Superconducting Electron Linacs

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DESY
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What's in Store

- Brief history of superconducting RF
- Choice of frequency (SCRF for pedestrians)
- RF Cavity Basics (efficiency issues)
- Wakefields and Beam Dynamics
- Emittance preservation in electron linacs

- Will generally consider only high-power high-gradient linacs
  - sc e^+e^- linear collider
  - sc X-Ray FEL

  } TESLA technology

Status 1992: Before start of TESLA R&D (and 30 years after the start)

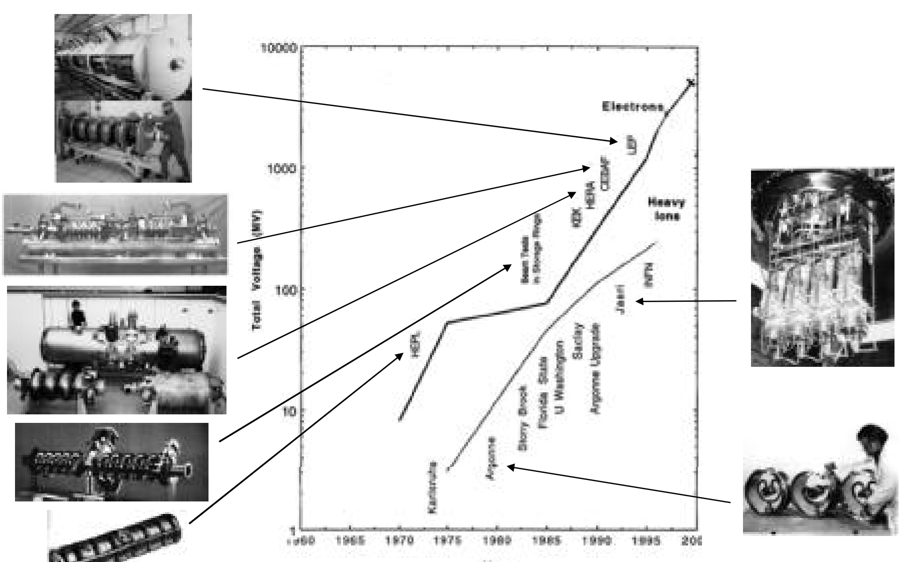
s.c. cavities in operation were ...

	Nbr. of cav.		MHz	m	MV/m	MV
MACSE	5	5-cell	1500	2.5	6.5	16
S-DALINAC	10	20-cell	3000	10.0	5.9	59
HERA	16	4-cell	500	19.2	3.6	69
HEPL				30.8	3.0	92
TRISTAN	32	5-cell	508	47.2	6.6	310
CEBAF	106	5-cell	1497	53.0	7.6	400
LEP	12	4-cell	352	20.4	3.7	75

and others....
CEBAF with an ogoing rate of 16 Cavities per month

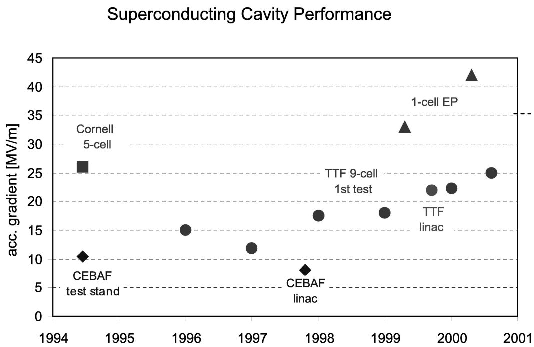
L Lilje

S.C. RF 'Livingston Plot'

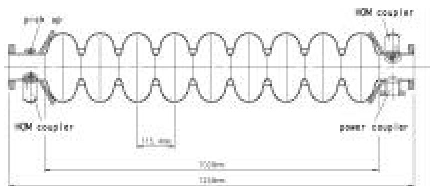


courtesy Hasan Padamsee, Cornell

TESLA R&D



2003
9 cell EP cavities



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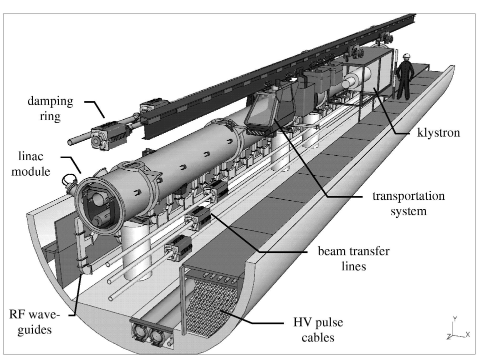
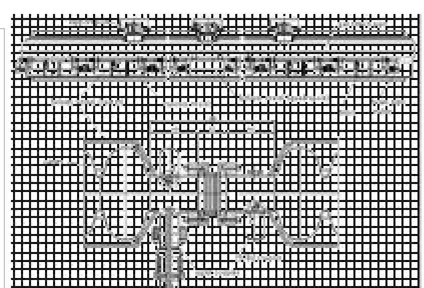
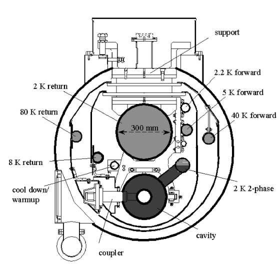
TESLA R&D







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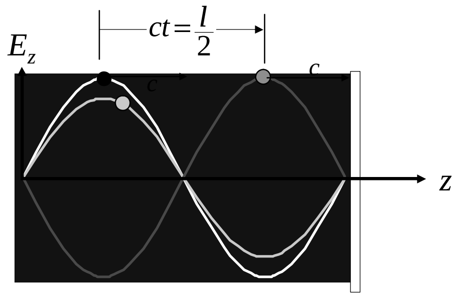
The Linear Accelerator (LINAC)



standing wave cavity:

bunch sees field:

$E_z = E_0 \sin(wt+f)\sin(kz)$

$= E_0 \sin(kz+f)\sin(kz)$

For electrons, life is easy since

- We will *only* consider relativistic electrons (*v*» *c*)
  we assume they have accelerated from the source by somebody else!
- Thus there is no longitudinal dynamics ($e^{\pm}$ do not move long. relative to the other electrons)
- No space charge effects

SC RF

Unlike the DC case (superconducting magnets), the surface resistance of a superconducting RF cavity is *not* zero:

$$R_{BCS} \propto \frac{f^2}{T} \exp\left(-1.76 T_c / T\right)$$

Two important parameters:

- residual resistivity
- thermal conductivity

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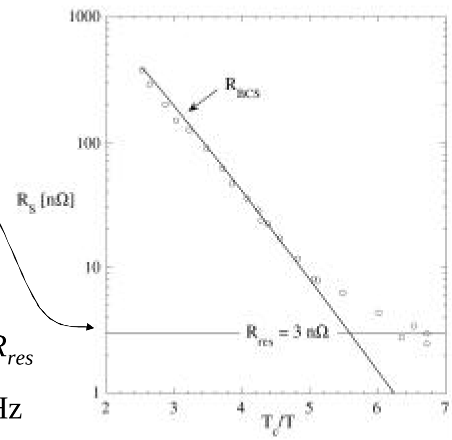
Two important parameters:

- residual resistivity R_{res}
- thermal conductivity

$$\text{losses}\begin{cases}\propto \text{surface area} \propto f^{-1} \\ R_s \propto f^{\,2} \text{ when } R_{BCS} > R_{res}\end{cases}$$

f_{TESLA} = 1.3 GHz $\qquad f$ > 3 GHz



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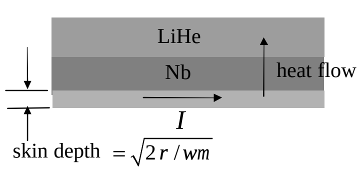
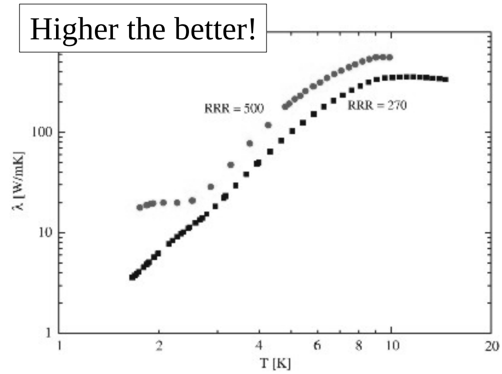


Higher the better!



RRR = Residual Resistivity Ratio

RF Cavity Basics
Figures of Merit

- RF power P_{cav}
- Shunt impedance r_s

$$\left.\begin{array}{l} P_{cav} \\ r_s \end{array}\right\} \quad V_{cav}^2 \equiv r_s P_{cav}$$

- Quality factor Q_0:

$$Q_0 \equiv 2p \frac{\text{stored energy}}{\text{energy lost per cycle}} = \frac{w_0 U_{cav}}{P_{cav}}$$

- *R-over-Q*

$$r_s / Q_0 = \frac{V_{cav}^2}{2 w_0 U_{cav}}$$

r_s/Q_0 is a constant for a given cavity geometry independent of surface resistance

Frequency Scaling

$$r_s \propto \begin{cases} f^{+1/2} & \text{normal} \\ f^{-1} & \text{superconducting} \end{cases}$$

$$Q_0 \propto \begin{cases} f^{-1/2} & \text{normal} \\ f^{-2} & \text{superconducting} \end{cases}$$

$$\frac{r_s}{Q_0} \propto \begin{cases} f & \text{normal} \\ f & \text{superconducting} \end{cases}$$

RF Cavity Basics
Fill Time

From definition of Q_0

$$Q_0 = \frac{w_0 U_{cav}}{P_{cav}}$$

Allow ‘ringing’ cavity to decay
(stored energy dissipated in walls)

$$P_{cav} = -\frac{dU_{cav}}{dt}$$

Combining gives eq. for U_{cav}

$$\frac{dU_{cav}}{dt} = -\frac{w_0}{Q_0} U_{cav}$$

Assuming exponential solution
(and that Q_0 and w_0 are constant)

$$U_{cav}(t) = U_{cav}(0)\, e^{-\frac{w_0}{Q_0}t}$$

Since $U_{cav} \propto V_{cav}^2$

$$V_{cav}(t) = V_{cav}(0)\, e^{-\frac{w_0}{2Q_0}t}$$

RF Cavity Basics
Fill Time

Characteristic ‘charging’ time:

$$\boxed{t = \frac{2Q_0}{w_0}}$$

time required to (dis)charge cavity voltage to 1/*e* of peak value.

Often referred to as the cavity *fill time*.

True fill time for a <u>pulsed linac</u> is defined slightly differently as we will see.

RF Cavity Basics
Some Numbers

f_{RF} = 1.3 GHz		S.C. Nb (2K)	Cu
Q_0		5×10^9	2×10^4
R/Q		1 kΩ	
R_0		5×10^{12} Ω	2×10^7 Ω
P_{cav} (5 MV)	*cw!*	5 W	1.25 MW
P_{cav} (25 MV)	*cw!*	125 W	31 MW
t_{fill}		1.2 s	5 μs

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t_{fill}		1.2 s	

Very high Q_0: the great advantage of s.c. RF

RF Cavity Basics
Some Numbers

- very small power loss in cavity walls
- all supplied power goes into accelerating the beam
- very high RF-to-beam transfer efficiency
- for AC power, must include cooling power

			Cu
			2×10^4
R_0			$2\times10^7\ \Omega$
P_{cav} (5 MV)	*cw!*	5 W	1.25 MW
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RF Cavity Basics
Some Numbers

- for high-energy higher gradient linacs (X-FEL, LC), *cw* operation not an option due to load on cryogenics
- pulsed operation generally required
- numbers now represent peak power
- $P_{cav} = P_{pk}\times$duty cycle
- (Cu linacs generally use <u>very</u> short pulses!)

f_{RF} = 1.3 GHz			
Q_0			
R/Q			
R_0			
P_{cav} (5 MV)	*cw!*	5 W	1.25 MW
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Cryogenic Power Requirements

Basic Thermodynamics: Carnot Efficiency (T_{cav} = 2.2K)

$$h_c = \frac{T_{cav}}{T_{room} - T_{cav}} = \frac{2.2}{300 - 2.2} \approx 0.7\%$$

System efficiency typically 0.2-0.3 (latter for large systems)

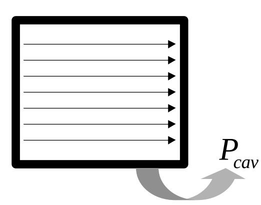
Thus total cooling efficiency is 0.14-0.2%

$$\boxed{P_{cooling} = 5\text{W}/0.002 \approx 2.5\text{kW}}$$

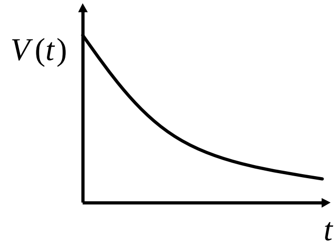
Note: this represents *dynamic load*, and depends on Q_0 and V
Static load must also be included (*i.e.* load at $V = 0$).

RF Cavity Basics
Power Coupling

- calculated 'fill time' was 1.2 seconds!
- this is time needed for field to decay to V/*e for a closed cavity* (i.e. only power loss to s.c. walls).



$$V(t) = V(0)\exp(-w_0 t / 2Q_0)$$



RF Cavity Basics
Power Coupling

- calculated 'fill time' was 1.2 seconds!
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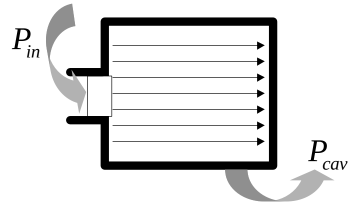
- however, we need a 'hole' (*coupler*) in the cavity to get the power in, and



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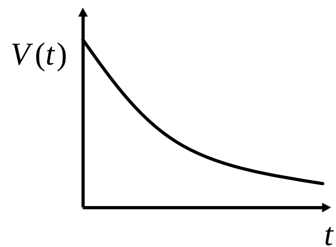
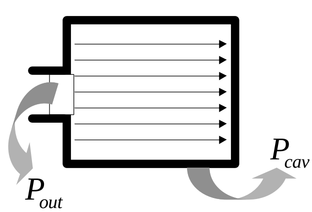
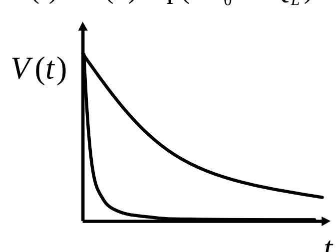


RF Cavity Basics
Power Coupling

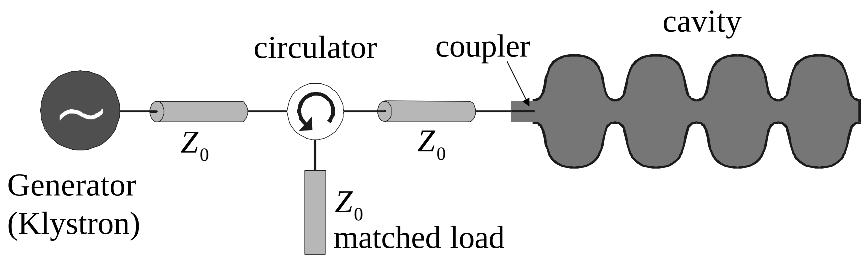
- calculated 'fill time' was 1.2 seconds!
- this is time needed for field to decay to *V/e for a closed cavity* (i.e. only power loss to s.c. walls).
- however, we need a 'hole' (*coupler*) in the cavity to get the power in, and
- this hole allows the energy *in* the cavity to leak out!



$$V(t) = V(0)\exp(-w_0 t / 2Q_L)$$

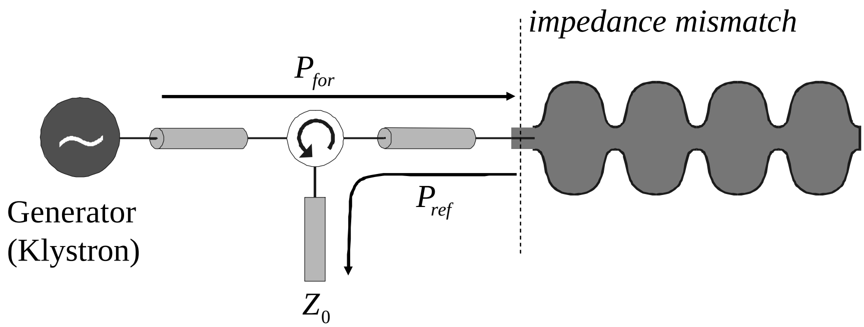


RF Cavity Basics



Z_0 = characteristic impedance of transmission line (waveguide)

RF Cavity Basics

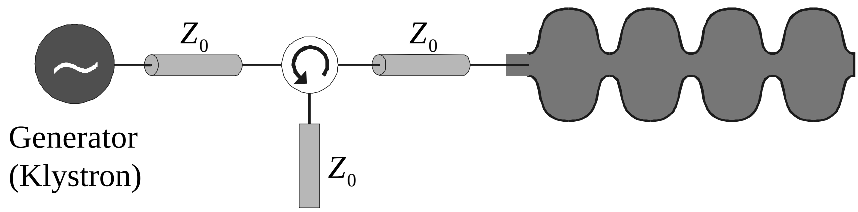


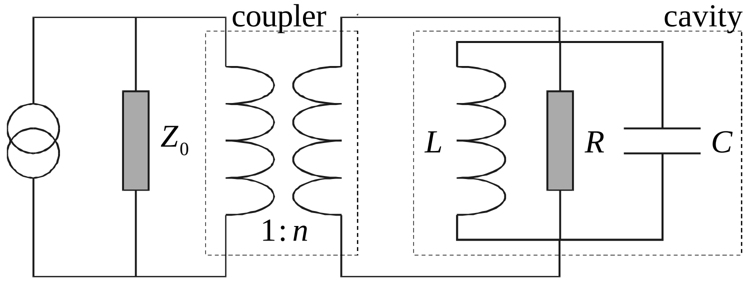
Klystron power P_{for} sees *matched impedance* Z_0

Reflected power P_{ref} from coupler/cavity is dumped in load

Conservation of energy: $P_{for} = P_{ref} + P_{cav}$

Equivalent Circuit

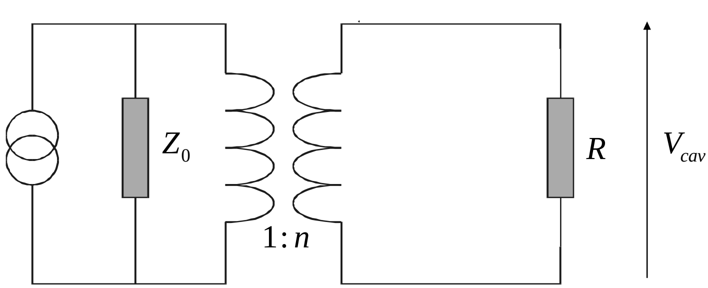




Equivalent Circuit

Only consider on resonance: $w_0 = \frac{1}{\sqrt{LC}}$

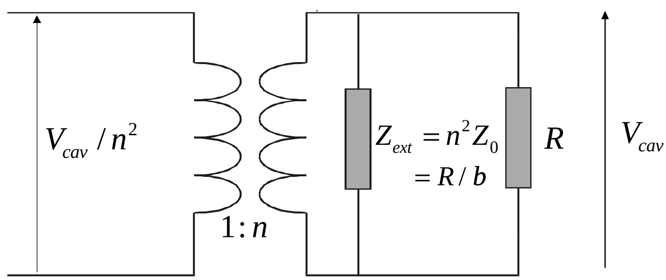
$note: \quad Q_0 = R\sqrt{\frac{C}{L}}$



Equivalent Circuit

Only consider on resonance: $w_0 = \frac{1}{\sqrt{LC}}$

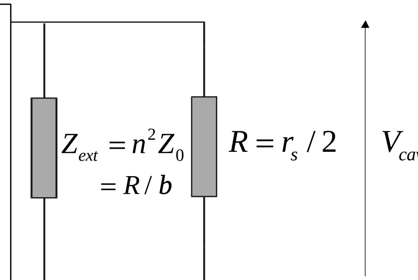
We can transform the matched *load* impedance Z_0 into the cavity circuit.



Equivalent Circuit

$$Q_0 = \frac{R}{w_0 L}$$

$$Q_{load} = \frac{\left(\frac{1}{R} + \frac{1}{Z_{ext}}\right)^{-1}}{w_0 L}$$



define external Q:

$$Q_{ext} = \frac{Z_{ext}}{w_0 L}$$

$$\frac{1}{Q_{load}} = \frac{1}{Q_{ext}} + \frac{1}{Q_0}$$

coupling constant:

$$b = \frac{P_{ext}}{P_{cav}} = \frac{R}{R_{ext}} = \frac{Q_0}{Q_{ext}}$$

$$Q_{load} = \frac{Q_0}{1+b}$$

Reflected and Transmitted RF Power

reflection coefficient
(seen from generator):

$$\Gamma = \frac{R - Z_{ext}}{R + Z_{ext}} = \frac{b - 1}{b + 1}$$

from energy conservation:

$$P_{cav} = P_{for} - P_{ref}$$
$$= P_{for}\left[1 - \Gamma^2\right]$$
$$= P_{for}\left[1 - \left(\frac{b - 1}{b + 1}\right)^2\right]$$

$$P_{ref} = \Gamma^2 P_{for} = \left(\frac{1 - b}{1 + b}\right)^2 P_{for} \qquad P_{cav} = \frac{4b}{(1 + b)^2} P_{for}$$

Transient Behaviour

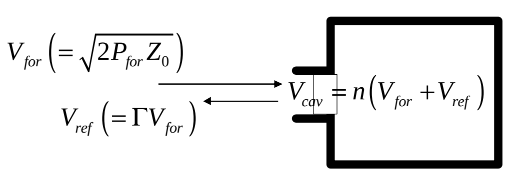
steady state cavity voltage:

$$\hat{V}_{cav} = \sqrt{P_{cav} r_s}$$
$$= \frac{2b^{1/2}}{1 + b}\sqrt{P_{for} r_s}$$

from before:

$$P_{cav} = \frac{4b}{(1 + b)^2} P_{for}$$

think in terms of (travelling) microwaves: remember: $w = w_0$



$$\Gamma = \frac{b - 1}{b + 1}$$

$$\hat{V}_{cav} = n\left(V_{for} + V_{ref}\right) = n\left(1 + \Gamma\right) V_{for} = \frac{2bn}{1 + b} V_{for}$$

steady-state result!

Reflected Transient Power

$$V_{cav}(t) = \left[1 - \exp\left(-w_0 t / 2Q_L\right)\right]\hat{V}_{cav}$$
$$= \left[1 - \exp\left(-w_0 t / 2Q_L\right)\right]\frac{2bn}{1+b}V_{for}$$

$$V_{ref}(t) = \frac{V_{cav}(t)}{n} - V_{for}$$
$$\frac{V_{ref}(t)}{V_{for}} = \left[1 - \exp\left(-w_0 t / 2Q_L\right)\right]\frac{2b}{1+b} - 1$$
$$\equiv \Gamma(t)$$

time-dependent reflection coefficient

Reflected Transient Power

$$\boxed{\frac{P_{ref}(t)}{P_{for}} = \left\{\left[1 - \exp\left(-w_0 t / 2Q_L\right)\right]\frac{2b}{1+b} - 1\right\}^2}$$

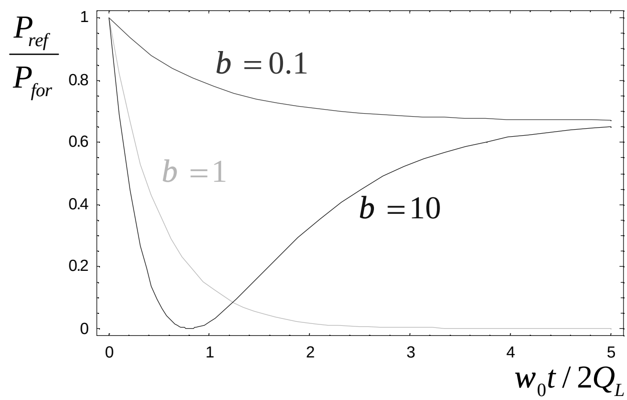
after RF turned off $V_{for} = 0 \quad t_{off} \gg 2Q_L / w_0$

$$V_{ref} = -\frac{\hat{V}_{cav}}{n}\exp\left(-w_0 t' / 2Q_L\right) \qquad t' = t - t_{off}$$
$$= -\frac{2b}{1+b}V_{for}\exp\left(-w_0 t' / 2Q_L\right)$$

$$\boxed{\frac{P_{ref}}{P_{for}} = \frac{4b^2}{\left(1+b\right)^2}\exp\left(-w_0 t' / Q_L\right)}$$

RF On

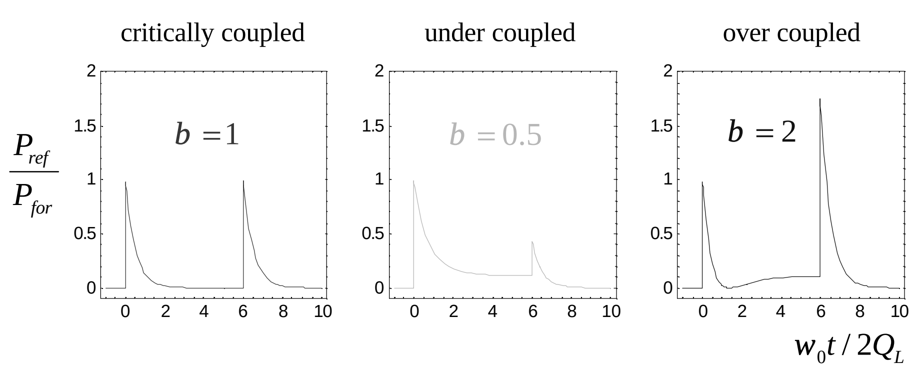
$$\frac{P_{ref}(t)}{P_{for}} = \left\{ \left[1 - \exp\left(-w_0 t / 2Q_L \right) \right] \frac{2b}{1+b} - 1 \right\}^2$$



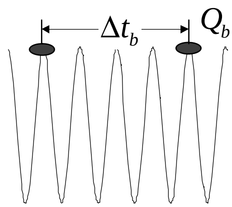
note:
No beam!

Reflected Power in Pulsed Operation

Example of square RF pulse with $\Delta t = 12 Q_0 / w_0$



Accelerating Electrons



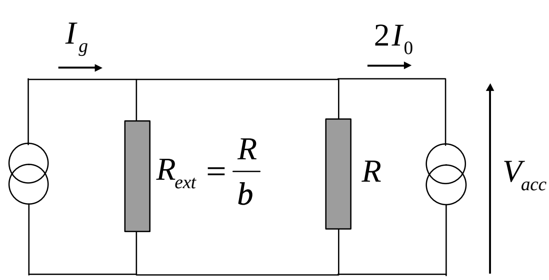
$$I_0 = \frac{Q_b}{\Delta t_b}$$

$$\Delta t_b = 2p\, h / w_0 \qquad t$$

- Assume bunches are very short
  $$s_z / c \ll l_{RF}$$
- model 'current' as a series of d functions:
  $$I_b(t) = Q_b \sum_n d(t - n\Delta t_b)$$
- Fourier component at w_0 is $2I_0$
- assume 'on-crest' acceleration (i.e. I_b is in-phase with V_{cav})

Accelerating Electrons

consider first steady state



$$V_{acc} = (I_g - 2I_0)\frac{R}{1+b}$$

what's I_g?

Accelerating Electrons

Consider power in cavity load R with I_b=0:

$$P_{cav} = \frac{4b}{(1+b)^2} P_{for}$$

steady state!

From equivalent circuit model (with I_b=0):

$$V_{cav} = \frac{I_g R}{1+b}$$

$$P_{cav} = \frac{V_{cav}^2}{2R} = \frac{I_g^2 R}{2(1+b)^2}$$

$$I_g = 2\sqrt{2}\sqrt{\frac{P_{for} b}{R}}$$

NB: I_g is actually twice the <u>true</u> generator current

Accelerating Electrons

$$V_{acc} = (I_g - 2I_0)\frac{R}{1+b}$$

substituting for I_g:

$$V_{acc} = \left(2\sqrt{2}\sqrt{\frac{P_{for} b}{R}} - 2I_0\right)\frac{R}{1+b}$$

$$= \left(1 - \sqrt{\frac{R}{2P_{for} b}} I_0\right) 2\sqrt{2}\sqrt{P_{for} R}\,\frac{\sqrt{b}}{1+b}$$

introducing $K = \sqrt{\frac{R}{2P_{for}}} I_0$

beam loading parameter

$$V_{acc} = 2\left(1 - \frac{K}{\sqrt{b}}\right)\frac{\sqrt{2P_{for} R b}}{1+b}$$

Accelerating Electrons

Now let's calculate the RF→beam efficiency

power fed to beam: $$P_{beam} = I_0 V_{acc} = \frac{4K\sqrt{b}}{1+b} P_{for}\left(1-\frac{K}{\sqrt{b}}\right)$$

hence:

$$h_{RF\to beam} = \frac{P_{beam}}{P_{for}} = \frac{4K\sqrt{b}}{1+b}\left(1-\frac{K}{\sqrt{b}}\right)$$

reflected power: $$P_{ref} = P_{for} - P_{beam} - P_{cav} = (1-h)P_{for} - \frac{V_{acc}^2}{2R}$$

$$= P_{for}\left(\frac{b-1-2K\sqrt{b}}{1+b}\right)^2$$

Accelerating Electrons

Note that if beam is off (K=0)

$$P_{ref} = P_{for}\left(\frac{b-1}{b+1}\right)^2$$ previous result

For zero-beam loading case, we needed b = 1 for maximum power transfer (i.e. P_{ref} = 0)

Now we require $$0 = b - 1 - 2K\sqrt{b}$$

$$K = \frac{b-1}{2\sqrt{b}}$$

Hence for a fixed coupler (b), zero reflection only achieved at one specific beam current.

A Useful Expression for *b*

efficiency: $$h_{RF\to beam} = \frac{P_{beam}}{P_{for}} = \frac{4K\sqrt{b}}{1+b}\left(1-\frac{K}{\sqrt{b}}\right)$$

voltage: $$V_{acc} = 2\left(1-\frac{1}{2}\sqrt{\frac{r_s}{P_{for}\,b}}I_0\right)\sqrt{P_{for}r_s}\,\frac{\sqrt{b}}{1+b}$$

optimum $$K = \frac{b-1}{2\sqrt{b}}$$ $$\left\{\begin{array}{l} h_{opt} = \dfrac{b-1}{b} \\[2ex] V_{acc} = \sqrt{\dfrac{r_s P_{for}}{b}} \end{array}\right\}$$

can show

$$\boxed{b_{opt} = 1+\frac{r_s}{r_{beam}}}$$

where

$r_{beam} \equiv V_{acc} / I_0$

Example: TESLA

beam current:

$N_b = 2\times 10^{10}$

$Q_b = 3.2\,\text{nC}$

$\Delta t_b = 337\text{ns}$

$I_0 = 9.5\text{mA}$

cavity parameters (at T = 2K):

$f = 1.3\text{GHz}$

$r_s / Q_0 \approx 1\text{kO}$

$Q_0 \approx 10^{10}$

$r_s \approx 10^{13}\,\Omega$

$V_{acc} = 25\text{MV}$

For optimal efficiency, $P_{ref} = 0$:

$$\left.\begin{array}{l} P_{for} = P_{beam} + P_{cav} \\ P_{beam} = I_0 V_{acc} = 237.5\text{kW} \\ P_{cav} = \dfrac{V_{acc}^2}{r_s} = 62.5\,\text{W} \\ h_{RF\to beam} = 99.97\% \end{array}\right\}\text{cw!}$$

From previous results:

$K \approx 30.8$

$b \approx 3804$

Unloaded Voltage

$$V_{loaded} = \sqrt{\frac{P_{for} r_s}{b}} \qquad \text{matched condition:} \quad K = \frac{b - 1}{2\sqrt{b}}$$

$$V_{unloaded} = \frac{2\sqrt{b}}{1+b}\sqrt{P_{for} r_s}$$

hence:

$$\frac{V_{unloaded}}{V_{loaded}} = \frac{2b}{1+b}$$

$$\approx 2 \qquad \text{for} \quad K, b \gg 1$$

Pulsed Operation

From previous discussions:

$$V_{cav}(t) = 2V_{acc}\left(1 - e^{-t/t}\right) \qquad t = \frac{2Q_L}{w_0} = \frac{2Q_0}{w_0\left(b + 1\right)}$$

$$V_{beam}(t) = \begin{cases} 0 & t \le t_{fill} \\ -V_{acc}\left(1 - e^{-(t - t_{fill})/t}\right) & t > t_{fill} \end{cases}$$

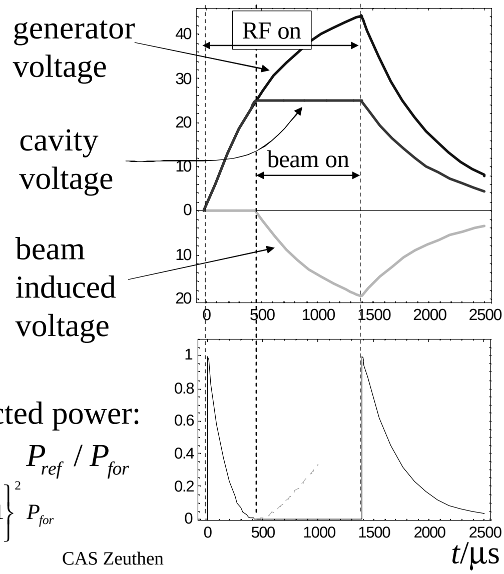
Allow cavity to charge for t_{fill} such that

$$V_{cav}(t_{fill}) = V_{acc} \Rightarrow t_{fill} = \ln(2)t$$

For TESLA example: $t = 645\,m\text{s} \quad t_{fill} = 447\,m\text{s}$

Pulse Operation

- After t_{fill}, beam is introduced
- exponentials cancel and beam sees constant accelerating voltage V_{acc} = 25 MV
- Power is reflected before and after pulse



reflected power:

$P_{ref} \,/\, P_{for}$

$$P_{ref}(t) = \left\{ \left[1 - \exp\left(-\omega_0 t / 2Q_L\right) \right] \frac{2\beta}{1+\beta} - 1 \right\}^2 P_{for}$$

Pulsed Efficiency

total <u>efficiency</u> must include t_{fill}:

$$\eta_{pulse} = \eta \left(\frac{t_{beam}}{t_{fill} + t_{beam}} \right)$$

for TESLA

$$= 99.97\% \times \frac{950\mu s}{446\mu s + 950\mu s}$$

$$\approx 68\%$$

Quick Summary

cw efficiency for s.c. cavity: $$h_{cw} = \frac{P_{beam}}{P_{for}} \approx 1$$

efficiency for pulsed linac: $$h = h_{cw}\left(\frac{t_{beam}}{t_{fill} + t_{beam}}\right)$$

fill time: $$t_{fill} = \ln(2) t_L \qquad t_L = \frac{2Q_L}{w_0} = \frac{2Q_0}{w_0(1+b)} \approx \frac{2}{w_0}\left(\frac{r_s}{Q_0}\right)^{-1}\frac{V_{acc}}{I_0}$$

Increase efficiency (reduce fill time):

- go to high I_0 for given V_{acc}
- longer bunch trains (t_{beam})

some other constraints:

- cyrogenic load $\propto V_{acc}^2 f_{rep} t_{pulse}$
- modulator/klystron

Lorentz-Force Detuning

In high gradient structures, *E* and *B* fields exert stress on the cavity, causing it to deform.
As a result:

- cavity off resonance by relative amount $D = dw/w_0$
- equivalent circuit is now complex
- voltage phase shift *wrt* generator (and beam) by $j = \tan^{-1}(2Q_L\Delta)$
- power is reflected

detuning of cavity

$$V'_{acc} = \mathrm{Re}\left\{\frac{V_{acc}}{1+2iQ_L\Delta}\right\}$$

$$= \frac{V_{acc}}{1+4Q_L^2\Delta^2}$$

require

$$\frac{\Delta V}{V_{acc}} \le 10^{-3} \Rightarrow \Delta^2 \le \frac{10^{-3}}{4Q_L^2}$$

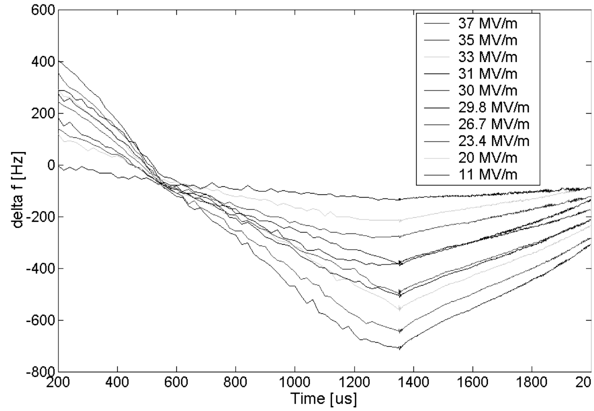
= few Hz for TESLA

For TESLA 9 cell at 25 MV, <u>*Df* ~ 900 Hz</u> !! (loaded BW ~500Hz)
[note: causes transient behaviour during RF pulse]

Lorentz Force Detuning cont.

$$\Delta f \approx k \cdot E^2$$

$$k \approx 1\text{Hz}/(\text{MV/m})^2$$



recent tests on TESLA high-gradient cavity

Lorentz Force Detuning cont.

Three fixes:

- mechanically stiffen cavity
- feed-forwarded (increase RF power during pulse)
- fast piezo tuners + feedback

Lorentz Force Detuning cont.

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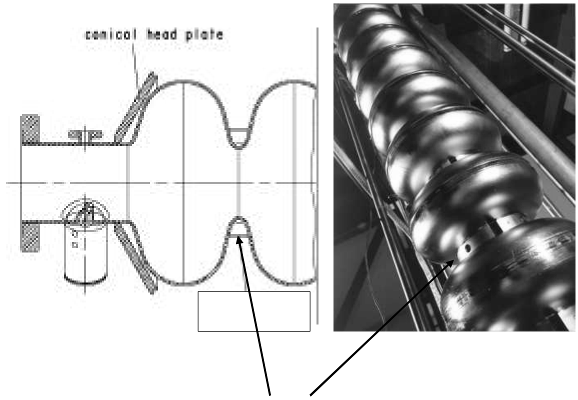


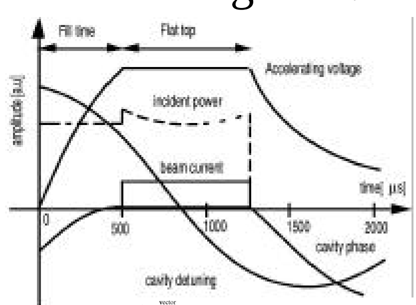
reduces effect by ~1/2

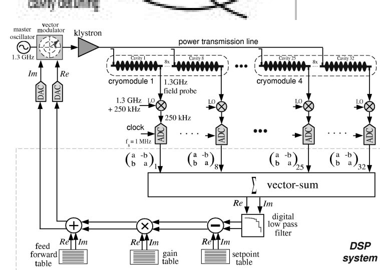
stiffening ring

Lorentz Force Detuning cont.

Three fixes:

- mechanically stiffen cavity
- feed-forwarded (increase RF power during pulse)
- fast piezo tuners + feedback



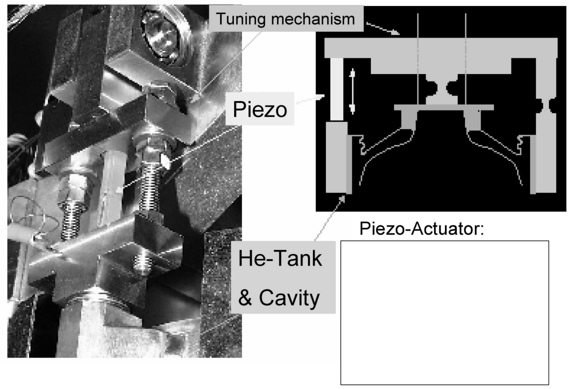


Low Level RF (LLRF) compensates.
Mostly feedforward (behaviour is repetitive)
For TESLA, 1 klystron drives 36 cavities,
thus ‘vector sum’ is corrected.

Lorentz Force Detuning cont.

Three fixes:

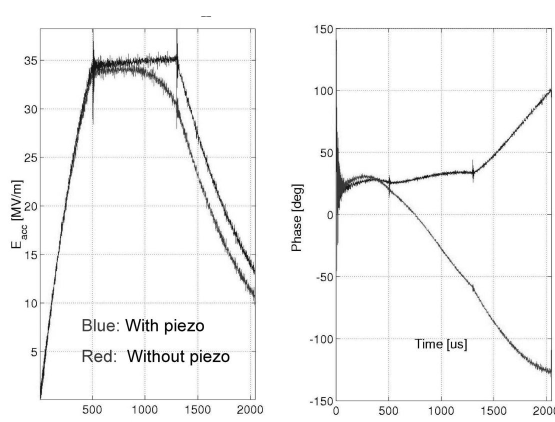
- mechanically stiffen cavity
- feed-forwarded (increase RF power during pulse)
- fast piezo tuners + feedback



Lorentz Force Detuning cont.

Three fixes:

- mechanically stiffen cavity
- feed-forwarded (increase RF power during pulse)
- fast piezo tuners + feedback



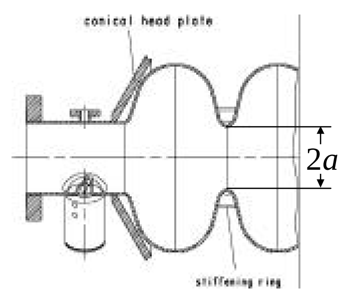
recent tests on TESLA high-gradient cavity

Wakefields and Beam Dynamics

- bunches traversing cavities generate many RF modes.
- Excitation of fundamental (w_0) mode we have already discussed (beam loading)
- higher-order (higher-frequency) modes (HOMs) can act back on the beam and adversely affect it.
- Separate into two time (frequency) domains:
  - long-range, bunch-to-bunch
  - short-range, single bunch effects (head-tail effects)

Wakefields: the (other) SC RF Advantage

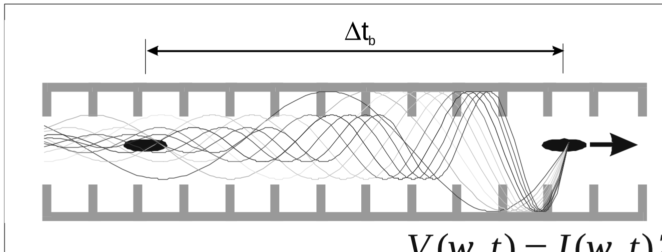
- the strength of the wakefield potential (W) is a strong function of the iris aperture a.
- Shunt impedance (r_s) is also a function of a.
- To increase efficiency, Cu cavities tend to move towards smaller irises (higher r_s).
- For S.C. cavities, since r_s is extremely high anyway, we can make a larger without loosing efficiency.



$$W \propto a^{-3}$$

significantly smaller wakefields

Long Range Wakefields



$$V(w,t) = I(w,t)Z(w,t)$$

Bunch 'current' generates wake that decelerates trailing bunches.

Bunch current generates transverse deflecting modes when bunches are not on cavity axis

Fields build up resonantly: latter bunches are kicked transversely

⇒ multi- and single-bunch beam break-up (MBBU, SBBU)

wakefield is the time-domain description of impedance

Transverse HOMs

wake is sum over modes: $W_{\perp}(t) = \sum_n \frac{2k_n c}{w_n} e^{-w_n t/2Q_n} \sin(w_n t)$

k_n is the *loss parameter* (units *V/pC/m*^{2}) for the n^{th} mode

Transverse kick of j^{th} bunch after traversing one cavity:

$$\Delta y'_j = \sum_{i=1}^{j-1} \frac{y_i q_i}{E_i} \frac{2k_i c}{w_n} e^{-w_n i \Delta t/2Q_n} \sin\left(w_i i \Delta t_b\right)$$

where y_i, q_i, and $E_{i,}$ are the offset *wrt* the cavity axis, the charge and the energy of the i^{th} bunch respectively.

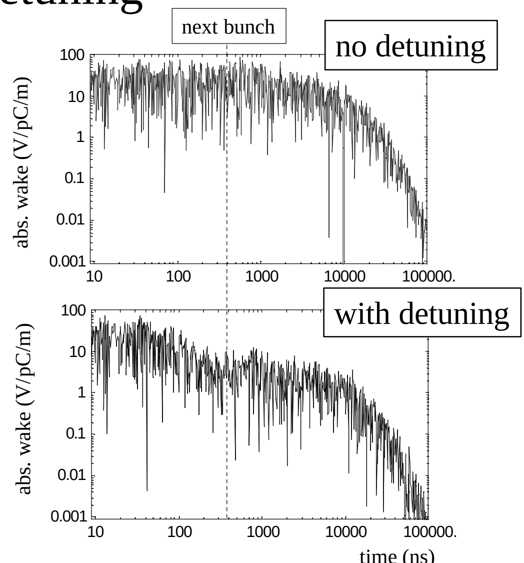
Detuning

HOMs cane be randomly detuned by a small amount.

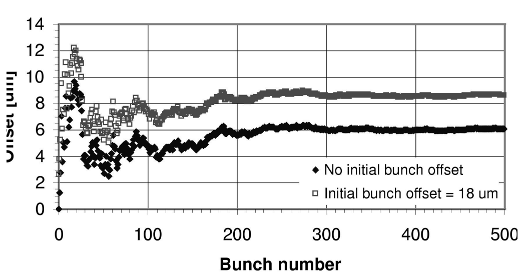
Over several cavities, wake 'decoheres'.

Effect of random 0.1% detuning (averaged over 36 cavities).

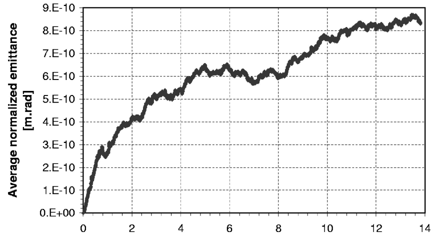
Still require HOM dampers



Effect of Emittance



vertical beam offset along bunch train (n_b = 2920)



Multibunch emittance growth for cavities with 500µm RMS misalignment

Single Bunch Effects

- Completely analogous to low-range wakes
- wake over a single bunch
- causality (relativistic bunch): head of bunch affects the tail
- Again must consider
  - longitudinal: effects energy spread along bunch
  - transverse: the emittance killer!
- For short-range wakes, tend to consider wake potentials (Greens functions) rather than ‘modes

Longitudinal Wake

Consider the TESLA wake potential $W_{||}(z = ct)$

$$W_{||}(z) \approx -38.1\left[\frac{\mathrm{V}}{\mathrm{pC}\cdot\mathrm{m}}\right]\left[1.165\exp\left(-\sqrt{\frac{s}{3.65\times10^{-3}[\mathrm{m}]}}\right)-0.165\right]$$

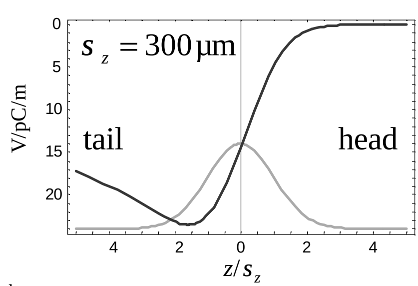
wake over bunch given by convolution: ($r(z)$ = long. charge dist.)

$$W_{||,bunch}(z) = \int_{z'=z}^{\infty} W_{||}(z'-z)\,r(z')dz'$$

average energy loss:

$$\langle \Delta E \rangle = q_b \int_{-\infty}^{\infty} W_{||,bunch}(z)\,r(z)dz$$

For TESLA LC: $\langle \Delta E \rangle \approx -46\,\mathrm{kV/m}$



RMS Energy Spread

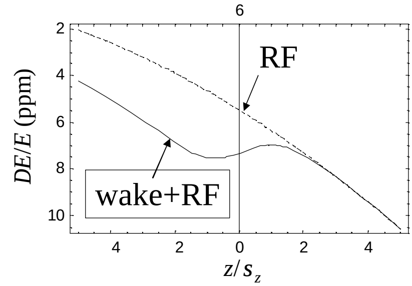
accelerating field along bunch:

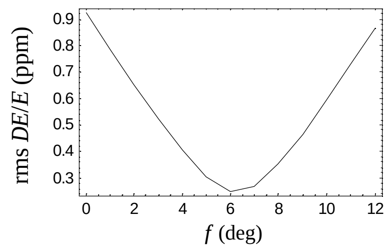
$$E(z) = q_b W_{\|,bunch}(z) + E_0 \cos(2p\, z / l_{RF} + f)$$

Minimum energy spread along bunch achieved when bunch rides ahead of crest on RF.

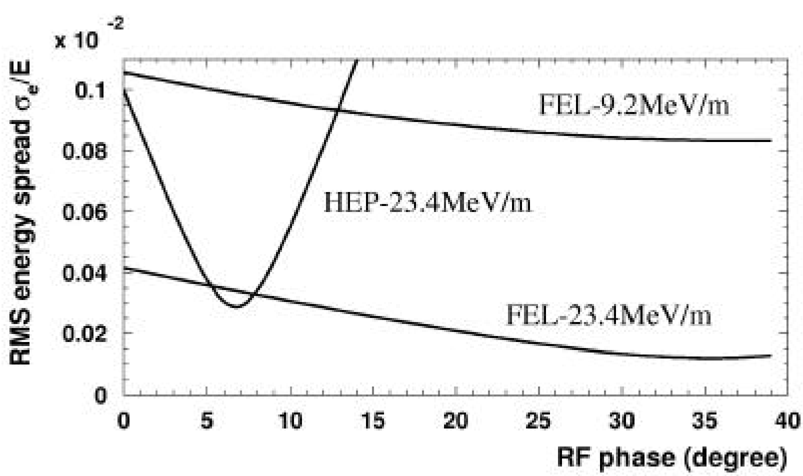
Negative slope of RF compensates wakefield.

For TESLA LC, minimum at about $f \sim +6^{o}$





RMS Energy Spread

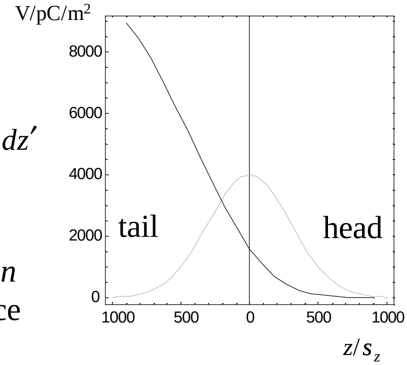


Transverse Single-Bunch Wakes

When bunch is offset wrt cavity axis, transverse (dipole) wake is excited.

'kick' along bunch:

$$\Delta y'(z) = \frac{q_b}{E(z)} \int_{z'=z}^{\infty} W_\perp(z'-z)\, r(z')\, y(s;z')\, dz'$$

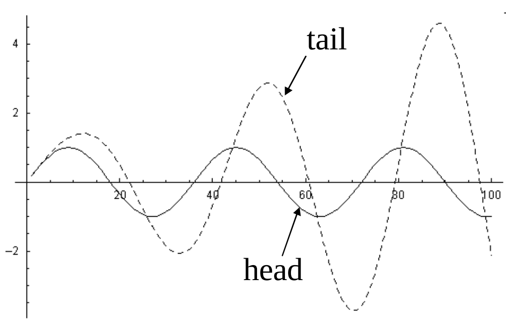


Note: $y(s; z)$ describes a free *betatron* oscillation along linac (FODO) lattice (as a function of s)

2 particle model

Effect of coherent betatron oscillation

- head *resonantly* drives the tail



head eom (Hill's equation):

$$y_1'' + k_b^2 y_1 = 0$$

solution:

$$y_1(s) = \sqrt{a b(s)} \sin\left(j(s) + j_0\right)$$

tail eom:

$$y_2'' + k^2 y_2 = y_1 \frac{W'_\perp \frac{q}{2} 2s_z}{E_{beam}}$$

resonantly driven oscillator

BNS Damping

If both macroparticles have an initial offset y_0 then particle 1 undergoes a sinusoidal oscillation, $y_1=y_0\cos(k_ß s)$. What happens to particle 2?

$$y_2 = y_0\left[\cos\left(k_b s\right)+s\sin\left(k_b s\right)\frac{W'_\perp qs_z}{2k_b E_{beam}}\right]$$

Qualitatively: an additional oscillation out-of-phase with the betatron term which grows monotonically with s.

How do we beat it? Higher beam energy, stronger focusing, lower charge, shorter bunches, or a damping technique recommended by Balakin, Novokhatski, and Smirnov *(BNS Damping)*

curtesy: P. Tenenbaum (SLAC)

BNS Damping

Imagine that the two macroparticles have different betatron frequencies, represented by different focusing constants $k_{ß1}$ and $k_{ß2}$

The second particle now acts like an undamped oscillator driven off its resonant frequency by the wakefield of the first. The difference in trajectory between the two macroparticles is given by:

$$y_2 - y_1 = y_0\left(1-\frac{W'_\perp qs_z}{E_{beam}}\frac{1}{k_{b2}^2-k_{b1}^2}\right)\left[\cos\left(k_{b2}s\right)-\cos\left(k_{b1}s\right)\right]$$

curtesy: P. Tenenbaum (SLAC)

BNS Damping

The wakefield can be locally cancelled (ie, cancelled at all points down the linac) if:

$$\frac{W'_{\perp} q s_z}{E_{beam}} \frac{1}{k_{b2}^2 - k_{b1}^2} = 1$$

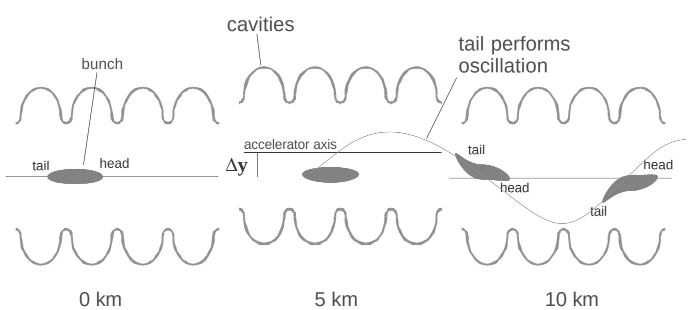
This condition is often known as “autophasing.”

It can be achieved by introducing an energy difference between the head and tail of the bunch. When the requirements of discrete focusing (ie, FODO lattices) are included, the autophasing RMS energy spread is given by:

$$\frac{s_E}{E_{beam}} = \frac{1}{16} \frac{W'_{\perp} q s_z}{E_{beam}} \frac{L_{cell}^2}{\sin^2(p n_b)}$$

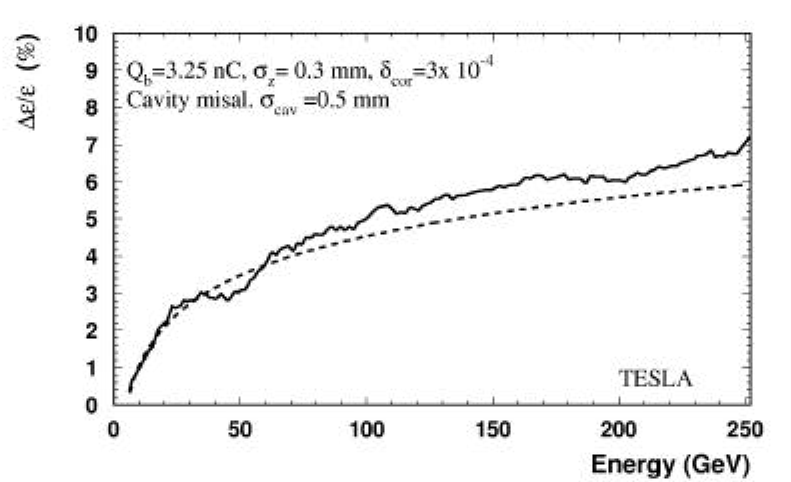
curtesy: P. Tenenbaum (SLAC)

Wakefields (alignment tolerances)



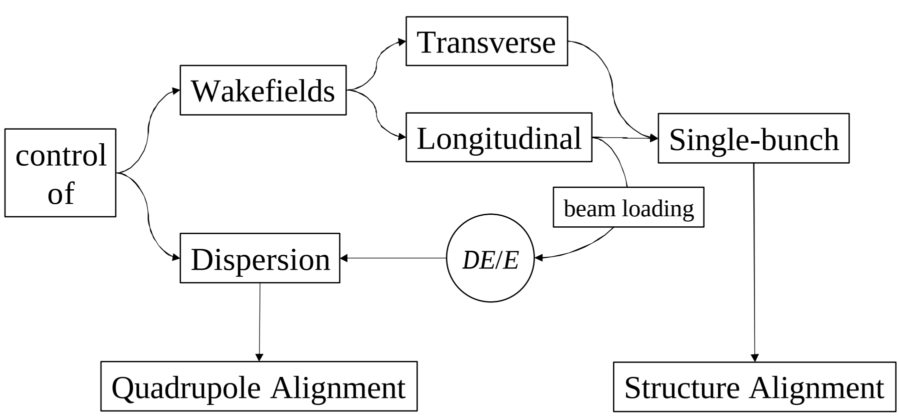
$$\frac{\Delta e}{e} \propto \frac{N^2 W_{\perp}^2}{e} b \left[\left(\frac{E_f}{E_i} \right) - 1 \right] \left\langle \Delta y_c^2 \right\rangle$$

Cavity Misalignments



Wakefields and Beam Dynamics

The preservation of (RMS) Emittance!



Emittance tuning in the Linac

Consider linear collider parameters:

- DR produces tiny vertical emittances ($ge_y \sim 20$nm)
- LINAC must preserve this emittance!
  - strong wakefields (structure misalignment)
  - dispersion effects (quadrupole misalignment)
- Tolerances too tight to be achieved by surveyor during installation

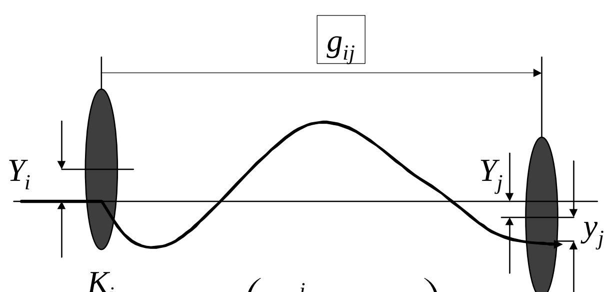
$\Rightarrow$ Need beam-based alignment

mma!

Basics (linear optics)

thin-lens quad approximation: $Dy' = -KY$

$$g_{ij} = \left.\frac{\partial y_i}{\partial y'_j}\right|_{y'_j=0} = R_{34}(i, j)$$



$$y_i = \left(-\sum_{i=1}^{j} g_{ij} K_i Y_j\right) - Y_i$$

linear system: just superimpose oscillations caused by quad kicks.

Introduce matrix notation

Original Equation $$y_i = \left(-\sum_{i=1}^{j} g_{ij} K_i Y_j \right) - Y_i$$

Defining *Response Matrix* $\mathbf{Q}$: $$\mathbf{Q} = \mathbf{G} \cdot \mathbf{diag(K)} + \mathbf{I}$$

Hence beam offset becomes $$\mathbf{y} = -\mathbf{Q} \cdot \mathbf{Y}$$

$\mathbf{G}$ is lower diagonal: $$\mathbf{G} = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ g_{21} & 0 & 0 & 0 & \cdots \\ g_{31} & g_{32} & 0 & 0 & \cdots \\ g_{41} & g_{42} & g_{43} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Dispersive Emittance Growth

Consider effects of finite energy spread in beam d_{RMS}

chromatic response matrix: $$\mathbf{Q}(d) = \mathbf{G}(d) \cdot \mathbf{diag}\left(\frac{\mathbf{K}}{1+d} \right) + \mathbf{I}$$

lattice chromaticity

dispersive kicks

$$\mathbf{G}(d) = \mathbf{G}(0) + \left. \frac{\partial \mathbf{G}}{\partial d} \right|_{d=0}$$

$$R_{34}(d) = R_{34}(0) + T_{346} d$$

dispersive orbit: $$?_y \approx \frac{?\,\mathbf{y}(d)}{d} = -\left[\mathbf{Q}(d) - \mathbf{Q}(0) \right] \cdot \mathbf{Y}$$

What do we measure?

BPM readings contain additional errors:

$\mathbf{b}_{\text{offset}}$ static offsets of monitors wrt quad centres

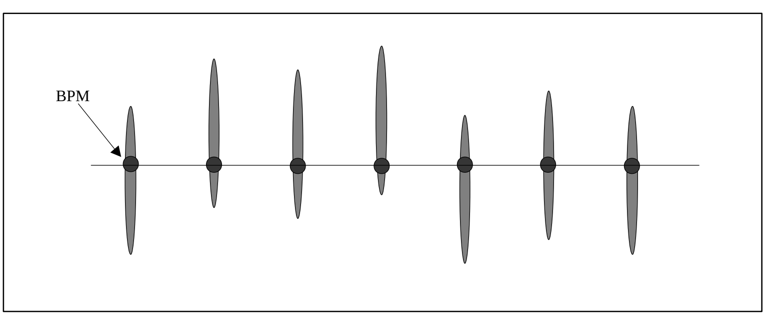
$\mathbf{b}_{\text{noise}}$ one-shot measurement noise (resolution s_{RES})

$$\mathbf{y}_{\text{BPM}} = -\mathbf{Q}\cdot\mathbf{Y} + \underbrace{\mathbf{b}_{\text{offset}}}_{\substack{\text{fixed from}\\ \text{shot to shot}}} + \underbrace{\mathbf{b}_{\text{noise}}}_{\substack{\text{random}\\ \text{(can be averaged}\\ \text{to zero)}}} + \mathbf{R}\cdot\mathbf{y}_0 \qquad \mathbf{y}_0 = \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$$

launch condition

In principle: all BBA algorithms deal with $\mathbf{b}_{\text{offset}}$

Scenario 1: Quad offsets, but BPMs aligned



Assuming:

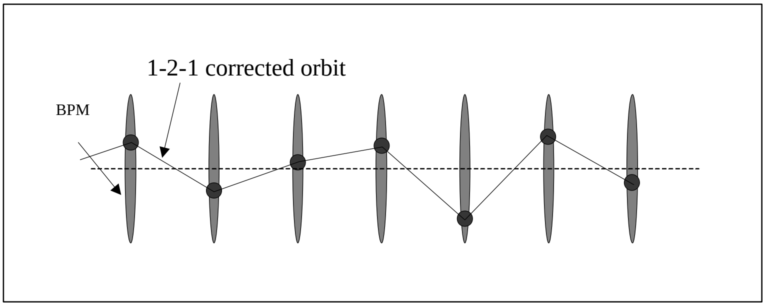
- a BPM adjacent to each quad

- a ‘steerer’ at each quad

steerer { quad mover / dipole corrector

simply apply one to one steering to orbit

Scenario 2: Quads aligned, BPMs offset



one-to-one correction BAD!

Resulting orbit not <u>Dispersion Free</u> $\Rightarrow$ <u>emittance growth</u>

Need to find a steering algorithm which effectively puts BPMs on (some) reference line

real world scenario: some mix of scenarios 1 and 2

BBA

- Dispersion Free Steering (DFS)
  - Find a set of steerer settings which minimise the dispersive orbit
  - in practise, find solution that minimises difference orbit when 'energy' is changed
  - Energy change:
    - true energy change (adjust linac phase)
    - scale quadrupole strengths
- Ballistic Alignment
  - Turn off accelerator components in a given section, and use 'ballistic beam' to define reference line
  - measured BPM orbit immediately gives $\mathbf{b}_{\text{offset}}$ wrt to this line

DFS

Problem:

$$? \mathbf{y} = -\left[\mathbf{Q}(\frac{\Delta E}{E}) - \mathbf{Q}(0) \right] \left(\frac{\Delta E}{E} \right) \cdot \mathbf{Y}$$

$$\equiv \mathbf{M} \left(\frac{\Delta E}{E} \right) \cdot \mathbf{Y}$$

Note: taking difference orbit D**y** removes $\mathbf{b}_{\text{offset}}$

Solution (trivial): $\mathbf{Y} = \mathbf{M}^{-1} \cdot ? \mathbf{y}$

Unfortunately, not that easy because of noise sources:

$$? \mathbf{y} = \mathbf{M} \cdot \mathbf{Y} + \mathbf{b}_{\text{noise}} + \mathbf{R} \cdot \mathbf{y}_0$$

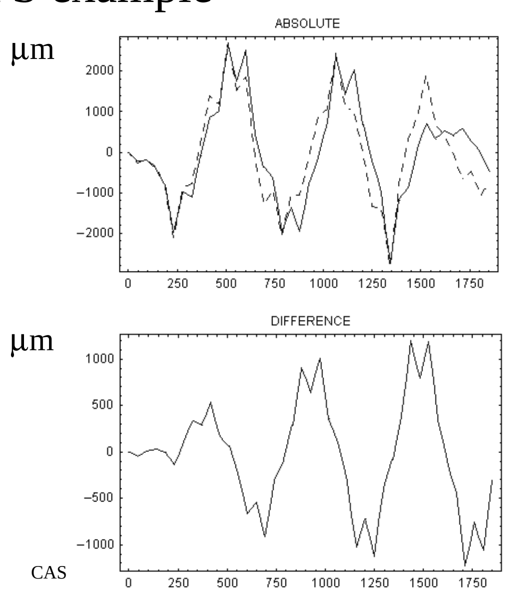
DFS example

300µm random quadrupole errors

20% *DE/E*

No BPM noise

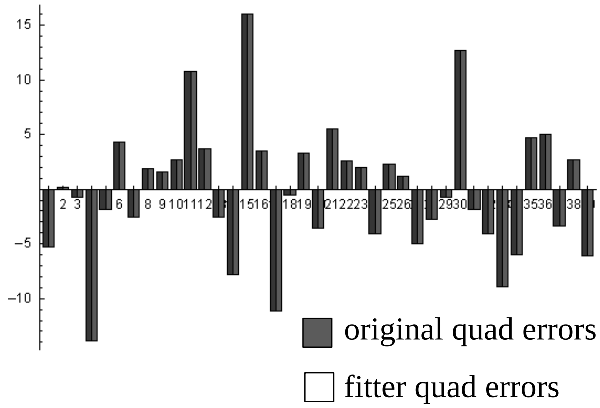
No beam jitter



DFS example

Simple solve

$$\mathbf{Y} = \mathbf{M}^{-1} \cdot \mathbf{?}\,\mathbf{y}$$



In the absence of errors, works exactly

Resulting orbit is flat

⇒Dispersion Free

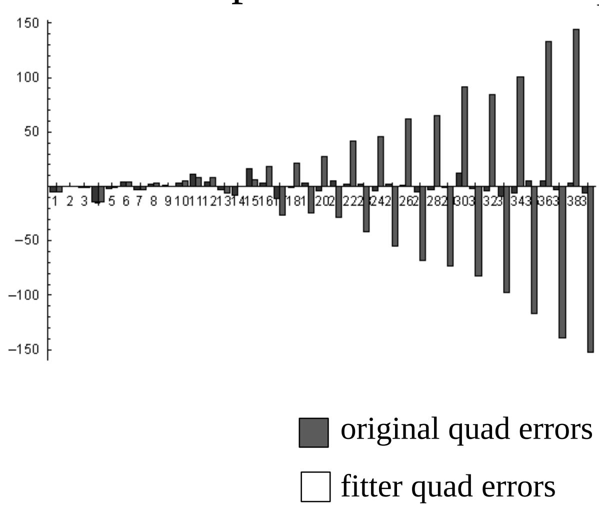
(perfect BBA)

Now add 1μm random BPM noise to measured difference orbit

DFS example

Simple solve

$$\mathbf{Y} = \mathbf{M}^{-1} \cdot \mathbf{?}\,\mathbf{y}$$

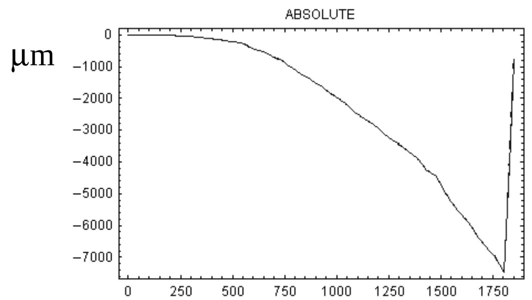


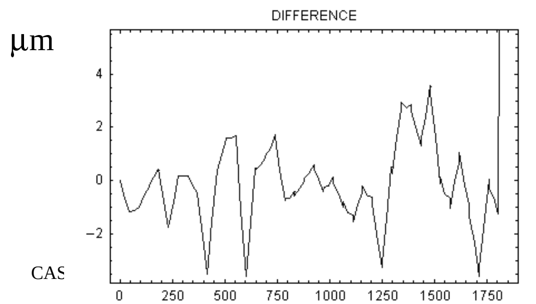
Fit is ill-conditioned!

DFS example

Solution is still Dispersion Free

but several mm off axis!





DFS: Problems

- Fit is ill-conditioned
  - with BPM noise DF orbits have very large unrealistic amplitudes.
  - Need to constrain the absolute orbit

$$\text{minimise} \quad \frac{?\mathbf{y}\cdot ?\mathbf{y}^{\mathrm{T}}}{2s_{\mathrm{res}}^{2}} + \frac{\mathbf{y}\cdot\mathbf{y}^{\mathrm{T}}}{s_{\mathrm{res}}^{2} + s_{\mathrm{offset}}^{2}}$$

- Sensitive to initial launch conditions (steering, beam jitter)
  - need to be fitted out or averaged away

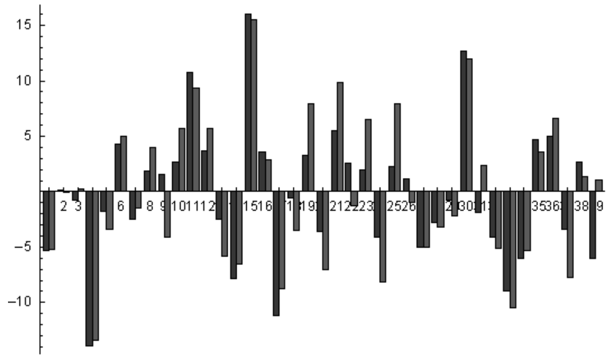
$\mathbf{R}\cdot\mathbf{y}_0$

DFS example

Minimise

$$\frac{\mathbf{?y \cdot ?y}^{T}}{2s_{\mathrm{res}}^{2}} + \frac{\mathbf{y \cdot y}^{T}}{s_{\mathrm{res}}^{2} + s_{\mathrm{offset}}^{2}}$$

absolute orbit now constrained



original quad errors

fitter quad errors

remember

$s_{\mathrm{res}} = 1\mu\mathrm{m}$

$s_{\mathrm{offset}} = 300\mu\mathrm{m}$

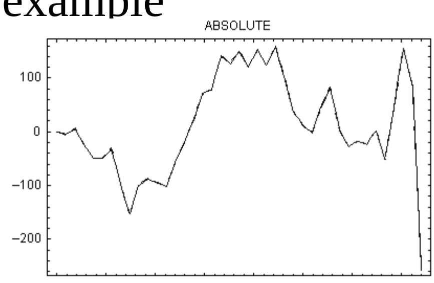
DFS example

Solutions much better behaved!

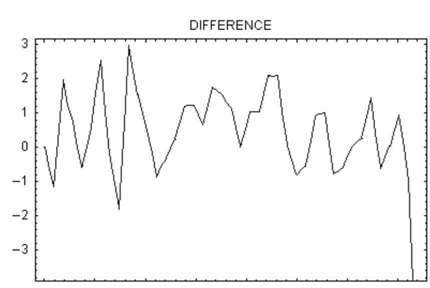
! Wakefields !

Orbit *not quite* Dispersion Free, but very close

μm



μm



DFS practicalities

- Need to align linac in sections (bins), generally overlapping.
- Changing energy by 20%
  - quad scaling: only measures dispersive kicks from quads. Other sources ignored (not measured)
  - Changing energy upstream of section using RF better, but beware of RF steering (see initial launch)
  - dealing with energy mismatched beam may cause problems in practise (apertures)
- Initial launch conditions still a problem
  - coherent β-oscillation looks like dispersion to algorithm.
  - can be random jitter, or RF steering when energy is changed.
  - need good resolution BPMs to fit out the initial conditions.
- Sensitive to model errors (**M**)

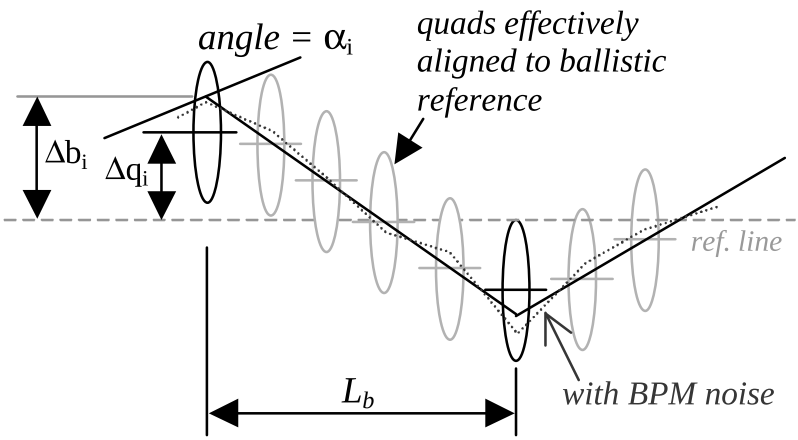
Ballistic Alignment

- Turn of all components in section to be aligned [magnets, and RF]
- use ‘ballistic beam’ to define straight reference line (BPM offsets)

$$y_{\mathrm{BPM},i} = y_0 + s_i y_0' + b_{\mathrm{offset},i} + b_{\mathrm{noise},i}$$

- Linearly adjust BPM readings to arbitrarily zero last BPM
- restore components, steer beam to adjusted ballistic line

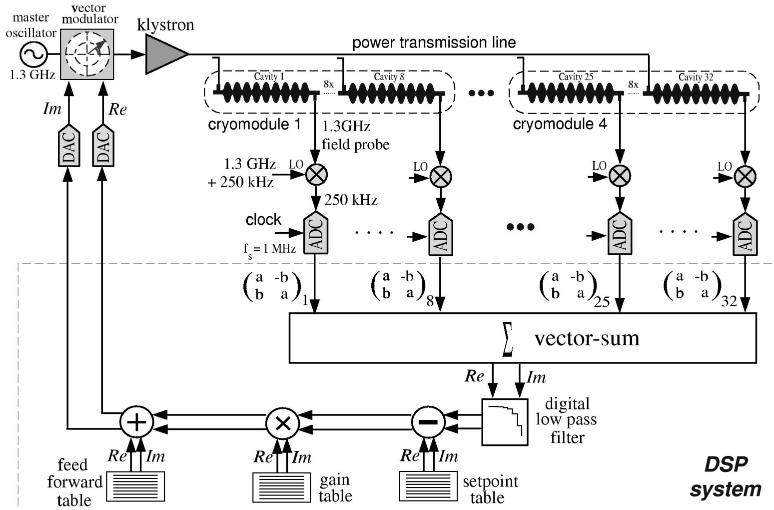
Ballistic Alignment



Ballistic Alignment: Problems

- Controlling the downstream beam during the ballistic measurement
  - large beta-beat
  - large coherent oscillation
- Need to maintain energy match
  - scale downstream lattice while RF in ballistic section is off
- use feedback to keep downstream orbit under control

TESLA LLRF


master
oscillator
1.3 GHz
vector
modulator
klystron
power transmission line
Cavity 1
8x
Cavity 8
Cavity 25
8x
Cavity 32
cryomodule 1
cryomodule 4
1.3GHz
field probe
Im
Re
DAC
DAC
1.3 GHz
+ 250 kHz
LO
250 kHz
clock
ADC
f_s = 1 MHz
vector-sum
Re
Im
digital
low pass
filter
feed
forward
table
gain
table
setpoint
table
DSP
system