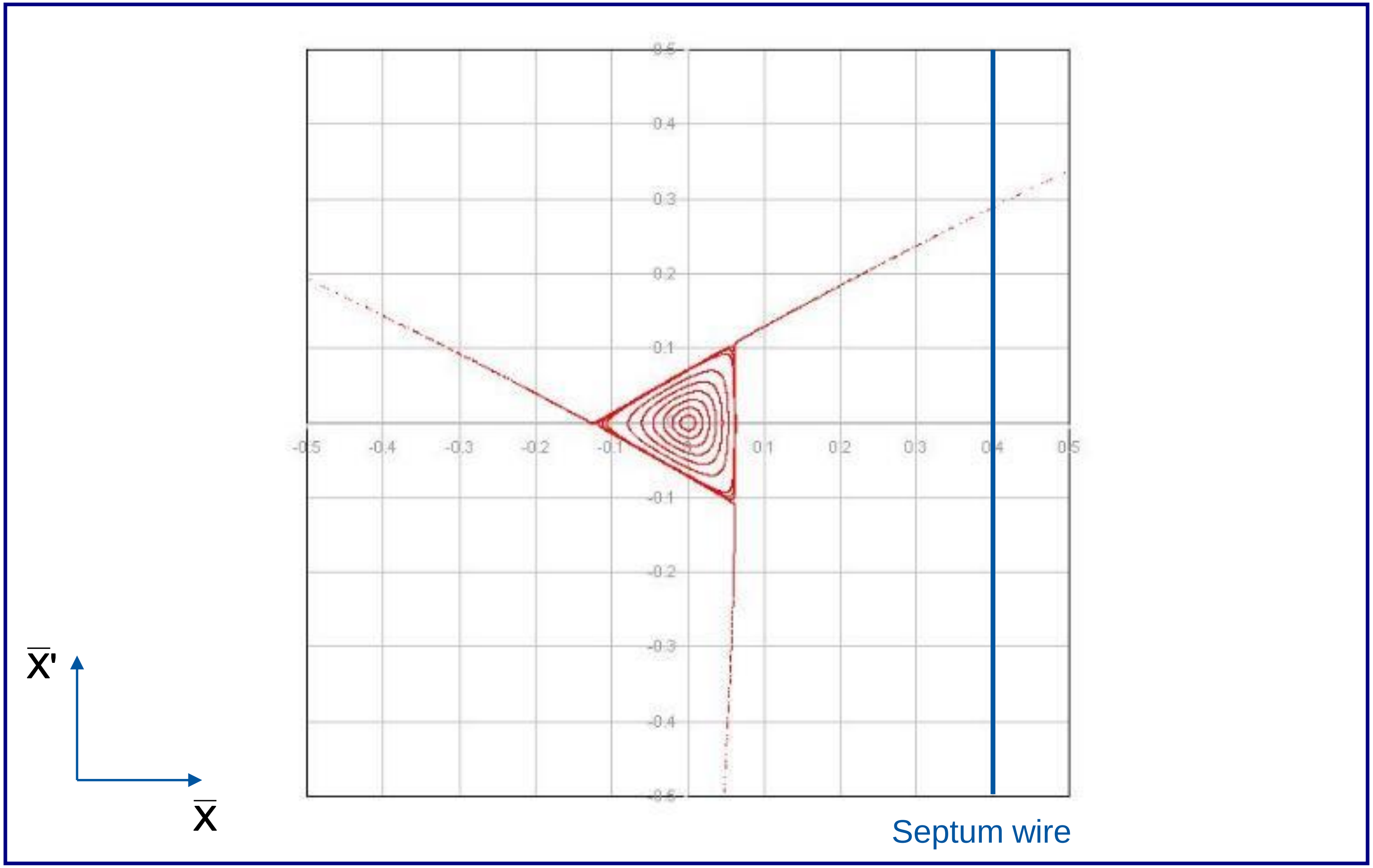
3rd-order resonant slow extraction



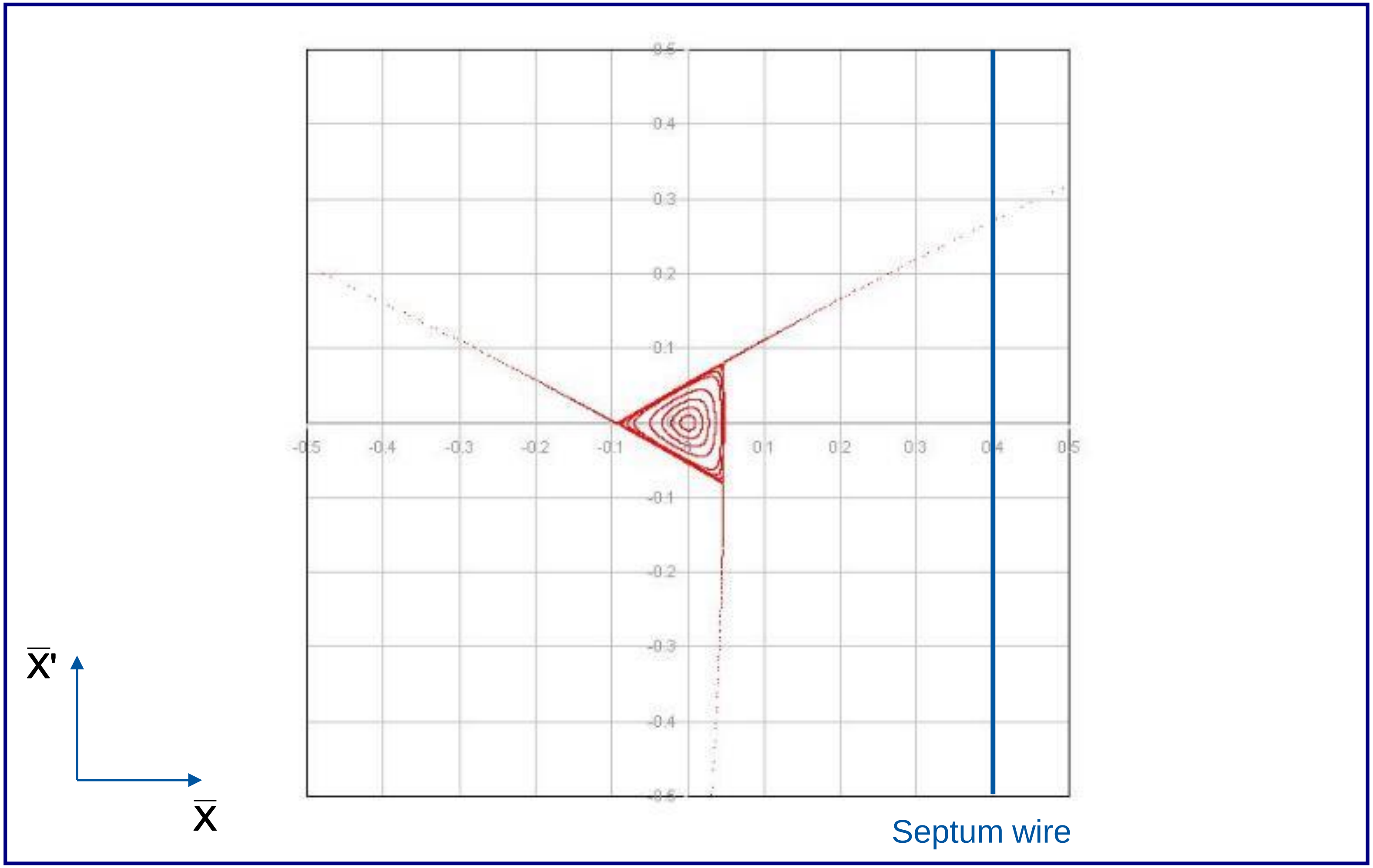
- Stable area shrinks as ΔQ becomes smaller

3rd order resonant slow extraction



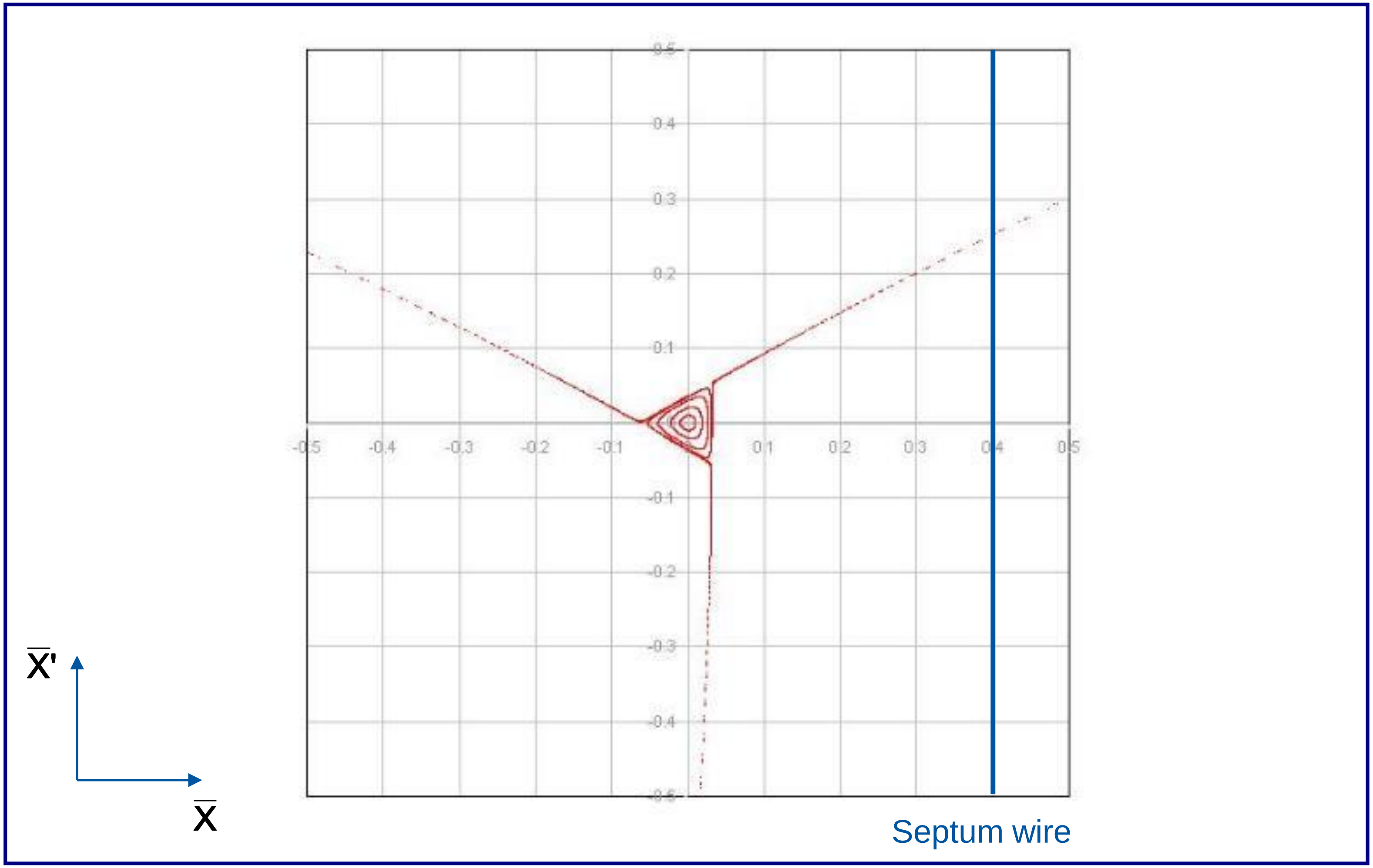
- Separatrix position in phase space shifts as the stable area shrinks

3rd order resonant slow extraction



- As the stable area shrinks, the circulating beam intensity drops since particles are being continuously extracted

3rd order resonant slow extraction



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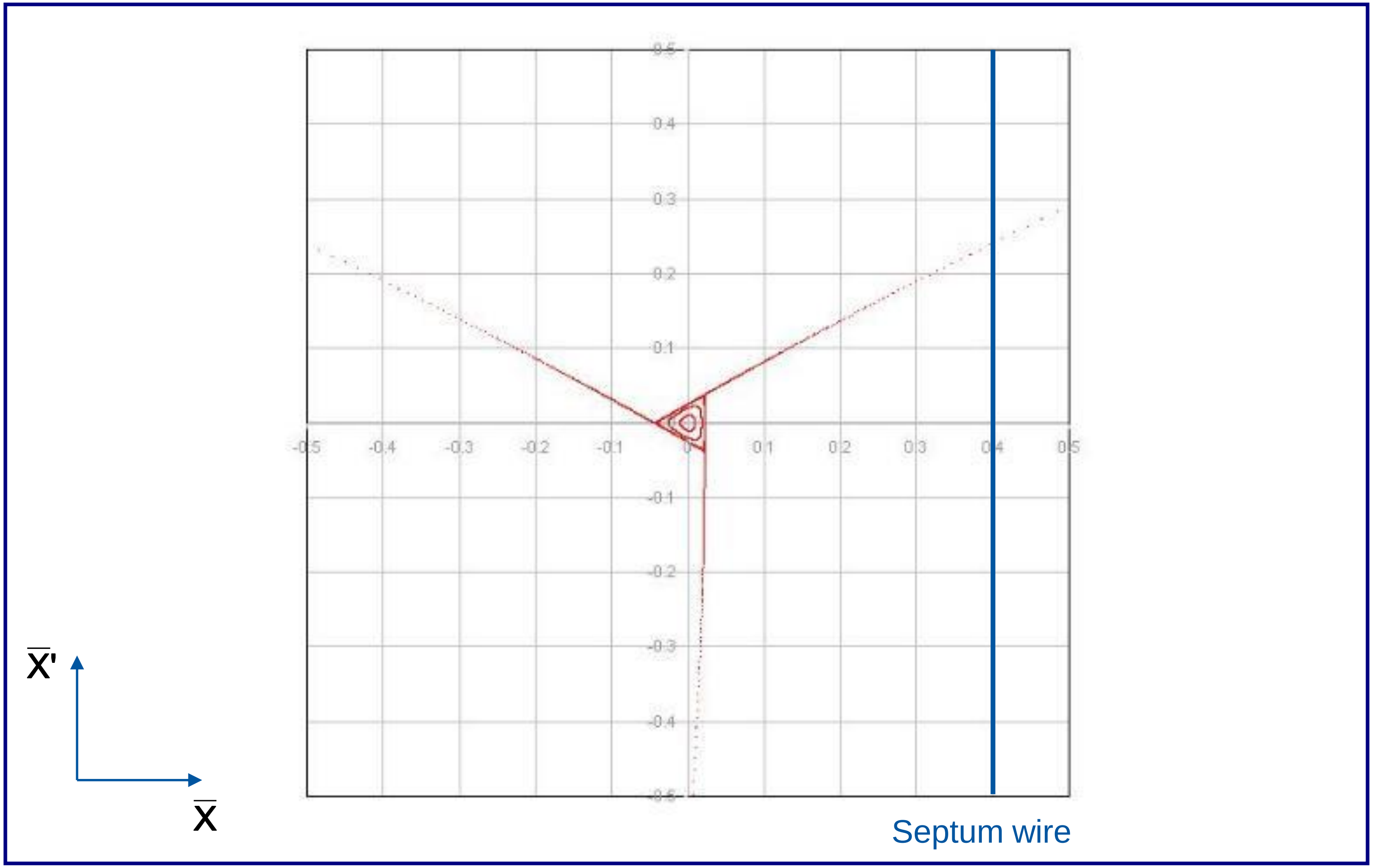
3rd order resonant slow extraction



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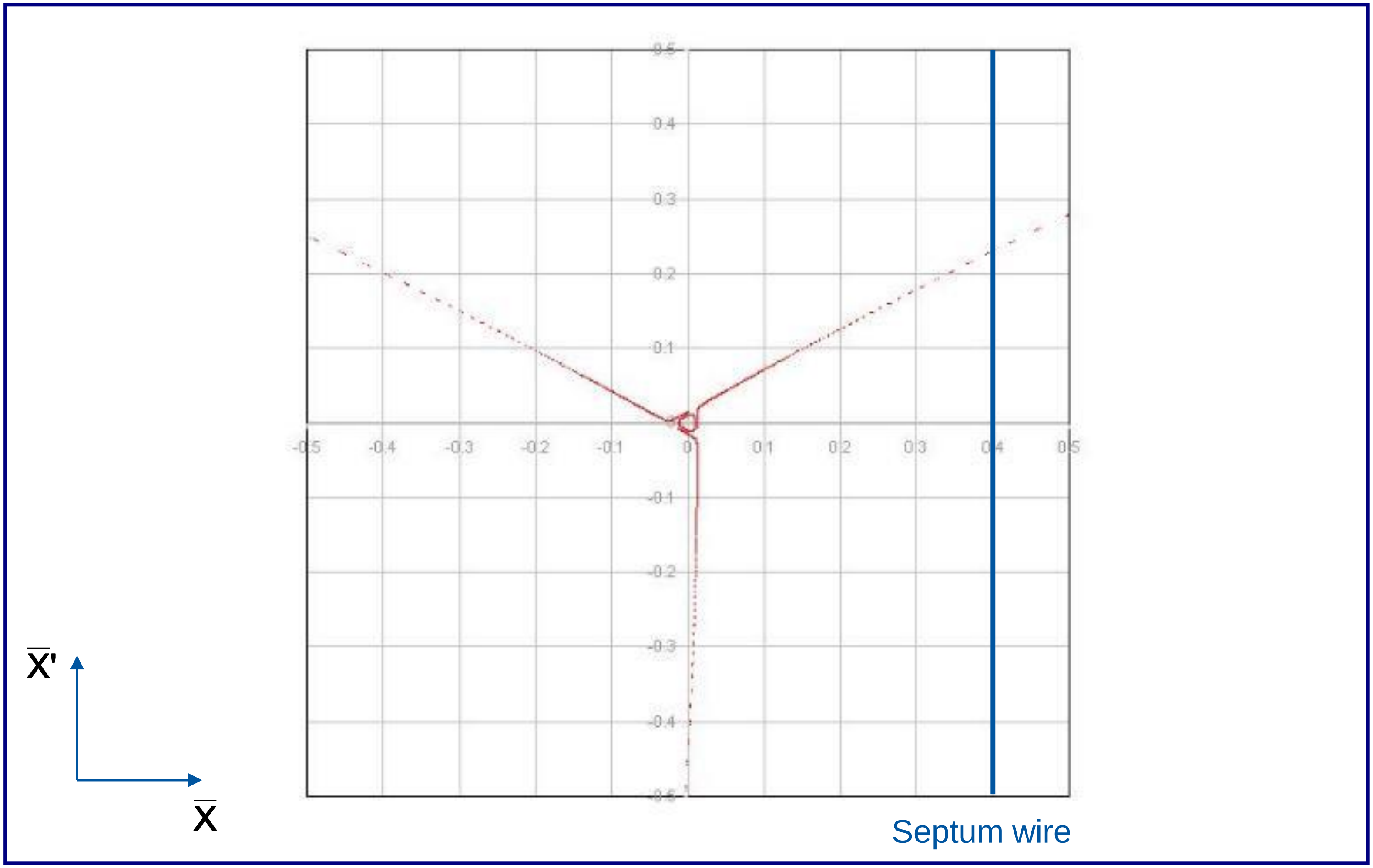
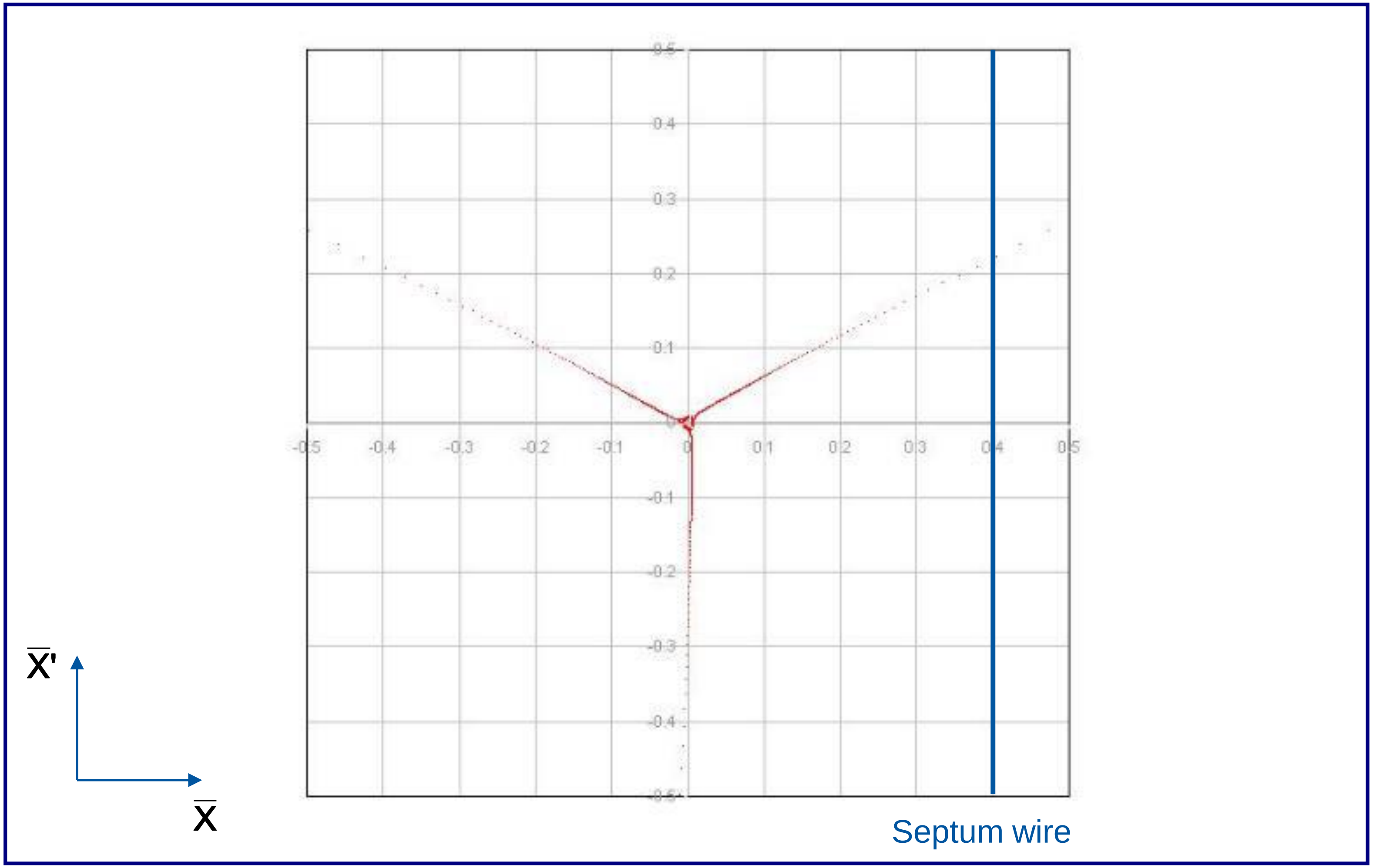
3rd order resonant slow extraction



- As the stable area shrinks, the circulating beam intensity drops since particles are being continuously extracted

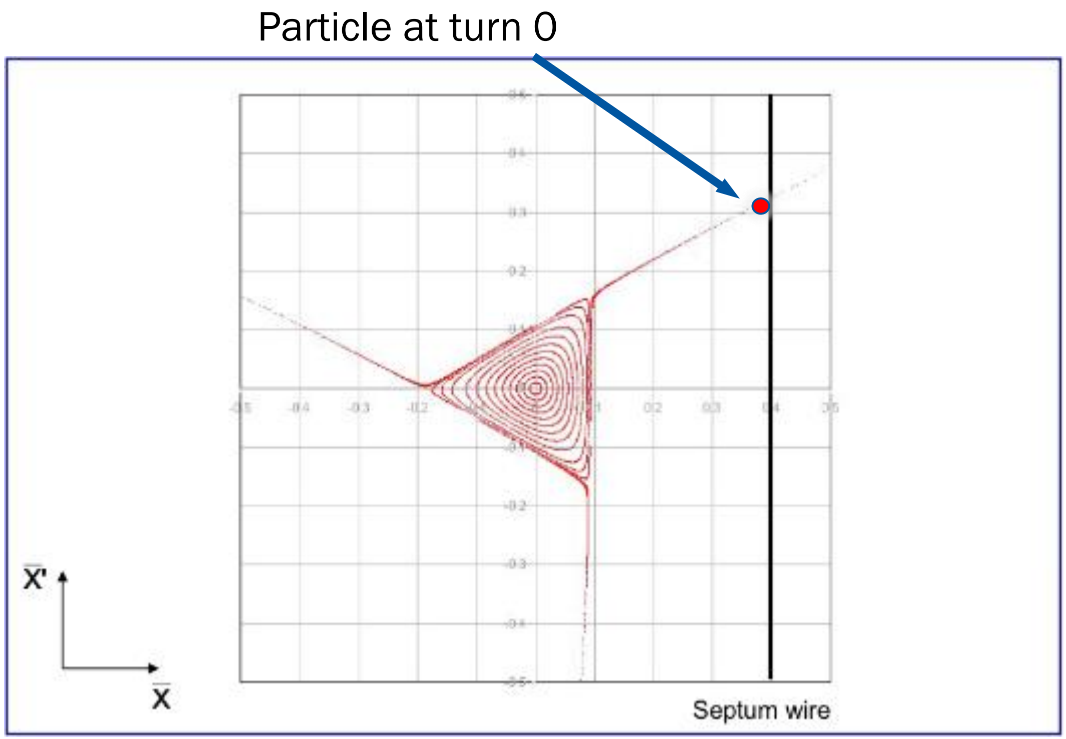
3rd order resonant slow extraction



- As ΔQ approaches zero, the particles with very small amplitude are extracted

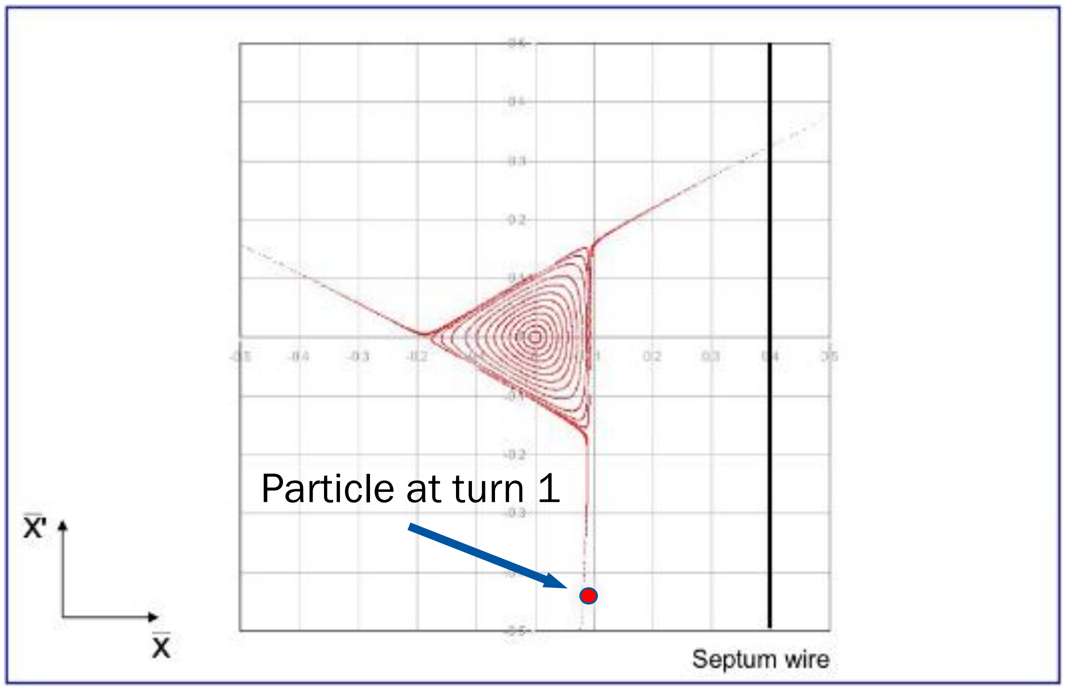
3rd order resonant slow extraction

- On resonance, sextupole kicks add-up driving particles over septum



3^{rd} order resonant slow extraction

- On resonance, sextupole kicks add-up driving particles over septum



3rd order resonant slow extraction

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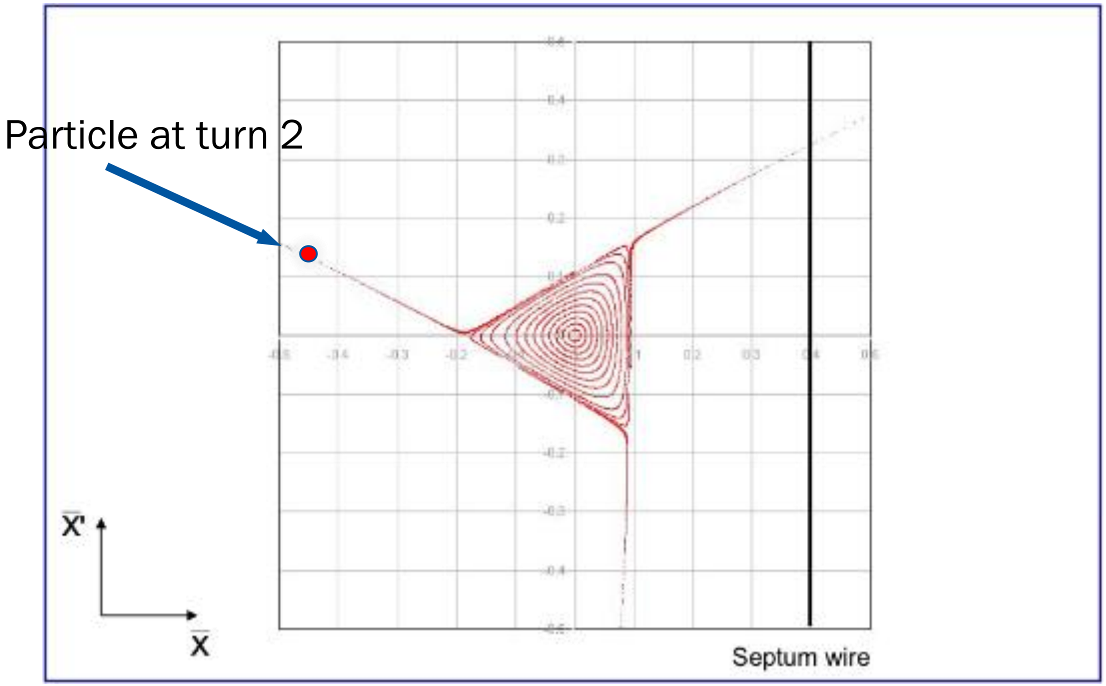
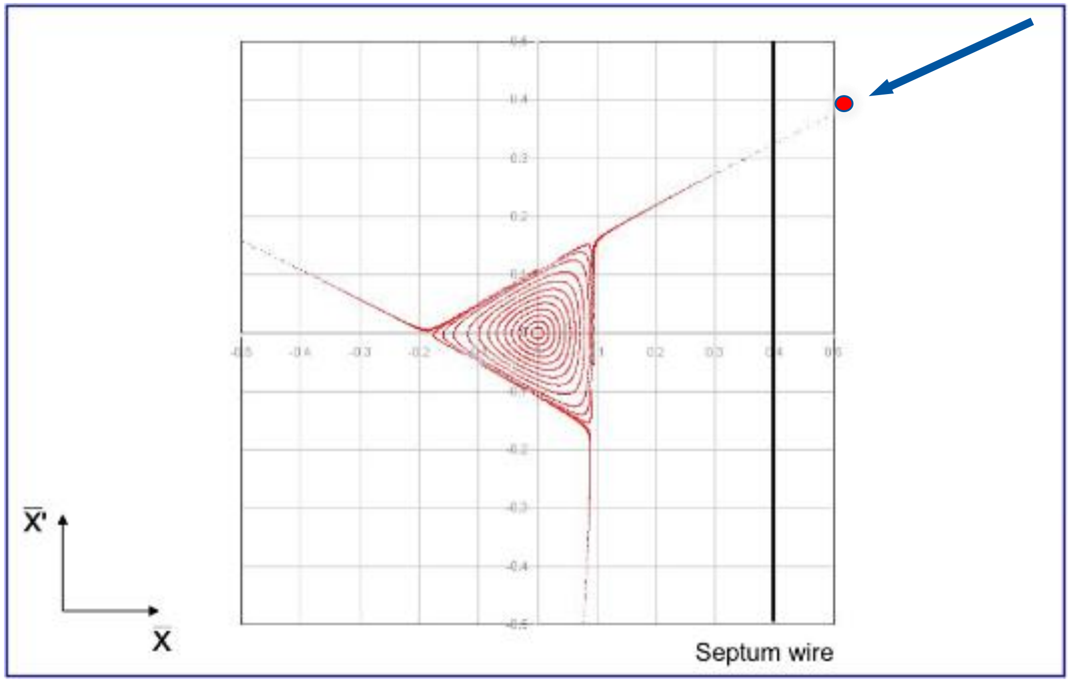


3^{rd} order resonant slow extraction

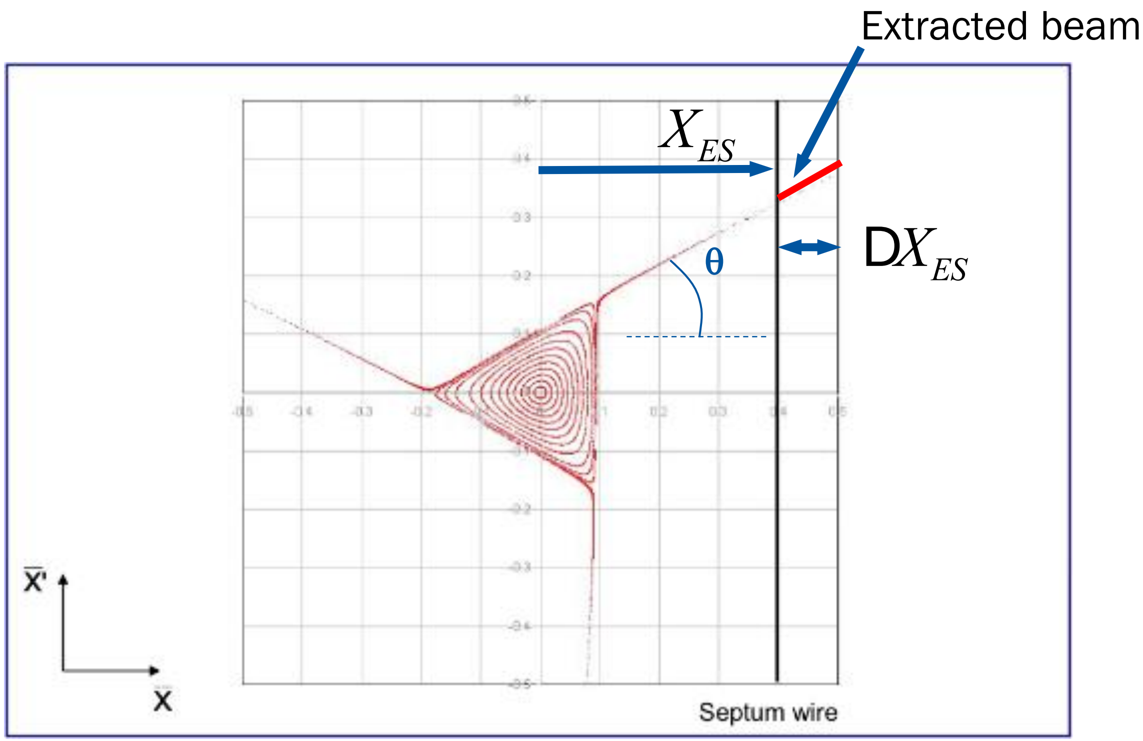
- On resonance, sextupole kicks add-up driving particles over septum

Particle at turn 3



3rd order resonant slow extraction

- On resonance, sextupole kicks add-up driving particles over septum



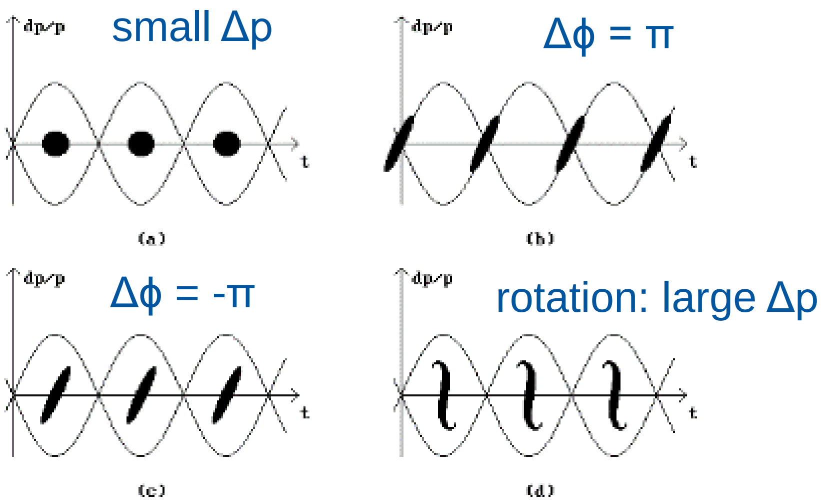
Distance travelled in these final three turns is termed the "spiral step," ΔX_{ES}

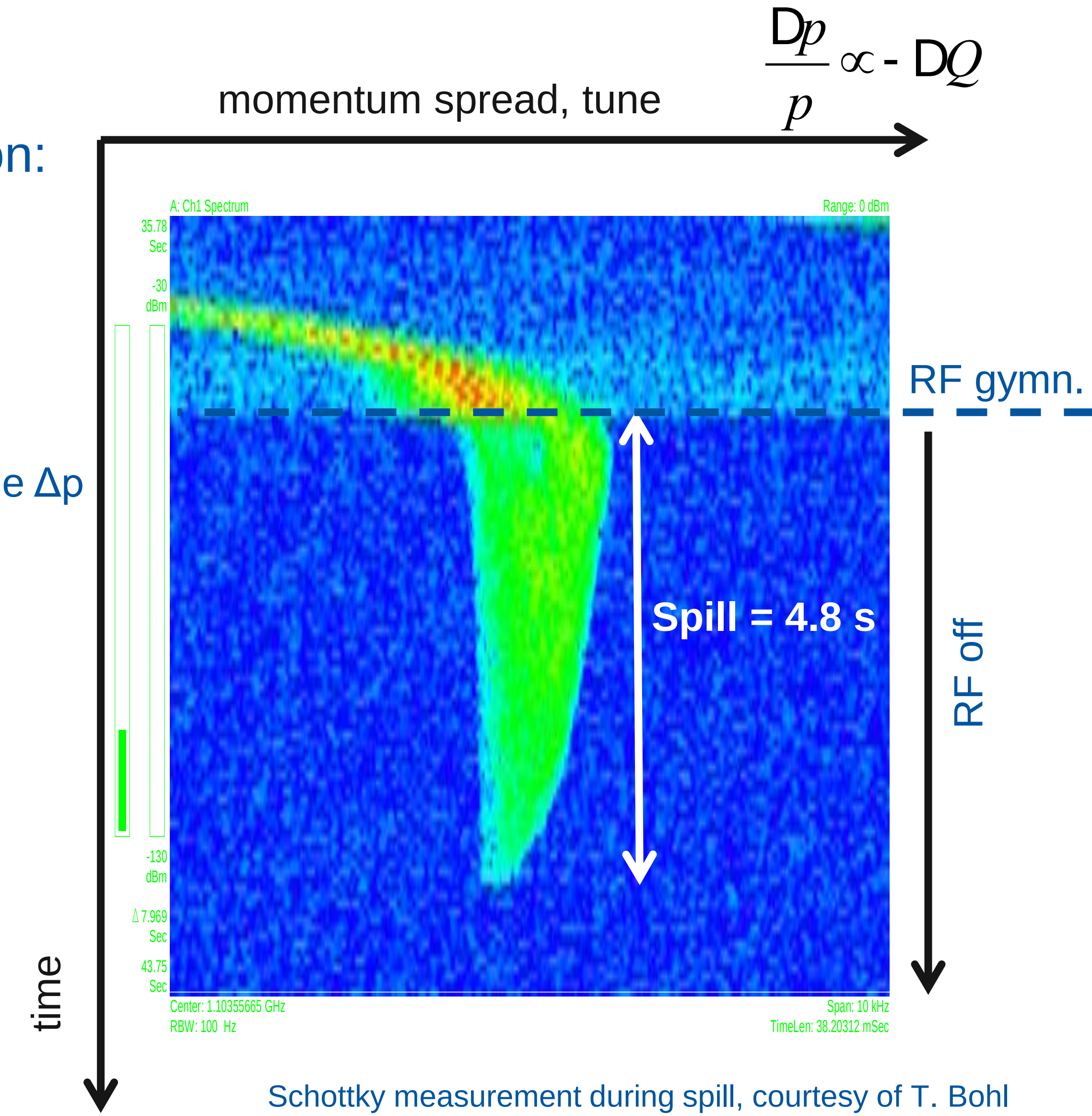
Extraction bump trimmed in the machine to adjust the spiral step

$$\Delta X_{ES} \propto \left|k_2\right| \frac{X_{ES}^2}{\cos\theta}$$

3rd order resonant slow extraction

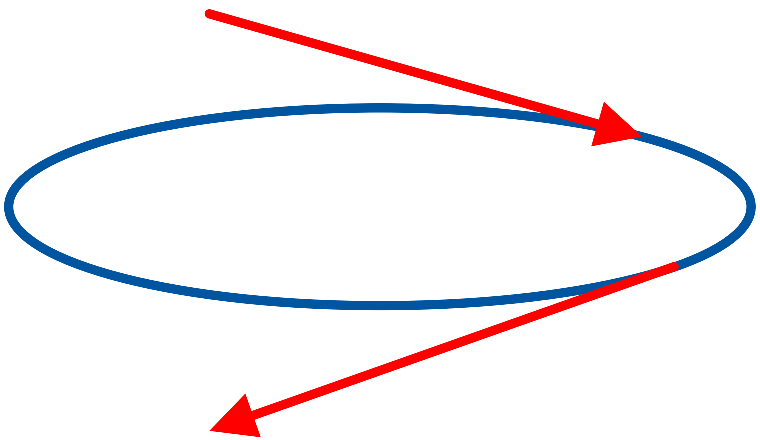
RF gymnastics before extraction:





Schottky measurement during spill, courtesy of T. Bohl

Outline



- Introduction
- Single-turn methods
  - Injection
  - Fast extraction
- Multi-turn methods
  - Multi-turn hadron injection
  - Charge-exchange H^- injection
  - Multi-turn extraction
- Resonant slow extraction
- **Summary**

Beam transfer - summary

Transfer lines transport beams between accelerators (from extraction of one to injection of the next) and onto experimental targets and beam dumps

- Requirements:
  - Geometric link between machines/experiment
  - Match optics between machines/experiment
  - Preserve emittance
  - Change particles' charge state (stripping foils)
  - Measure beam parameters (measurement lines)
  - Protect downstream machine/experiment

Injection - summary

Several different techniques using kickers, septa and bumpers

- Single-turn injection for hadrons
  - Boxcar stacking: transfer between machines in accelerator chain
  - Angle / position errors $\Rightarrow$ injection oscillations
  - Uncorrected errors $\Rightarrow$ filamentation $\Rightarrow$ emittance increase

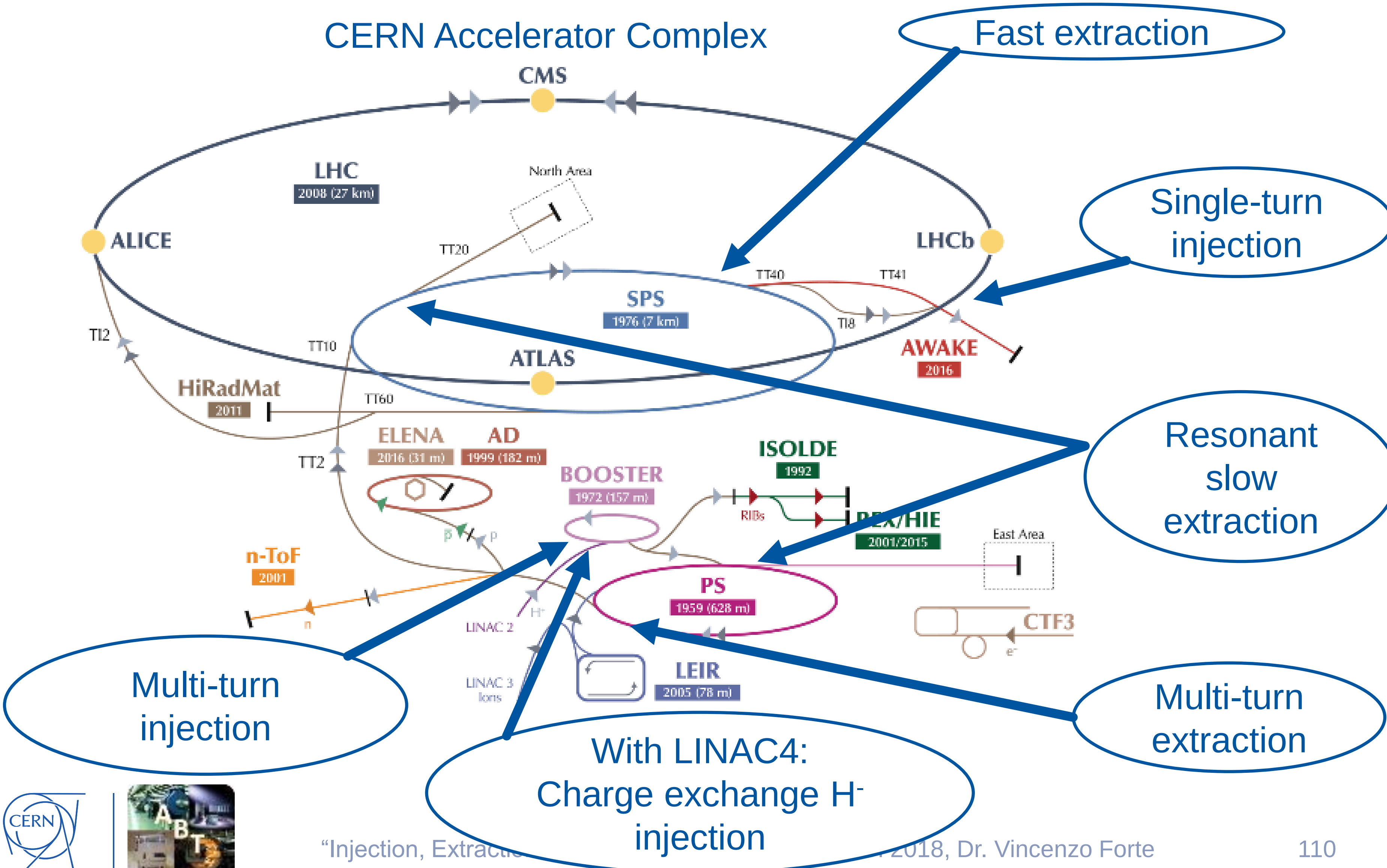
- Multi-turn injection for hadrons
  - Phase space painting to increase intensity
  - H^- injection allows injection into same phase space area

Extraction - summary

Extraction technique is chosen depending on “receivers” requirements

- Single-turn fast extraction:
  - for transfer between machines in accelerator chain, beam abort or experiments with requests for short pulses
- Non-resonant multi-turn extraction
  - slice beam into equal parts for transfer between machine over a few turns.
- Resonant low-loss multi-turn extraction
  - create stable islands in phase space: extract over a few turns.
- Resonant slow extraction
  - create stable area in phase space ➔ slowly drive particles into resonance ➔ long spill over many thousand turns.

Linking the machines



Help! What do I do when???

Application	Method
Direct transfer between machines with minimal effect on beam emittance	Single-turn injection and extraction
Fill up acceptance of receiving machine to create **higher beam intensity**	Multi-turn injection with phase space painting
Create **higher particle density** *(and beam intensity)*	Charge exchange injection *(with phase space painting)*
Fill following machine with minimum number of extractions *(with minimized losses)*	Multi-turn extraction *(using a resonance to split beam into several stable islands in phase space)*
Send long ”quasi-constant” spills to fixed target experiment	Resonant slow extraction, using a resonance to drive particles into the extraction channel

Literature

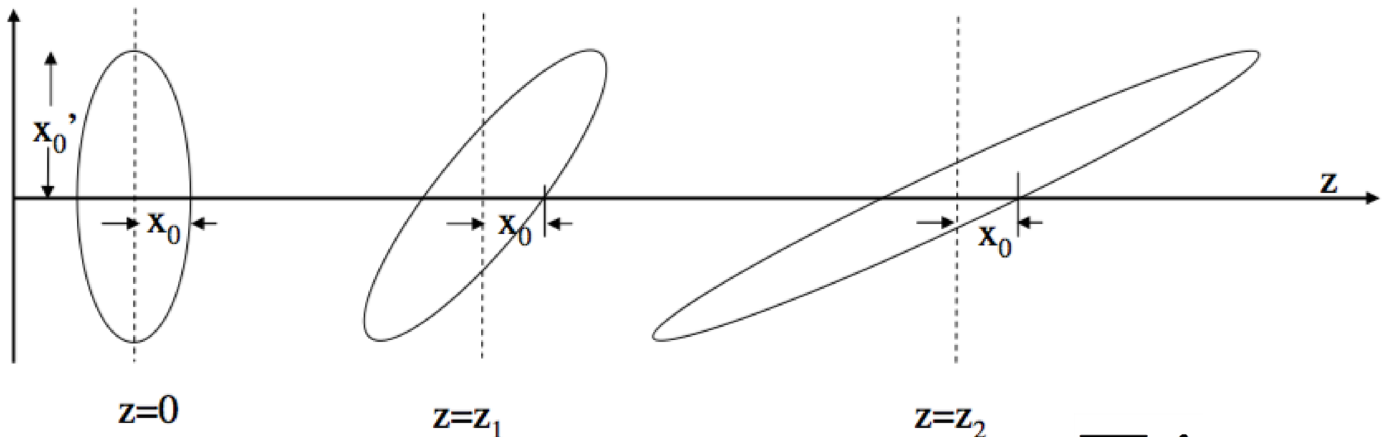
- **Lectures of beam injection, extraction and transfer CAS in Erice (Italy) 2017**
- General accelerator physics course of the CAS
- Lectures of Brennan Goddard at CAS and Rende Steerenberg at OP AXEL lectures
- M. Pullia, “Beamlines and matching to gantries” CAS 2015, Vösendorf, Austria
- K. Wille, ”The Physics of Particle Accelerators: An Introduction”, 2000
- H. Wiedemann, ”Particle Accelerator Physics”
- LHC design report (Vol.III), CERN-2004-003
- A. Hilaire, “Beam transfer to and injection into LHC”, EPAC'98 Conference, Stockholm, June 1998
- C. Bovet, “The fast shaving ejection for beam transfer from the CPS to the CERN 300 GeV machine”, IEEE 1973
- R. Cappi, M. Giovannozzi, “Multiturn extraction:performance analysis of old and new approaches”, Phys. Rev. Spec. Top. Accel. Beams 7 (2004) 024001
- V. Forte, ”Performance of the CERN PSB at 160 MeV with H- charge exchange injection”, PhD thesis 2016
- E. Benedetto et al, “Space Charge Effects and Mitigation in the CERN PS Booster, in View of the Upgrade”, HB2016 Workshop, CERN-ACC-2016-0108
- M. Benedikt, “Optical design of a synchrotron with optimisation of the slow extraction for hadron therapy”, PhD thesis 1997
- F. M. Velotti, “Higher Brightness Beams from the SPS for the HL-LHC Era”, PhD thesis 2016

Thank you

Appendix

Liouville's Theorem

- "… under the influence of conservative forces the particle density in phase space stays constant" (H. Wiedemann, Particle Accelerator Physics)



$$\frac{\partial \Psi}{\partial \tau} + \overbrace{\boldsymbol{\nabla}_r \Psi \; \dot{\boldsymbol{r}} + \boldsymbol{\nabla}_p \Psi \; \dot{\boldsymbol{p}}}^{\boldsymbol{\nabla} \boldsymbol{j}} = \frac{\mathrm{d}\Psi}{\mathrm{d}\tau} = 0$$

Normalised phase space

- Transform real transverse coordinates (x, x', s) to normalised coordinates ($\bar{X}$,$\bar{X}'$,μ) where the independent variable becomes the phase advance μ:

$$\begin{bmatrix} \bar{X} \\ \bar{X}' \end{bmatrix} = \mathbf{N} \cdot \begin{bmatrix} x \\ x' \end{bmatrix} = \sqrt{\frac{1}{\beta(s)}} \cdot \begin{bmatrix} 1 & 0 \\ \alpha(s) & \beta(s) \end{bmatrix} \cdot \begin{bmatrix} x \\ x' \end{bmatrix}$$

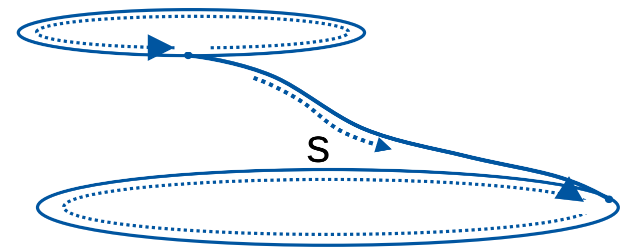
$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos\left[\mu(s) + \mu_0\right]$$

$$\mu(s) = \int_0^s \frac{ds}{\beta(s)}$$

$$\bar{X}(\mu) = \sqrt{\frac{1}{\beta(s)}} \cdot x = \sqrt{\varepsilon}\cos\left[\mu + \mu_0\right]$$

$$\bar{X}'(\mu) = \sqrt{\frac{1}{\beta(s)}} \cdot \alpha(s)x + \sqrt{\beta(s)}x' = -\sqrt{\varepsilon}\sin\left[\mu + \mu_0\right] = \frac{d\bar{X}}{d\mu}$$

Transfer matrix



One pass: $\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \mathbf{M}_{1\to 2} \cdot \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}$

$$\mathbf{M}_{1\to 2} = \begin{bmatrix} \sqrt{\beta_2/\beta_1}\left(\cos\Delta\mu + \alpha_1 \sin\Delta\mu\right) & \sqrt{\beta_1\beta_2}\sin\Delta\mu \\ \sqrt{1/\beta_1\beta_2}\left[\left(\alpha_1-\alpha_2\right)\cos\Delta\mu - \left(1+\alpha_1\alpha_2\right)\sin\Delta\mu\right] & \sqrt{\beta_1/\beta_2}\left(\cos\Delta\mu - \alpha_2 \sin\Delta\mu\right) \end{bmatrix}$$

- Straight propagation of TWISS parameters (no periodic conditions)
- At any point in line, α(s) β(s) are functions of α_1 and β_1

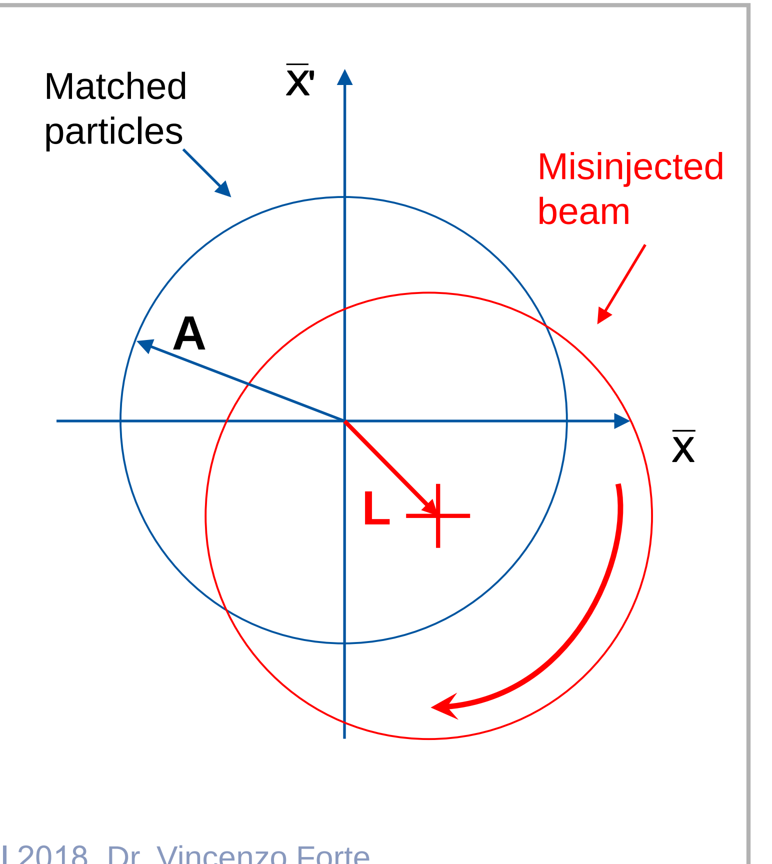
- For a ring the transfer matrix can be simplified (periodic conditions) ➔

$$\Delta\mu = 2\pi Q$$

$$\mathbf{M}_{1\to 2} = \mathbf{M}_{0\to L} = \begin{bmatrix} \cos 2\pi Q + \alpha \sin 2\pi Q & \beta \sin 2\pi Q \\ -1/\beta \left(1+\alpha^2\right)\sin 2\pi Q & \cos 2\pi Q - \alpha \sin 2\pi Q \end{bmatrix}$$

Blow-up from steering error

- Consider a collection of particles with max. amplitudes A
- The beam can be injected with an error in angle and position.
- For an injection error Δa, in units of $\sigma = \sqrt{(\beta\varepsilon)}$, the mis-injected beam is offset in normalised phase space by an amplitude $L = \Delta a\sqrt{\varepsilon}$



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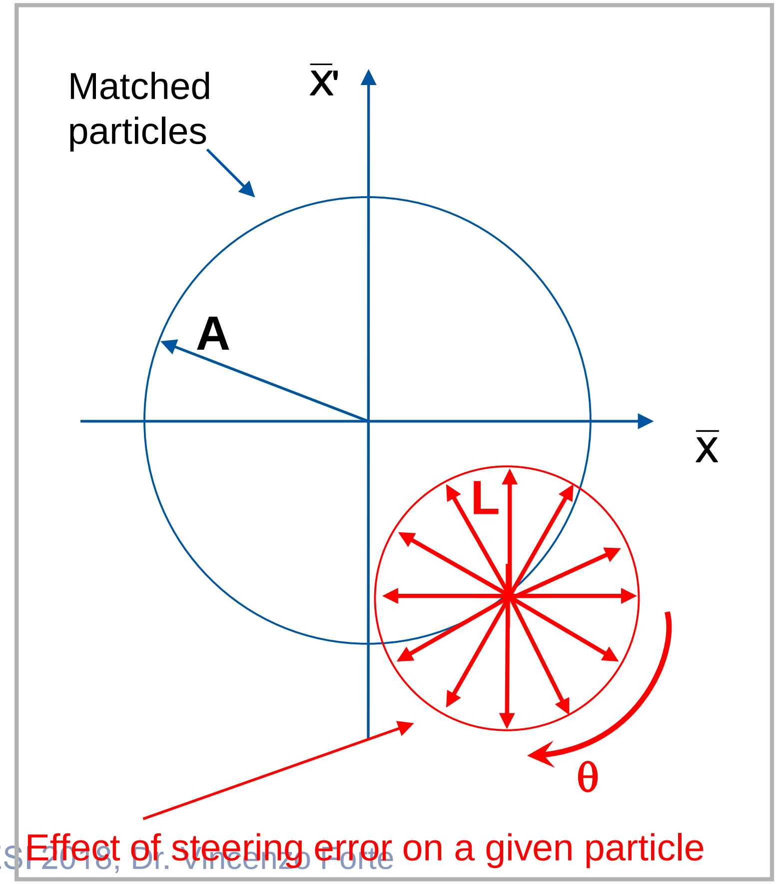
- Any given point on the matched ellipse is randomised over all phases after filamentation due to the steering error:



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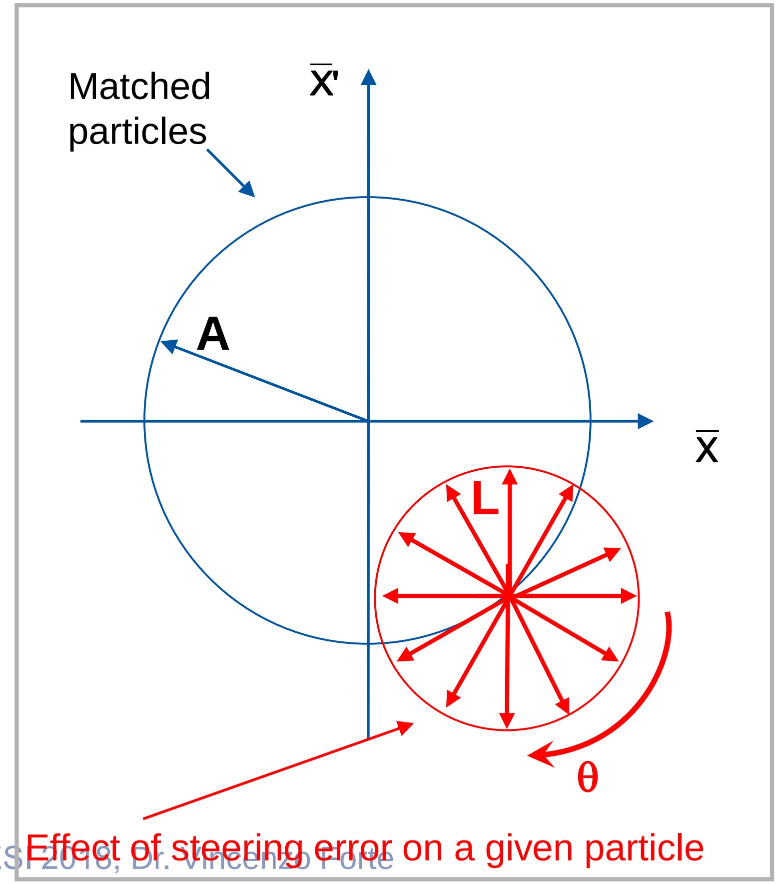
- For a general particle distribution, where A_i denotes amplitude in normalised phase of particle i:

$$\varepsilon_{matched} = \langle \boldsymbol{A}_i^2 \rangle / 2$$



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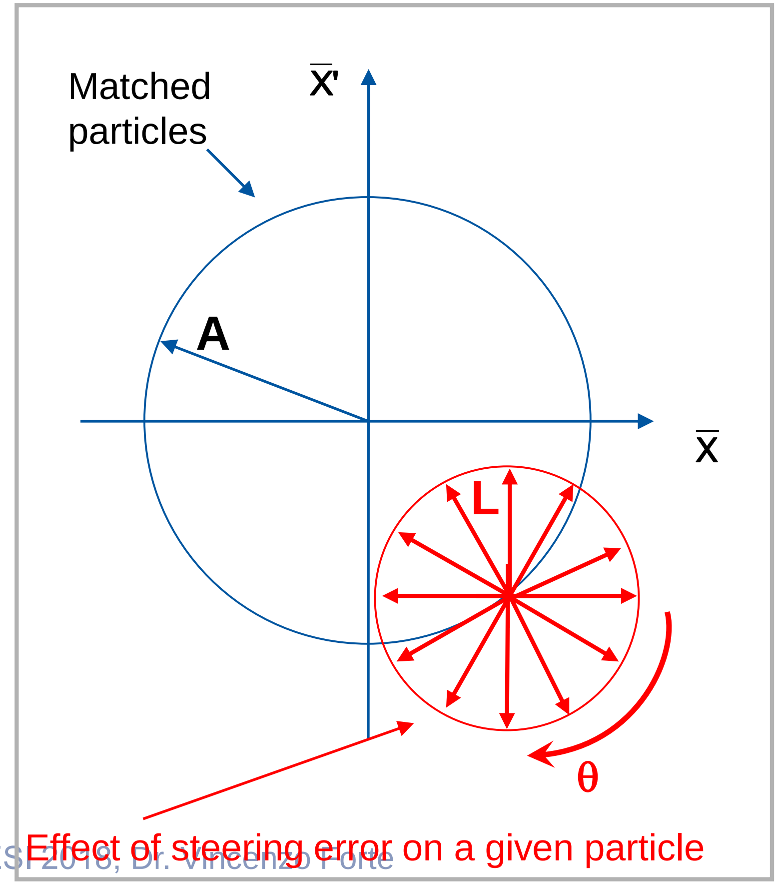
- After filamentation:

$$e_{diluted} = e_{matched} + \frac{L^2}{2}$$



Blow-up from steering error

- The new particle coordinates in normalised phase space are:

$$\bar{X}_{error} = \bar{X}_0 + L\cos\theta$$

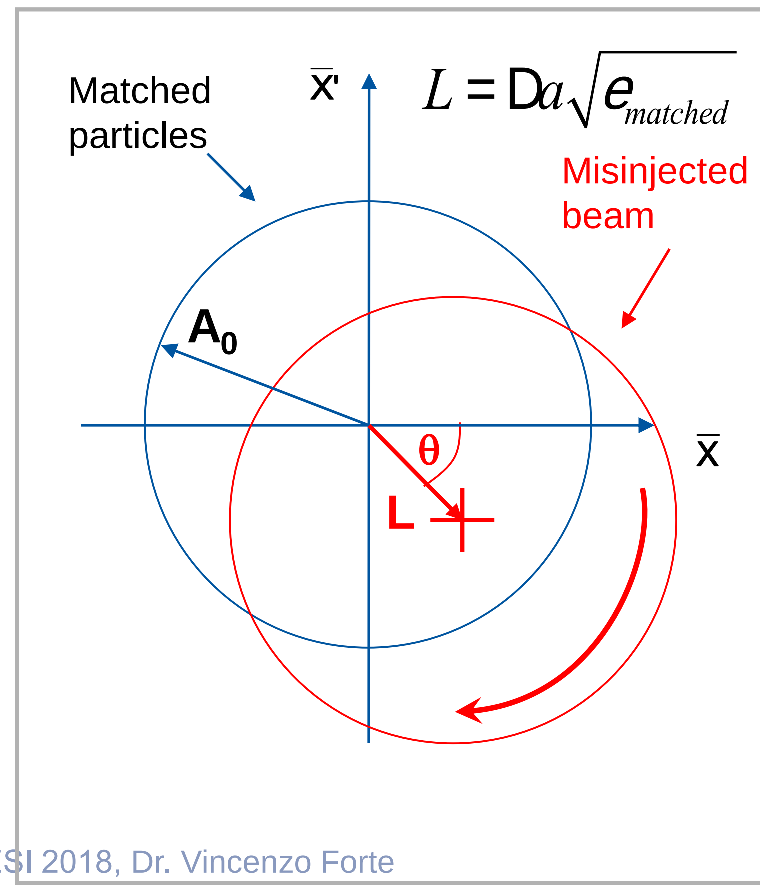
$$\bar{X}'_{error} = \bar{X}'_0 + L\sin\theta$$

- For a general particle distribution, where A_i denotes amplitude in normalised phase of particle i:

$$A_i^2 = \bar{X}_{0,i}^2 + \bar{X}'^2_{0,i}$$

- The emittance of the distribution is:

$$\varepsilon_{matched} = \langle A_i^2 \rangle / 2$$



Blow-up from steering error

- So we plug in the new coordinates:

$$\cos^2\theta + \sin^2\theta = 1$$

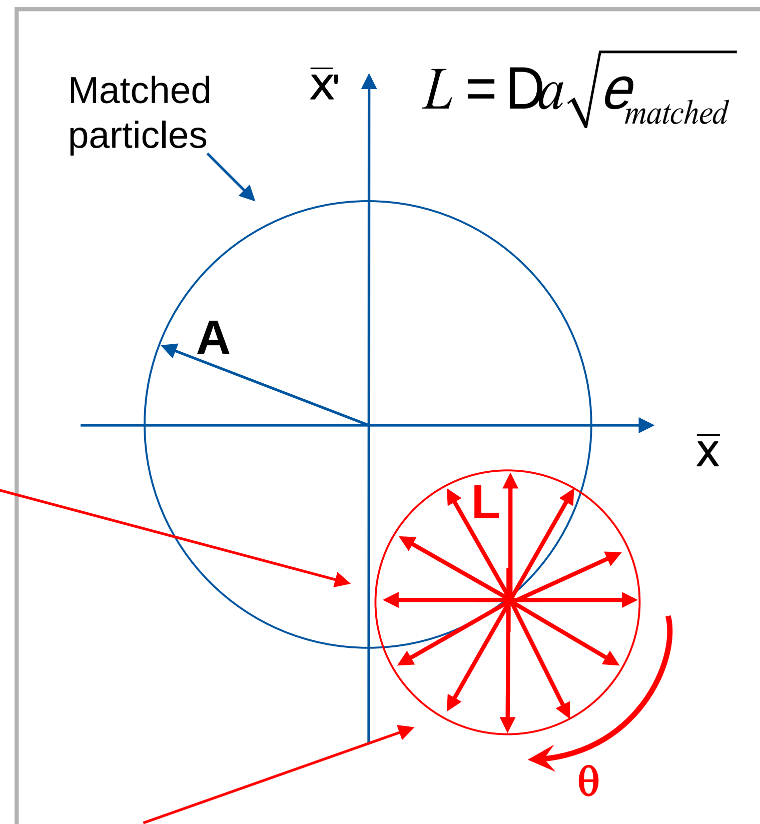
$$\begin{aligned}
A_{error}^2 &= \bar{X}_{error}^2 + \bar{X}'^2_{error} \\
&= (\bar{X}_0 + L\cos\theta)^2 + (\bar{X}'_0 + L\sin\theta)^2 \\
&= \bar{X}_0^2 + \bar{X}'^2_0 + 2L(\bar{X}_0\cos\theta + \bar{X}'_0\sin\theta) + L^2
\end{aligned}$$

- Taking the average over distribution:

$$\begin{aligned}
\langle A_{error}^2 \rangle &= \langle A_0^2 \rangle + 2L(\underbrace{\langle \bar{X}_0\cos\theta \rangle}_{0} + \underbrace{\langle \bar{X}'_0\sin\theta \rangle}_{0}) + \langle L^2 \rangle \\
&= 2\varepsilon_{matched} + L^2
\end{aligned}$$

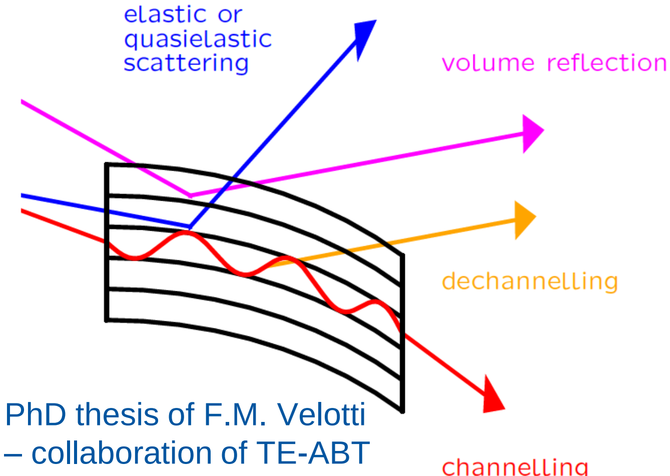
- Giving the diluted emittance as:

$$\begin{aligned}
\varepsilon_{diluted} &= \varepsilon_{matched} + \frac{L^2}{2} \\
&= \varepsilon_{matched}\left[1 + \frac{\Delta\alpha^2}{2}\right]
\end{aligned}$$

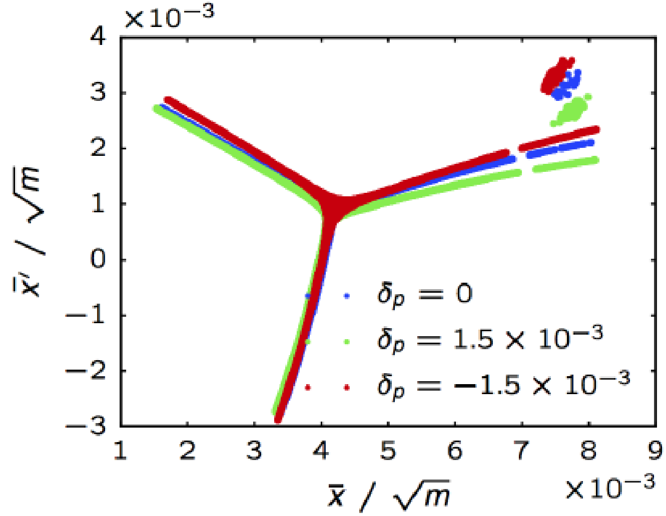


Effect of steering error on a given particle

Reduction of losses on the septum – shadowing with crystals

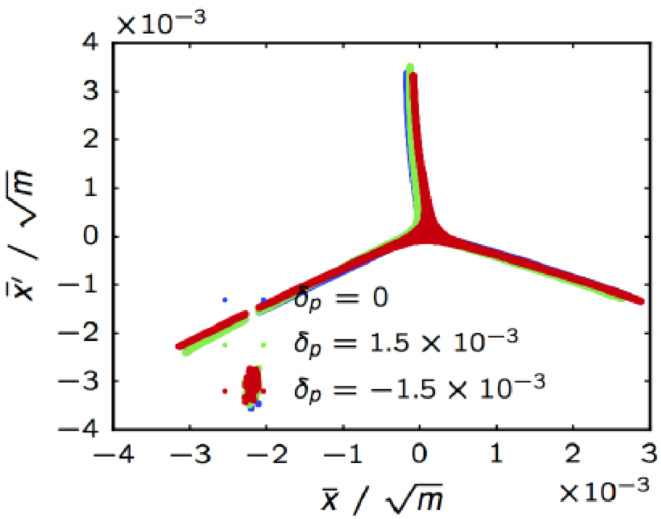


PhD thesis of F.M. Velotti – collaboration of TE-ABT group with UA9



Extraction septum

Particles, which were kicked by the crystal have entered the extraction channel



Crystal location

Particles, which hit the thin crystal are kicked