Components of High power protons linacs

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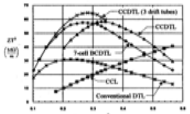
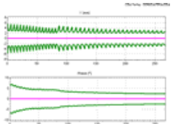
Topics

Some common structures

- The Radio Frequency Quadrupole (RFQ) 
- The Drift-Tube Linacs (DTLs) 
- The Coupled Cavity Linac (CCL) 
- The Superconducting Cavity Linac (SCL) 

Design of the linac

- Choice of the structure 

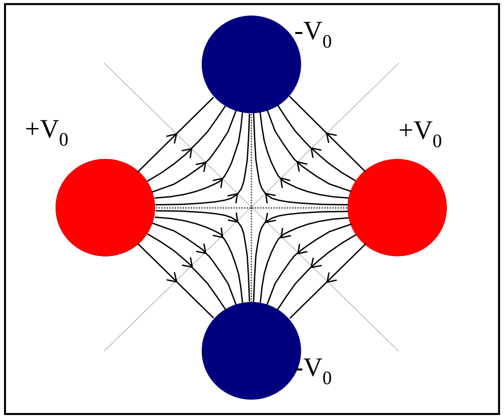
- Matching 
- Space-charge and emittance growth

The RFQ (RadioFrequency Quadrupole)

Goals of a RFQ

• Bunch the beam with the maximum efficiency and the lower longitudinal emittance as possible,

• Accelerate the beam with the lowest particle losses and emittance growth as possible.

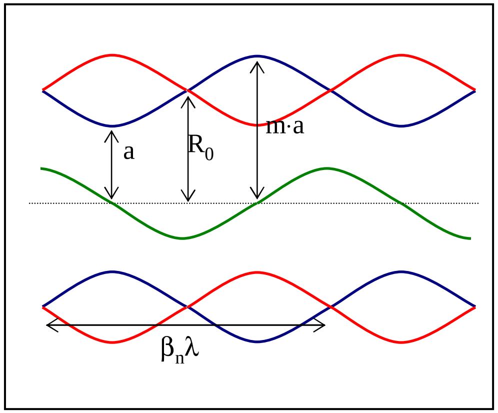


Principles

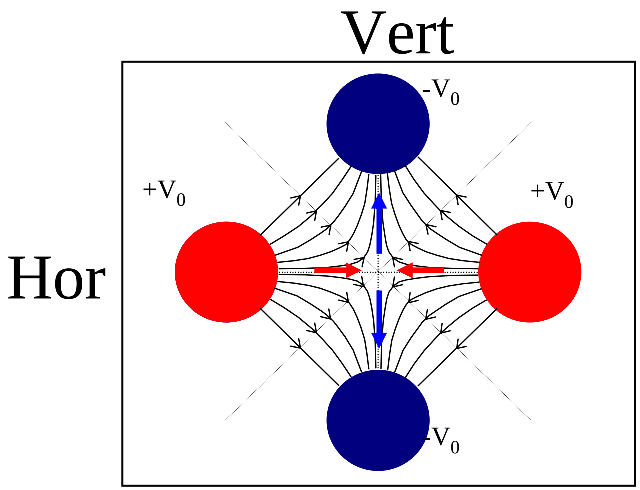
• The RFQ tank is a resonnant cavity.

• Close to the beam axis, a RF electric field is created between poles with transverse quadrupolaire symmetry. The electric field contributes to beam transverse confinement (a time varying FODO channel).

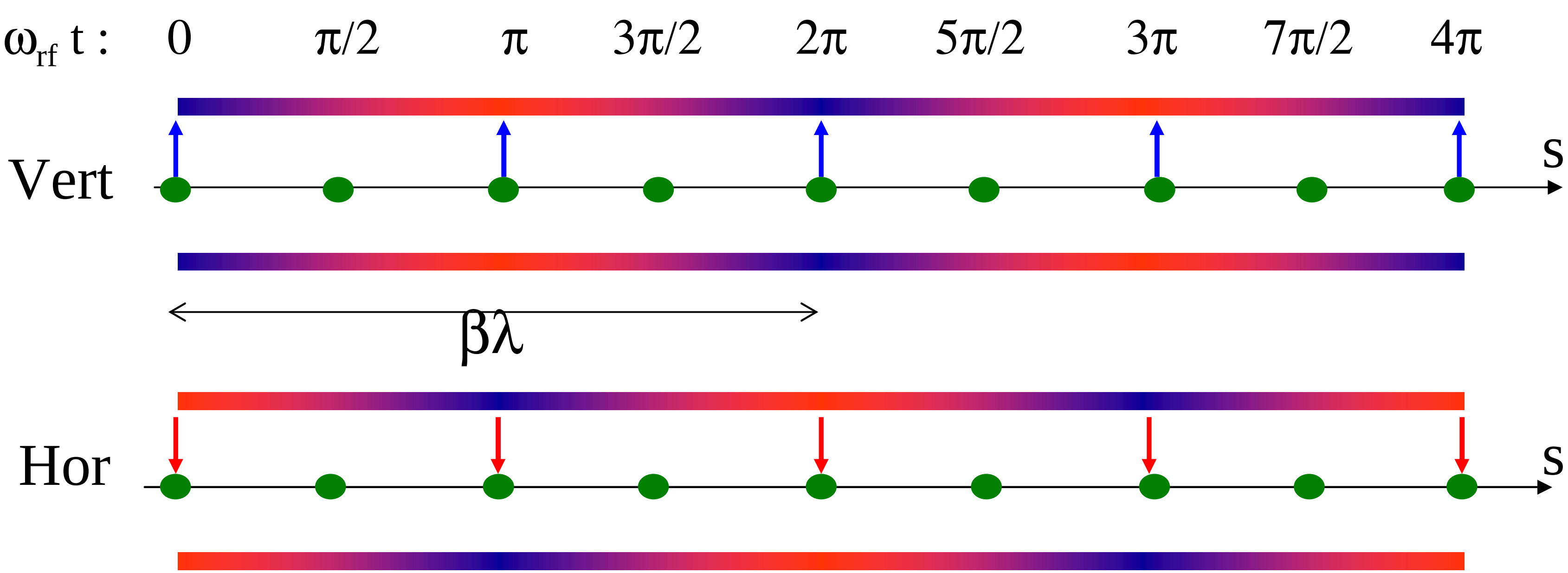
• The poles are progressively (longitudinally) modulated to add a longitudinal component to the field. This field contributes to 1, the beam bunching, and 2, the beam acceleration, once the beam is bunched.



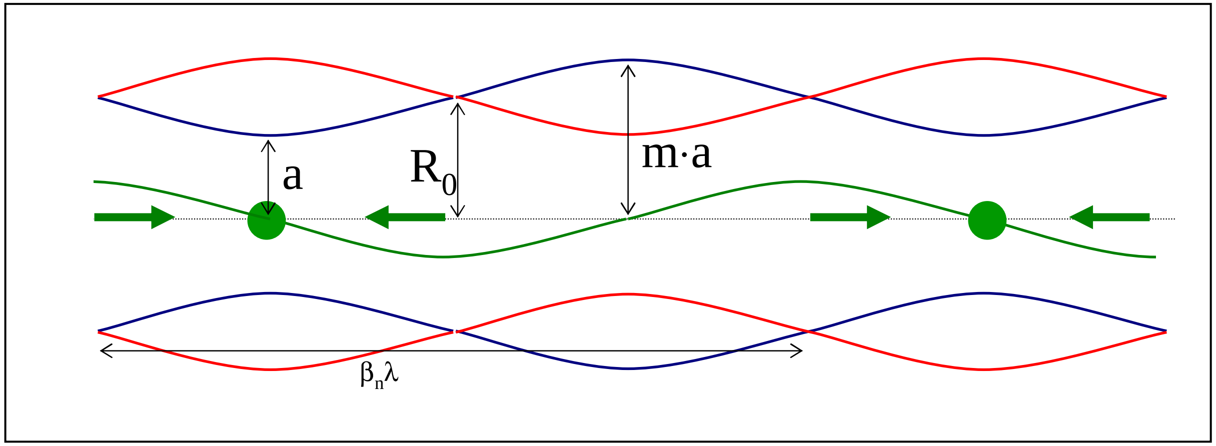
Transverse focusing



$$V_0 \Rightarrow V_0 \cdot \cos(\omega_{rf}\, t)$$



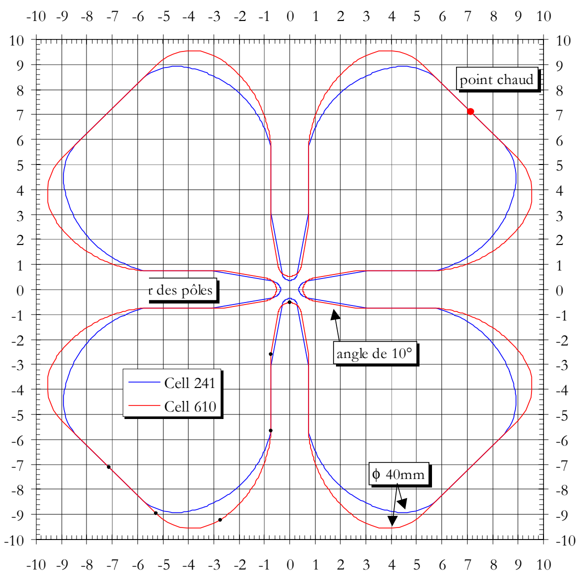
Longitudinal focusing (bunching)

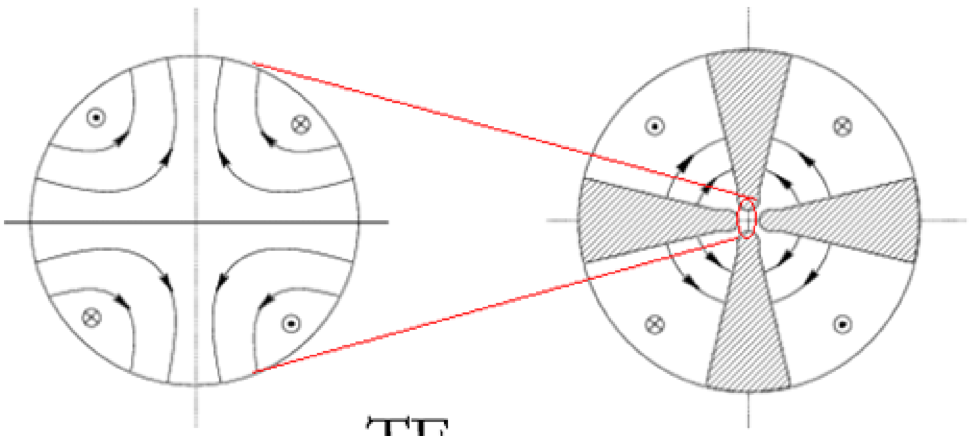


• Pole modulation induces longitudinal electric field component,
• Electric field amplitude is quasi-sinusoidal with *s*,
• By progressively increasing the modulation, the beam is adiabatically bunched.

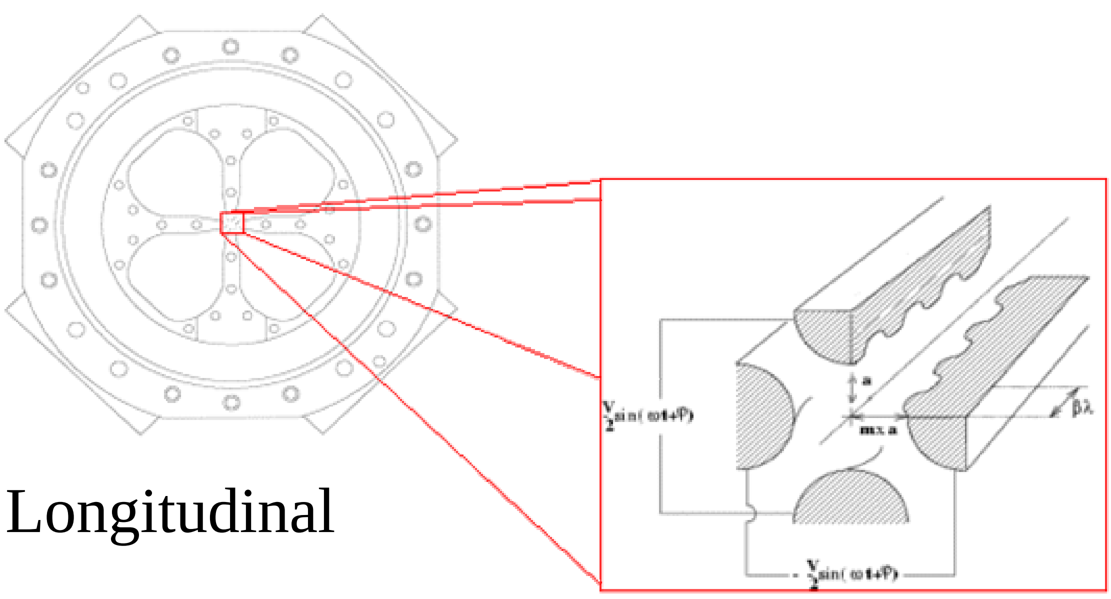
The Pole profile (1)

Transverse



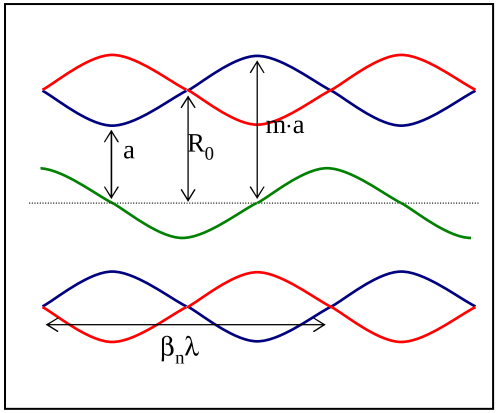


TE_{210}



Longitudinal

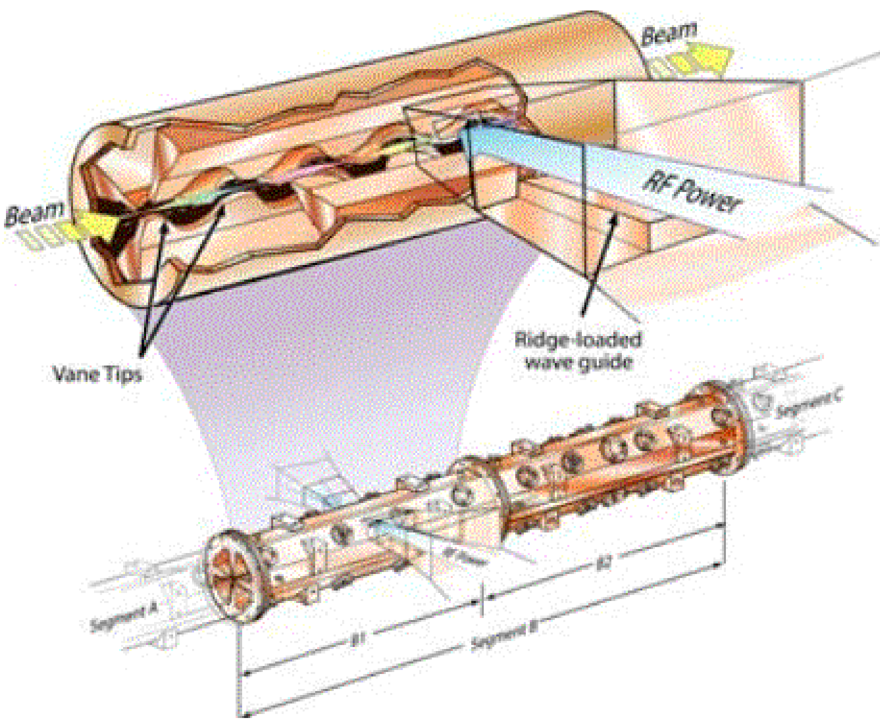
The Pole profile (2)

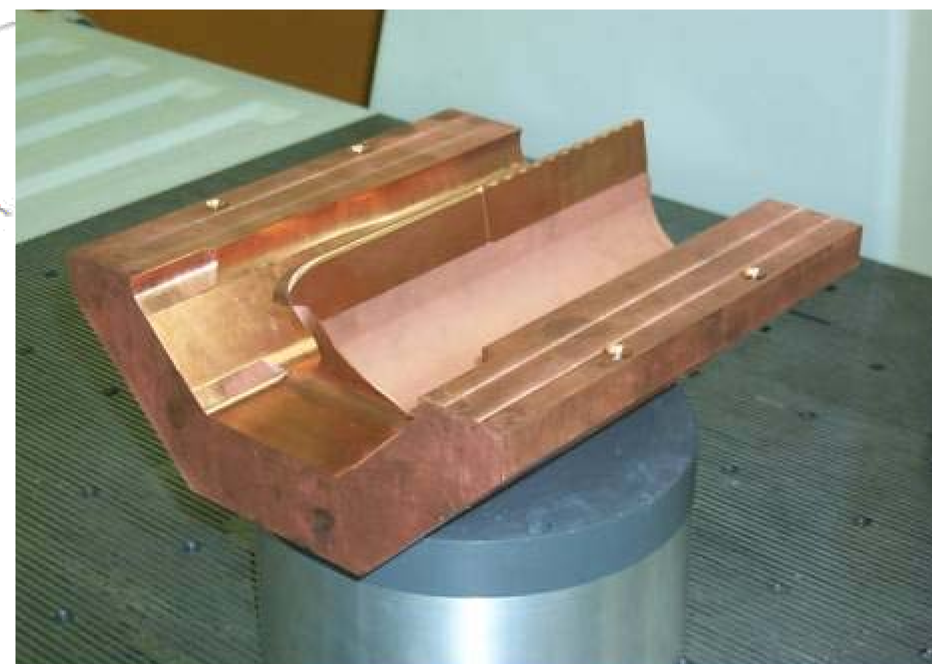


m is the modulation ($1 \rightarrow 2$)

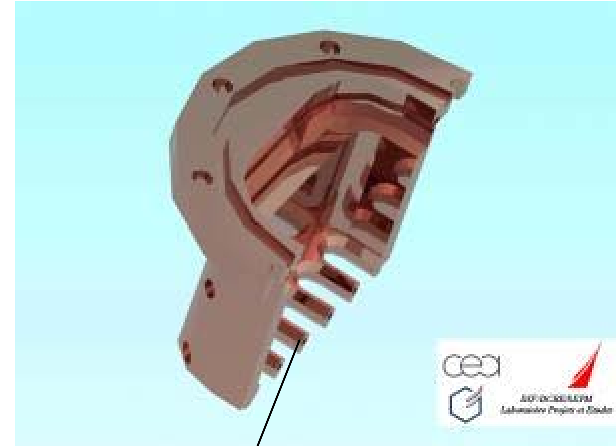
R_0 is the pole mean radius

$\beta_0\lambda$ is the pole modulation period



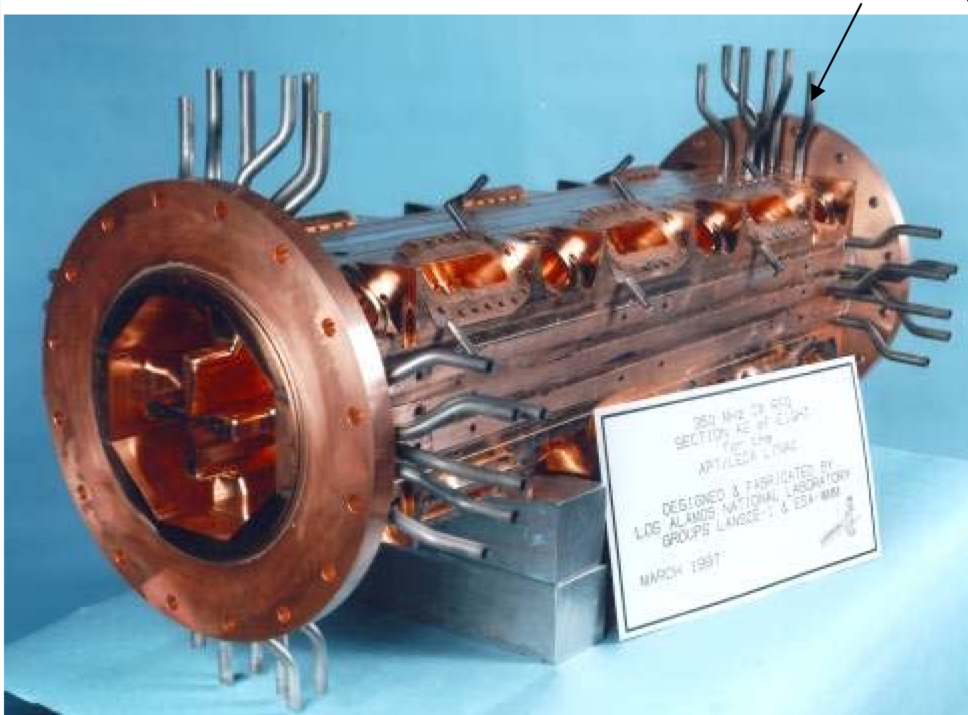


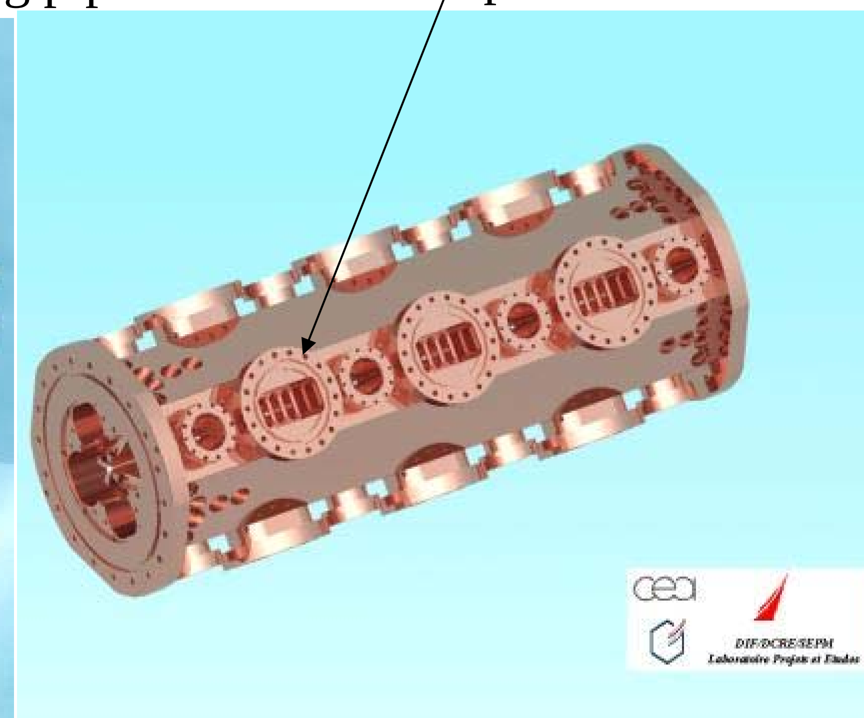
The RF Tank



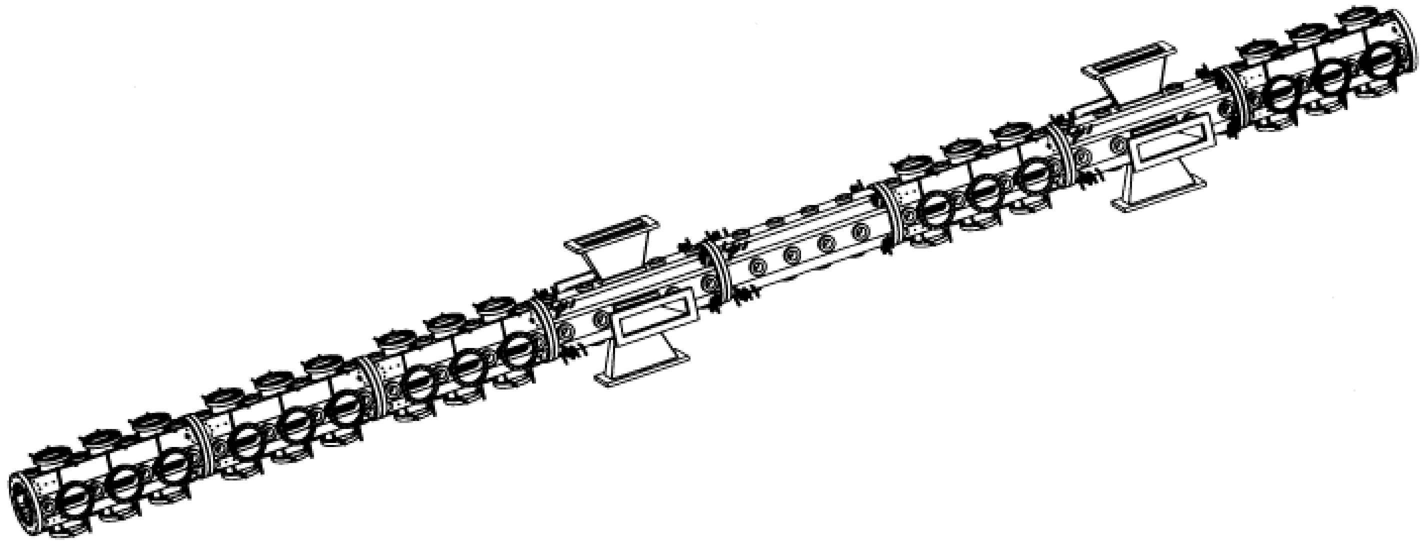
Cooling pipe

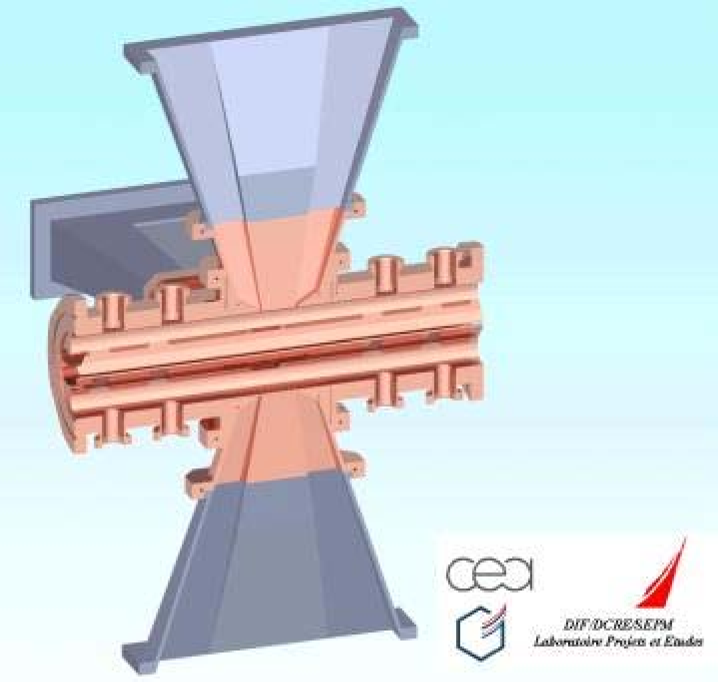
Vacuum port





The RF Input





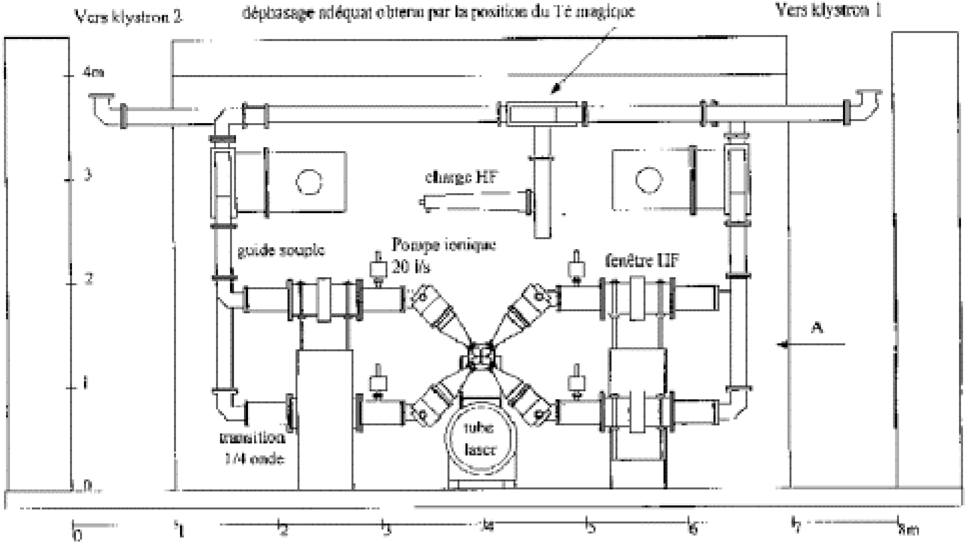


fig 2a

Vue du canon des guides RF d'alimentation du RFQ dans le cas d'une alimentation des quatre cavités par chaque klystron

Thomson CSF /HEA 215/97

Electric field calculation

Helmoltz equation : $$\vec{\nabla}^2 \vec{E} + \left(\frac{2\pi}{\lambda}\right)^2 \cdot \vec{E} = \vec{0}$$

If $\lambda >> R_0 \Rightarrow$ quasistatic approximation : $$\begin{cases} \vec{E} = -\vec{\nabla} U \\ \vec{\nabla}^2 U = 0 \end{cases}$$

$$U(r,\theta,s,t) = V(r,\theta,s) \cdot \sin(\omega t + \phi)$$

In cylindrical coordinates

$$V(r,\theta,s) = \sum_{i=0}^{\infty} X_i \cdot r^{2\cdot(2i+1)} \cdot \cos(2\cdot(2i+1)\cdot\theta)$$

No modulation

$$+ \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} A_{ni} \cdot I_{2j}(knr) \cdot \cos(2j\theta) \cdot \cos(kns)$$

Modulation

$$k = \frac{2\pi}{\beta_s \lambda}$$

Coefficients X_i and A_{ni} depend on the electrodes geometry

Two-terms potential description

$$V(r,\theta,s)=V_0\cdot\left(X\cdot\frac{r^2}{a^2}\cdot\cos 2\theta+A\cdot I_0(kr)\cdot\cos ks\right)$$

V_0 : pole voltage

$$X=\frac{I_0(ka)+I_0(kma)}{m^2 I_0(ka)+I_0(kma)}$$

The focusing efficiency (X=1 if m=1)

$$A=\frac{m^2-1}{m^2 I_0(ka)+I_0(kma)}$$

The accelerating efficiency (A=1 if m=1)

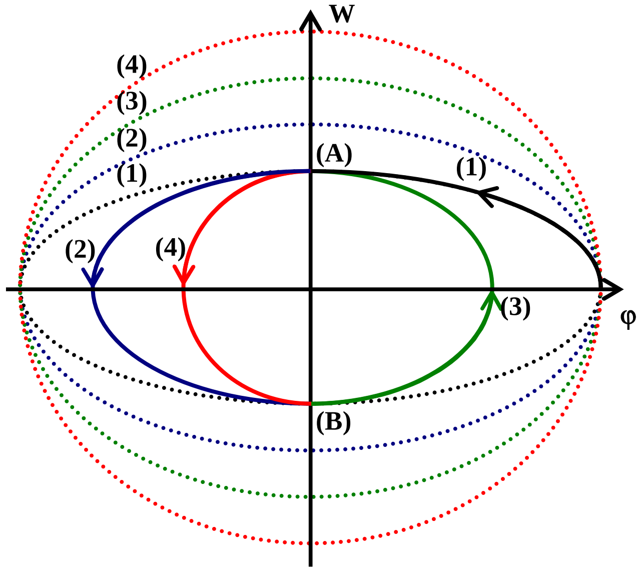
Electric field :

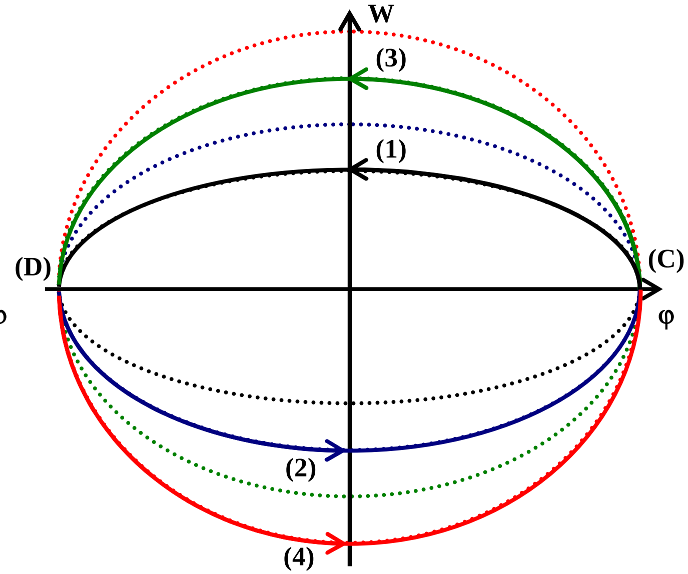
$$\left\{\begin{array}{l} E_x=-V_0\cdot\left(\frac{2\cdot X}{a^2}+A\cdot k\frac{I_1(kr)}{r}\cdot\cos ks\right)\cdot x \\ E_y=V_0\cdot\left(\frac{2\cdot X}{a^2}-A\cdot k\frac{I_1(kr)}{r}\cdot\cos ks\right)\cdot y \\ E_z=V_0\cdot A\cdot k\cdot I_0(kr)\cdot\sin ks \end{array}\right\}\cdot\sin(\omega t+\phi)$$

The bunching process (1)

Lets study the phase oscillation amplitude of a particle in an continuous focusing channel with an increasing confinement force .

(1), (2), (3), (4) : increasing confinement force





The field is increased when the particle is in (A) or (B)
⇒ phase amplitude ↓

The field is increased when the particle is in (C) or (D)
⇒ phase amplitude →

The bunching process (2)

In the RFQ, the pole modulation is "adiabatically" increased with a constant period (no average acceleration) resulting in an increased of the electric field component. The effect is then a longitudinal bunching of the beam.

The motion Hamiltonian is :

$$H_{\varphi w} = B \cdot \frac{w^2}{2} - C \cdot E_z(s) \cdot (\cos\varphi - 1)$$

Giving the phase evolution equation :

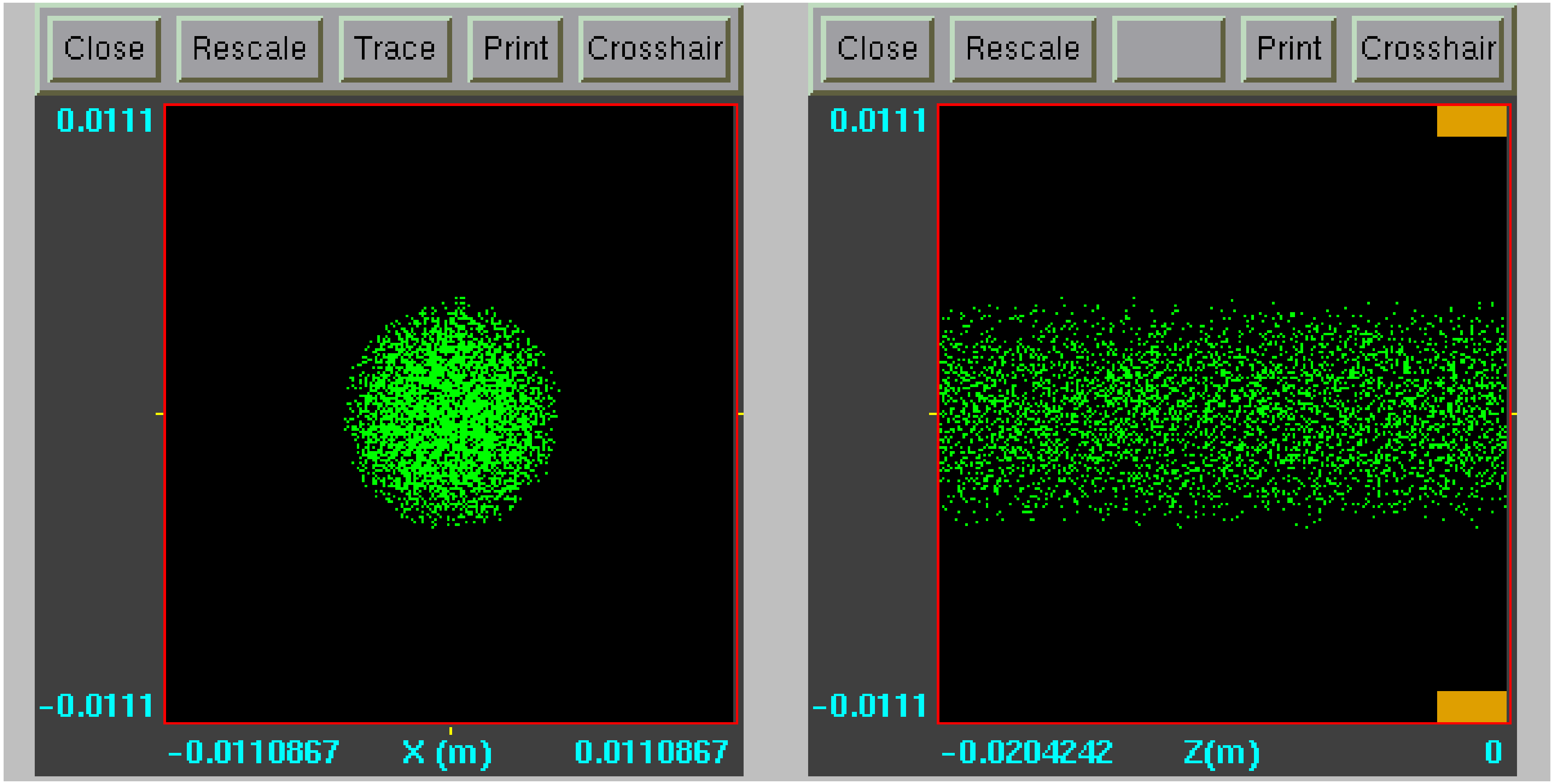
$$\frac{d^2\varphi}{ds^2} + BC \cdot E_z(s) \cdot \sin\varphi = 0$$

If $E_z(s)$ increases adiabatically : $\left(i.e.\ 0 < \frac{dE_z}{ds} << \sqrt{BC \cdot E_z} \cdot \frac{\sin\varphi}{\varphi} \right)$

the phase amplitude decreases in : $\frac{1}{\sqrt{E_z}}$

Which cannot be when E_z close to zero or φ close to π.

The bunching process (3)



Animation from TOUTATIS code (Author : Romuald Duperrier)

The acceleration process

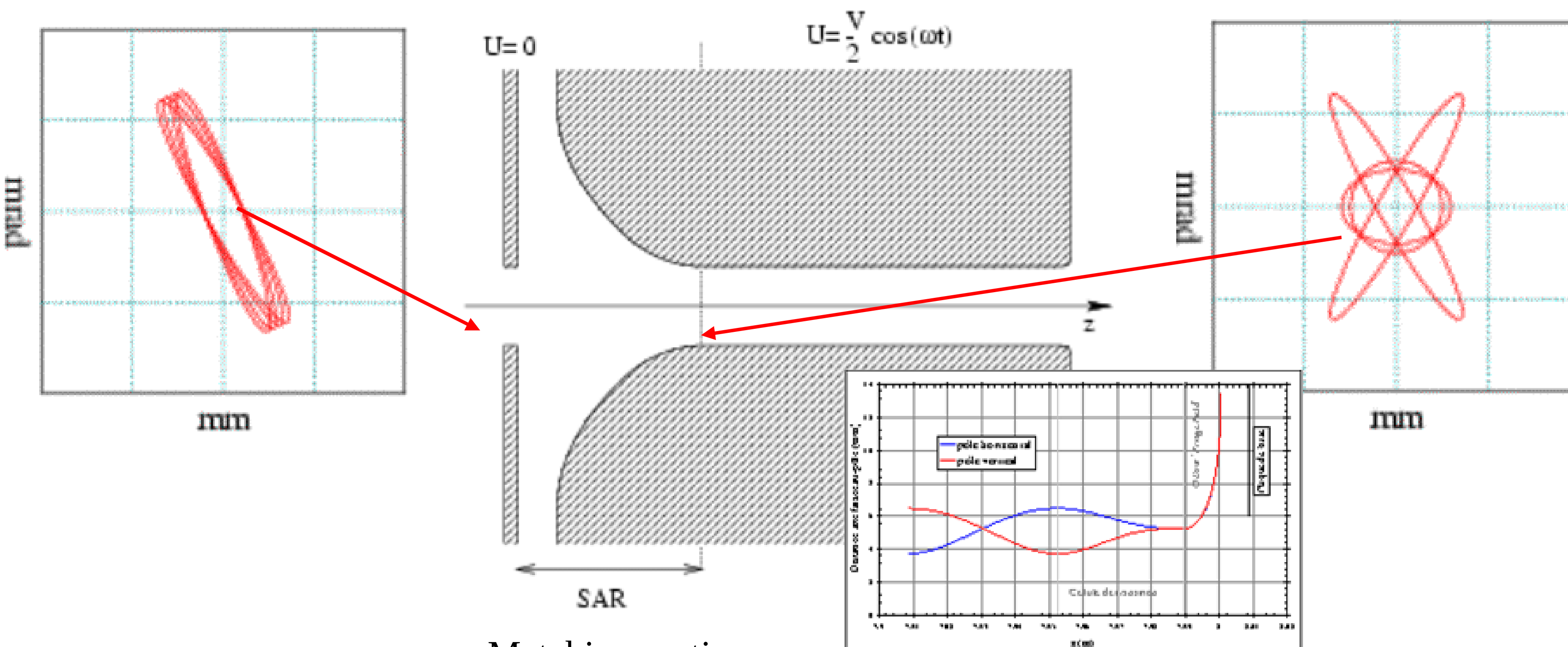
- The acceleration is obtained by progressively increasing the modulation period of the poles.
- This results in the change of the RF phase of the synchronous particle from -90° (no acceleration) to higher values (until ~-30°).
- Acceleration is accompanied by bucket reduction. Beam should be bunched before acceleration process to accelerate more particles as possible.

Matching a beam in a RFQ

The RFQ is some sort of FODO channel with focalisation <u>changing with time</u> rather than with space as in classical FODO channel. The consequence is that at a given position in the RFQ, the matched beam has time-dependant Twiss parameters.

The input beam, coming from the source, is time-independant.

A matching section is then added between the RFQ input and its first cell.



Matching section

Multiparticle simulation codes

PARMTEQ (LANL)
- z is independent variable,
- Fields description using multipolar expansion,
- SCHEFF as space-charge routine,
- Image charge using superposition of point and line charge

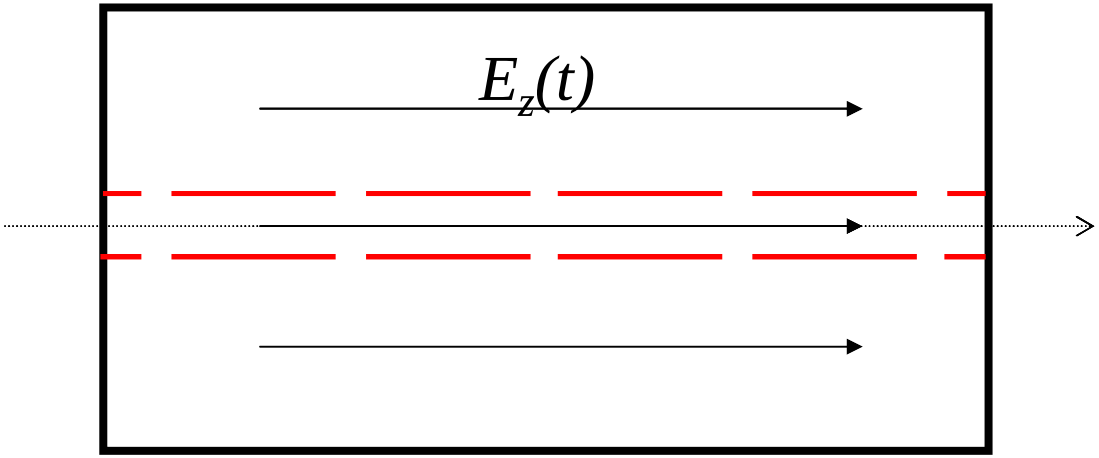
TOUTATIS (CEA, R. Duperrier)
- t is independent variable,
- Fields calculation with multigrid relaxation scheme

LIDOS (Russia, B. Bondarev)
- t is independent variable,
- Fields calculation with Chebyshev accelerator relaxation scheme

Main Challenge in RFQ Design

- Low peak field (< 2 kilp.),
- Number of particles accelerated as high as possible,
- Transmission as good as possible,
- Control of beam losses (< 2MeV is preferred),
- Low emittance growth,
- Low sensitivity to errors (misalignment …) : tolerances calculations,
- Good vacuum properties,
- Easy cooling,
- Good RF stability,
- Easy tuning.

The DTL (Drift Tube Linac) Principle

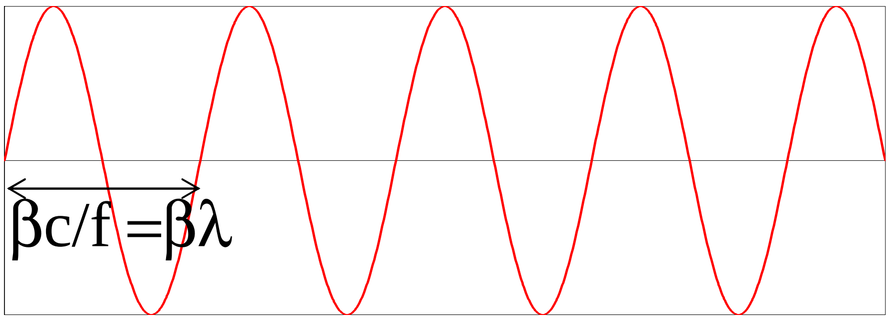


f : RF frequency

ω : RF pulsation

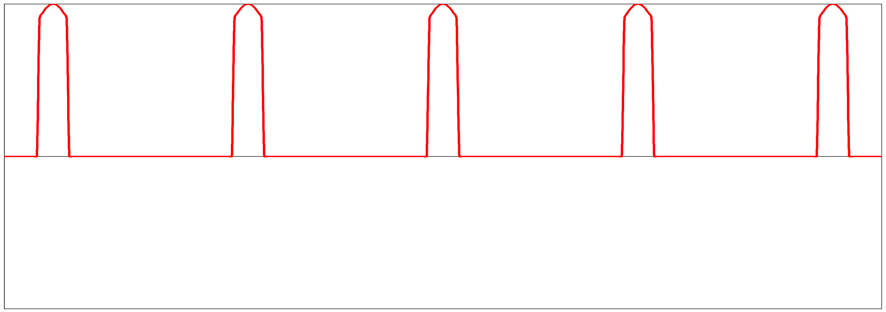
$$E_z(t) = E_0 \cdot cos(\omega t + \varphi_0)$$

βc : beam velocity



$$E_z(t) = E_0 \cdot cos\left(\omega \cdot \frac{s}{\beta c} + \varphi_0\right)$$

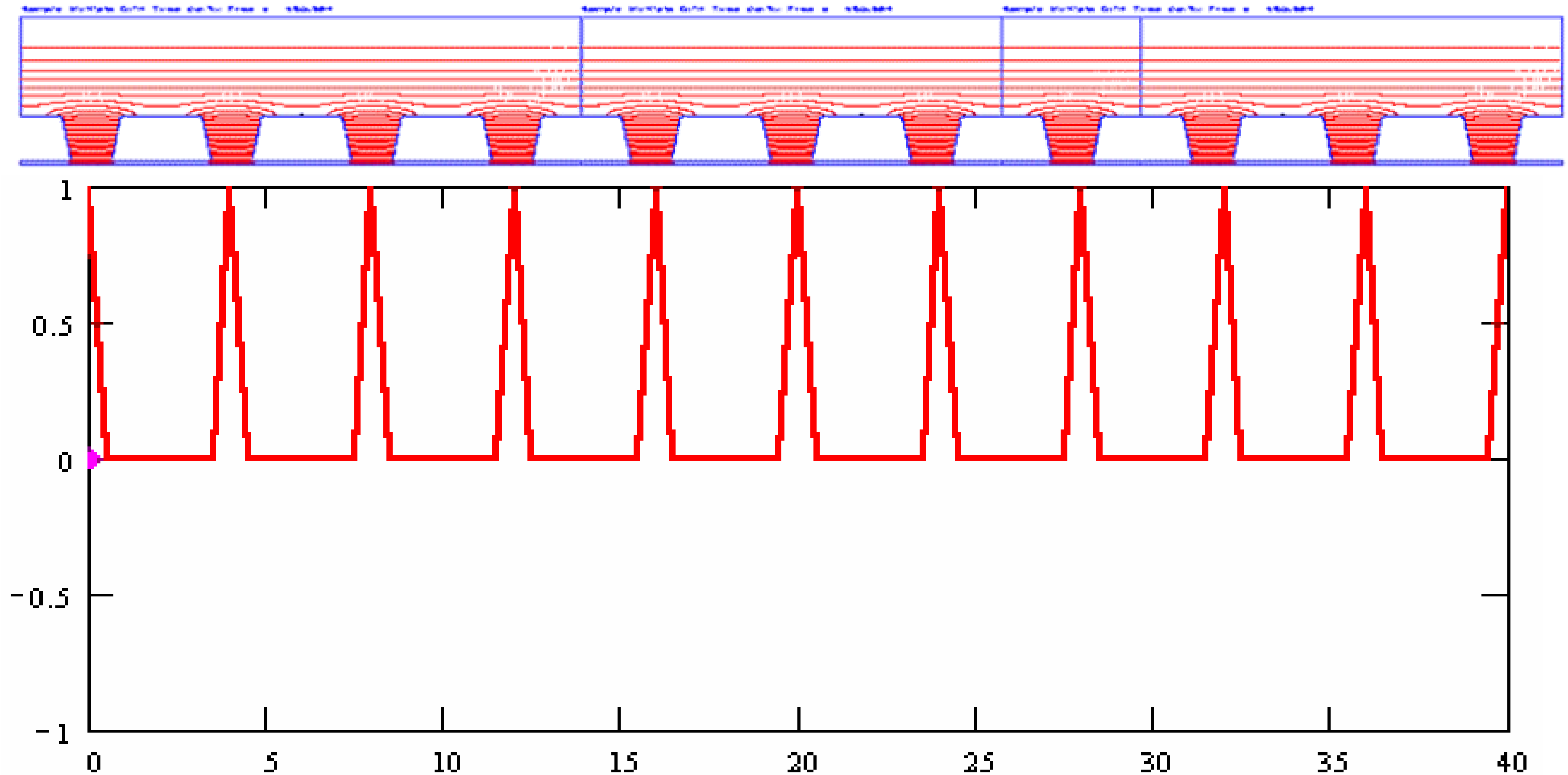
$$\Delta W = q \int E_z(s) \cdot ds \rightarrow 0$$



$$\Delta W = q \int E_z(s) \cdot ds > 0$$

① Linac with coupled cavities (DTL)

Gaps have the same phase. Distances between them are adjusted for synchronism.



Field in cavities

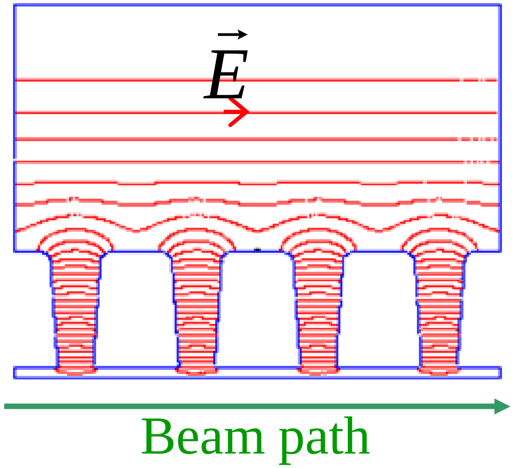
Particle synchronous with the field — Its energy gain

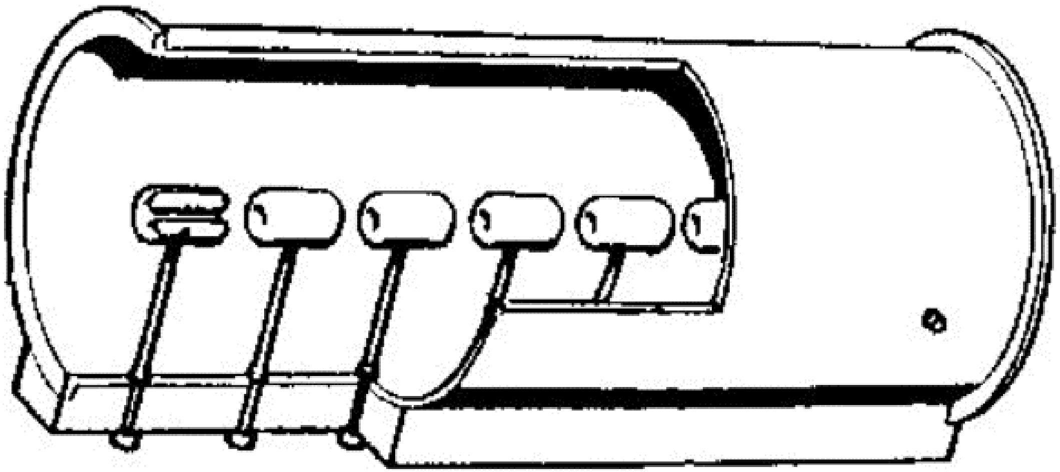
Particle not synchronous with the field — Its energy gain

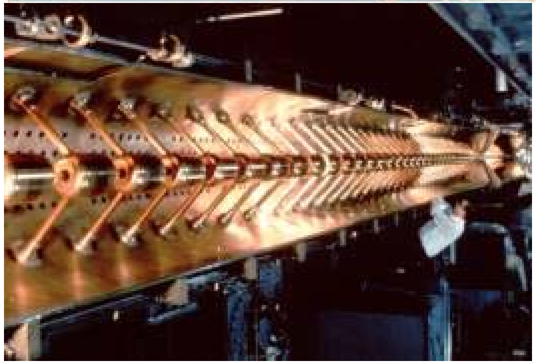
The DTL (Drift Tube Linac)

Principles

- The DTL tank is a resonant cavity.
- The excited mode is the TM_{010} ($TM_{\theta rz}$).
- In the tank, a succession of drift tubes and gaps.
- Drift tube : No field.
- Gap : Accelerating field.
- The drift tubes are linked to the tank wall by stems.
- Quadrupoles can be inserted in drift tubes.

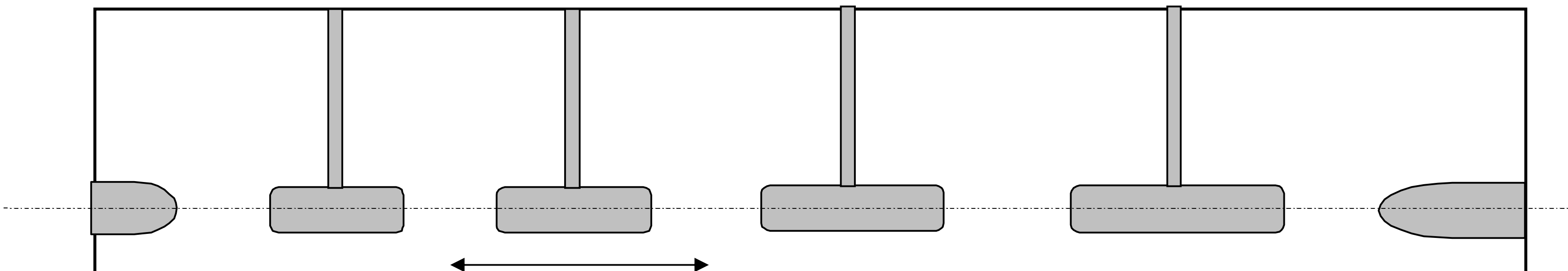






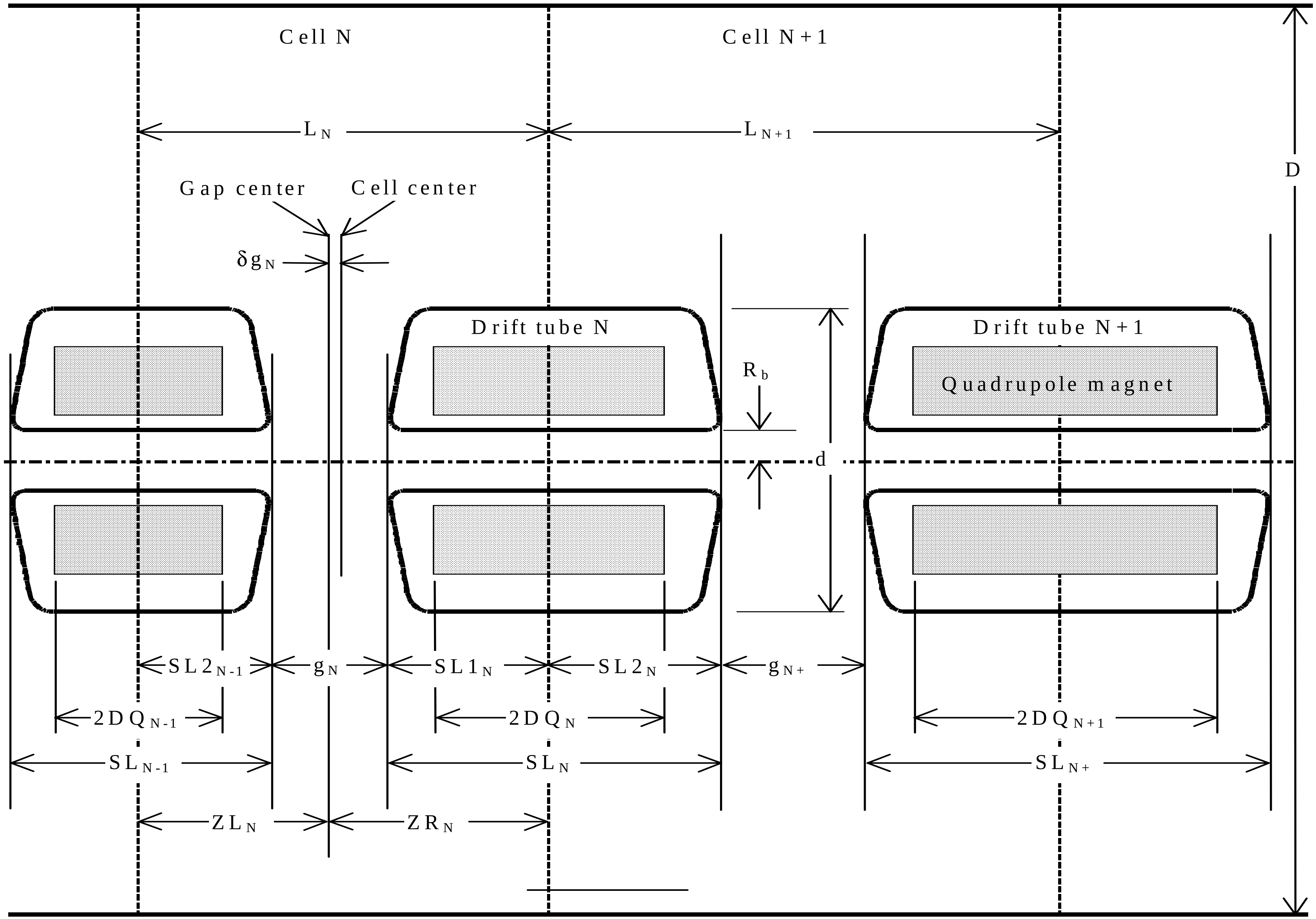
The Synchronism

DTL are used to accelerate beams from ~2 MeV to ~100 MeV.
At this energy, velocity is changing (β from 6.5 % to 40%).
The synchronism is set by adjusting the drift length in order to have a distance of ~$\beta\lambda$ between the gaps.
Energy gain through the gap is given either by integration of the motion equation in the field calculated by SUPERFISH, or by using transit time factor of the cell given by SUPERFISH.



$$D_i = \overline{\beta_{s\,i}}\,\lambda \cdot \left(1 + \frac{\phi_{si+1} - \phi_{si}}{2\pi}\right)$$

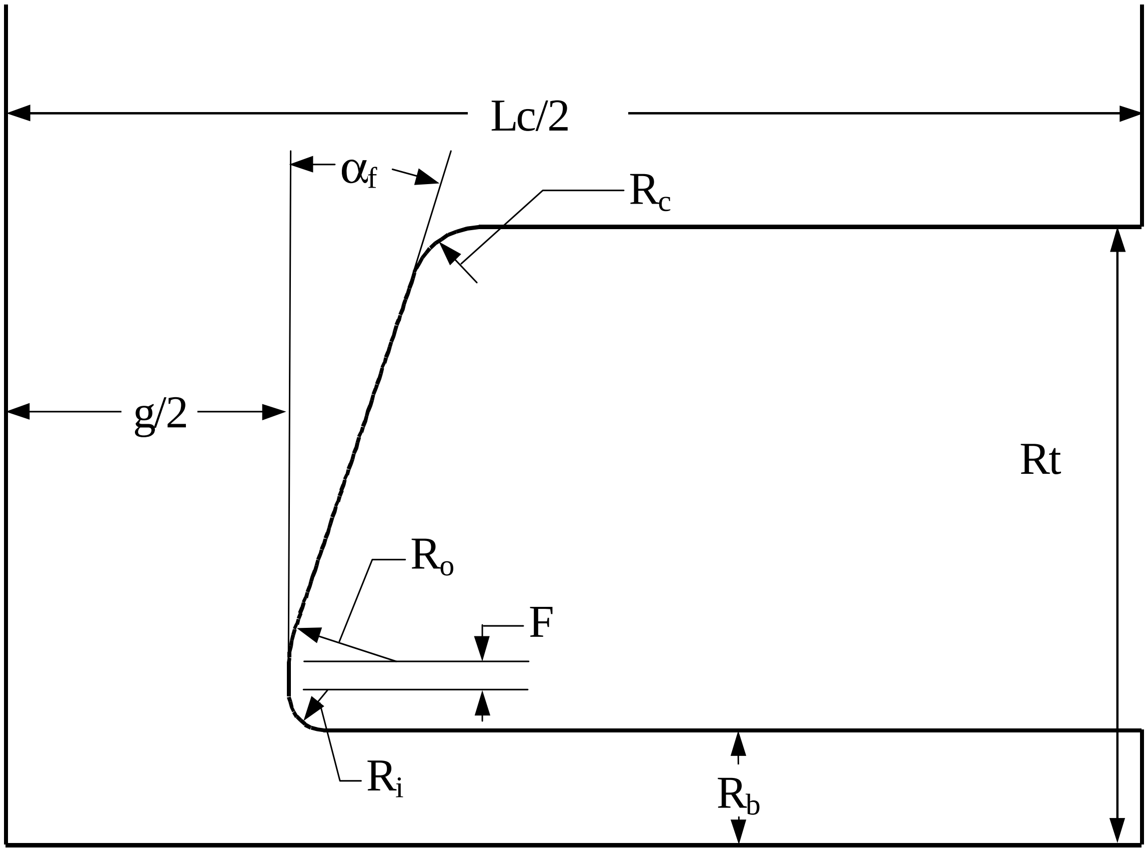
The DTL cells



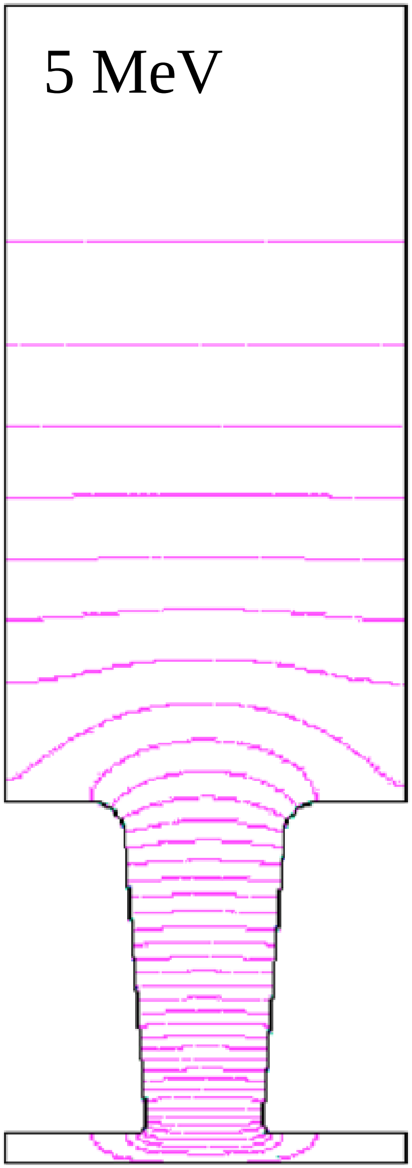
The Cell design with SUPERFISH

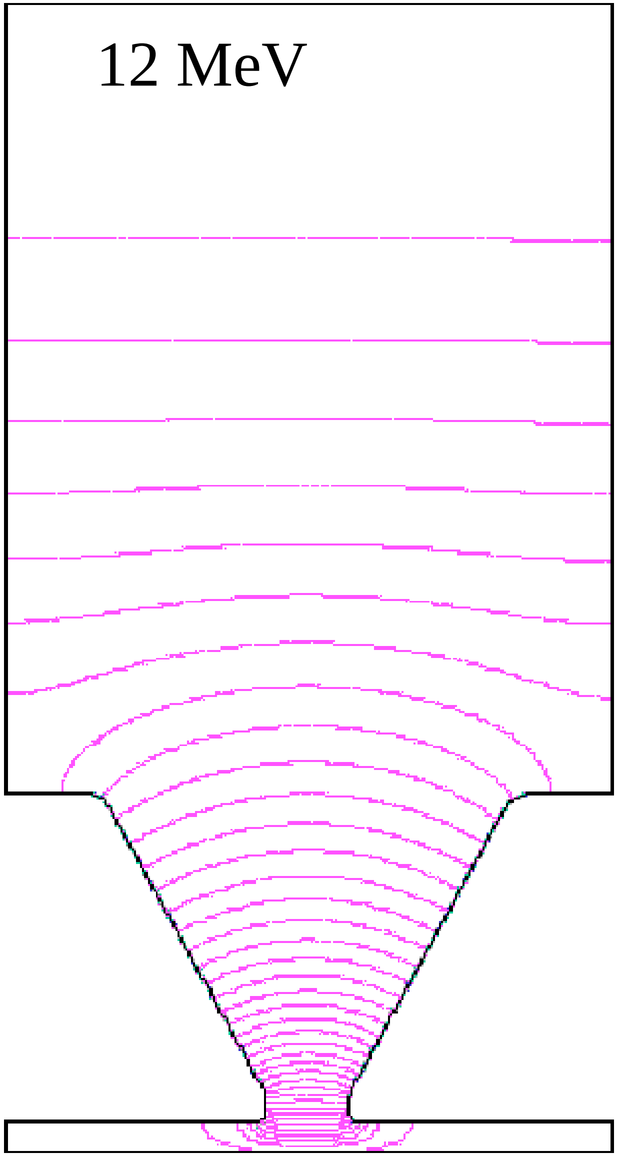
Parameters are adjusted to

- get the good resonance frequency
- limite the peak field
- let enough room for quadrupoles
- keep the synchronism
- let enough room for beam
- minimise the shunt impedance



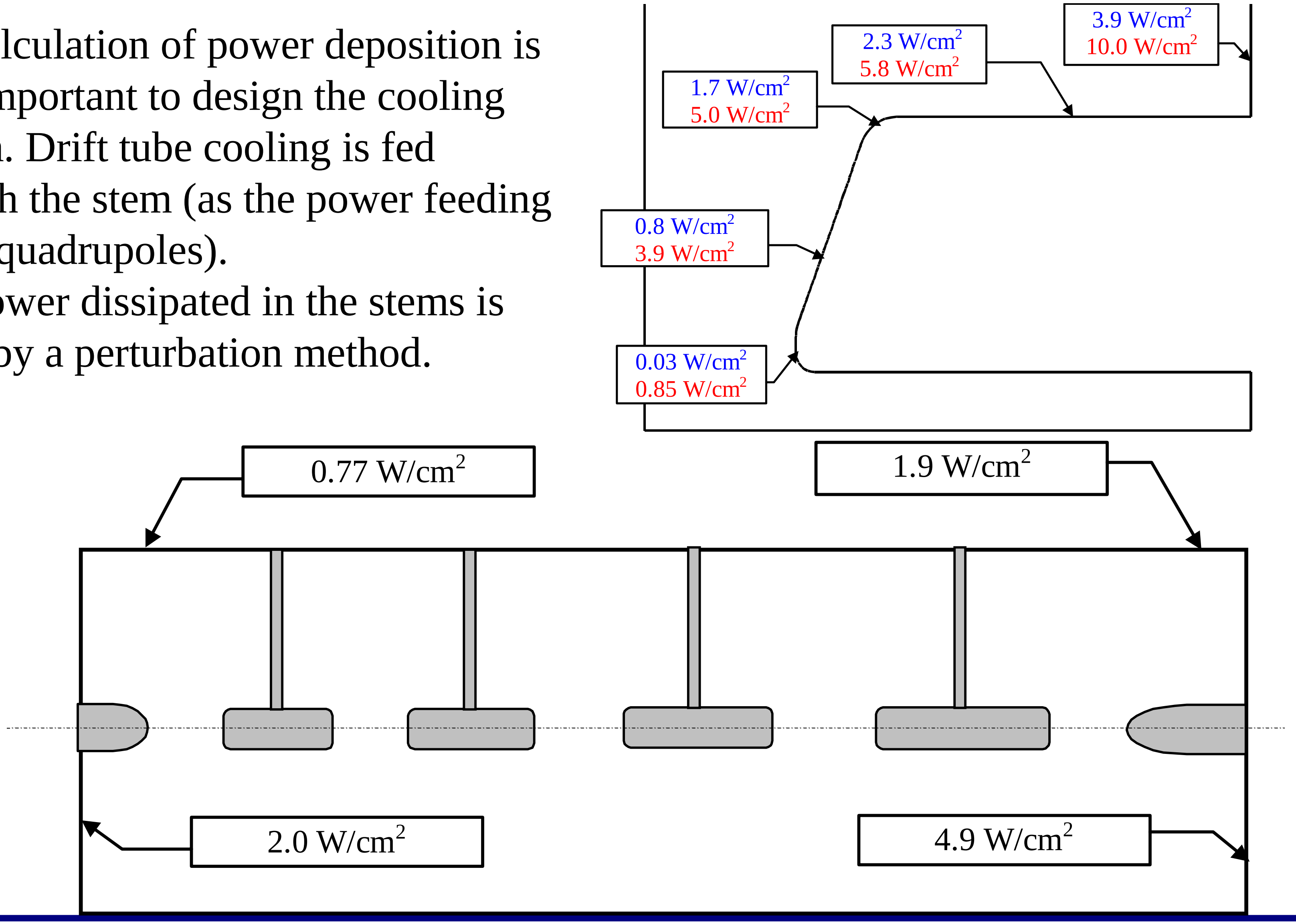
Examples of cells (IPHI)





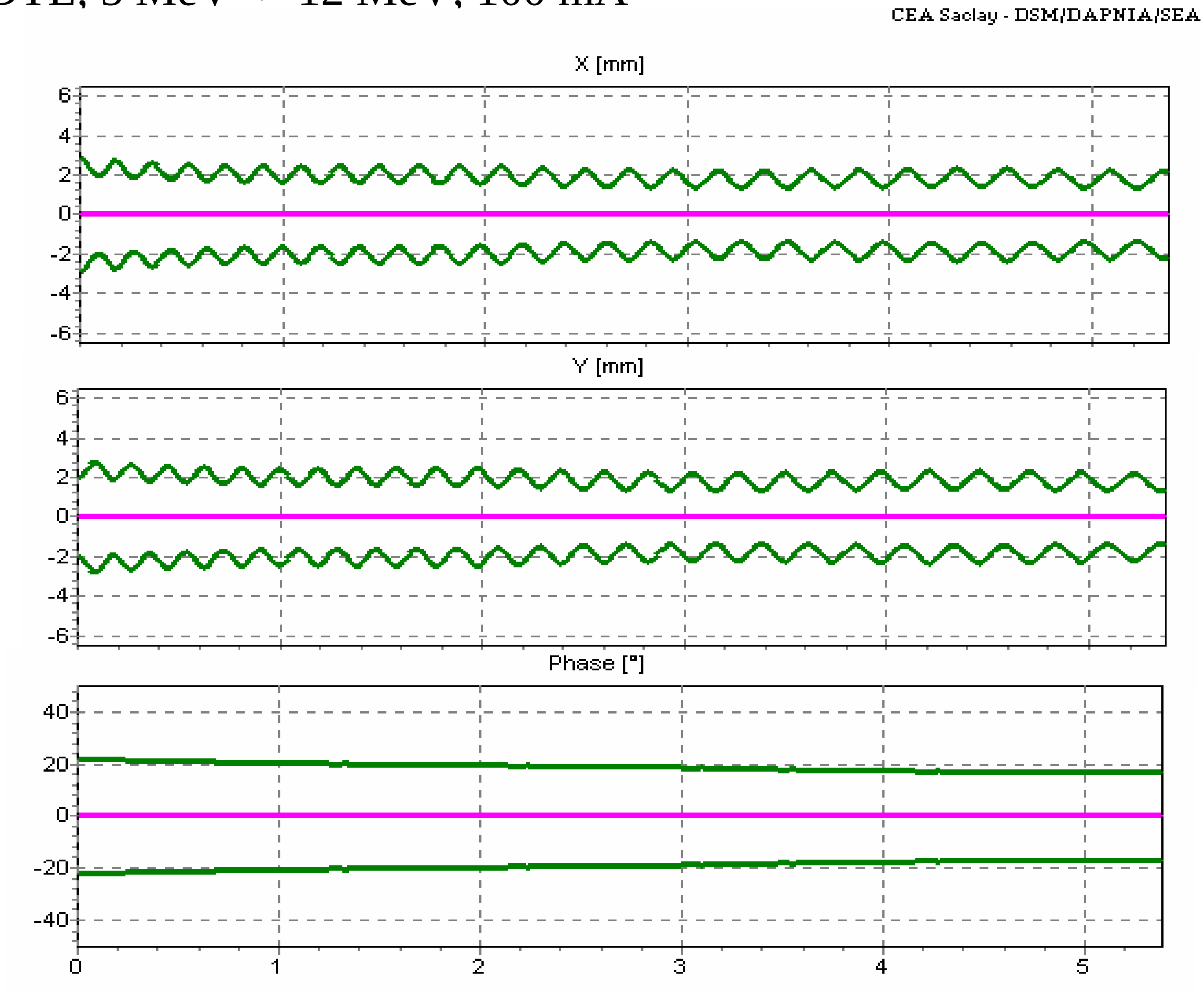
Power deposition

The calculation of power deposition is very important to design the cooling system. Drift tube cooling is fed through the stem (as the power feeding of the quadrupoles).
The power dissipated in the stems is given by a perturbation method.



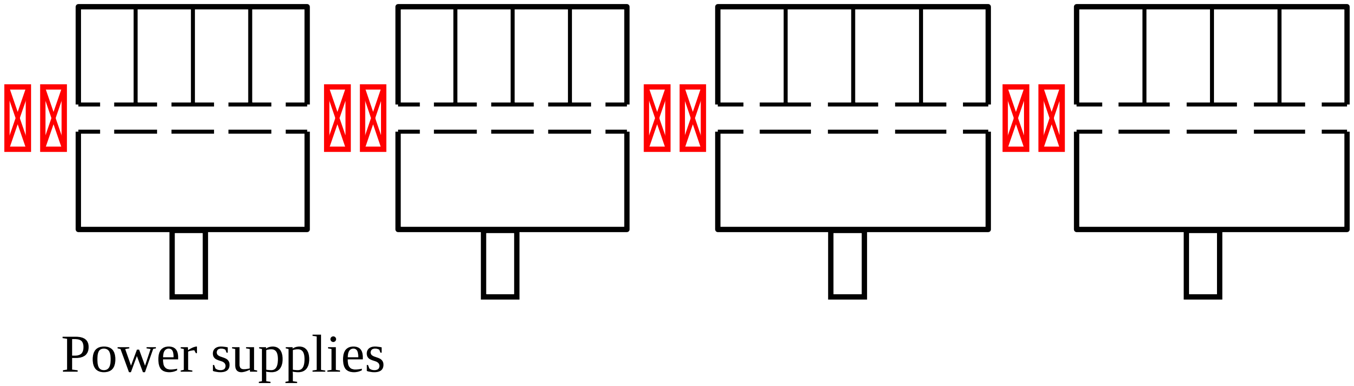
Beam envelopes in a classical DTL tank

IPHI DTL, 5 MeV -> 12 MeV, 100 mA



S-DTL

This structure is constituted of small tanks containing a few cells without any quadripoles in drift tubes. Quadripoles are outside the tanks.
Classical DTL drawbacks are then cancelled, but A1-3 are now disadvantages of this kind of structure.



Generally, this kind of structure can be used from ~20 MeV. Before, to avoid a too large beam size, small number of cells per tank should be used, increasing the losses in the tank end walls, reducing the packing factor and multiplying the power sources

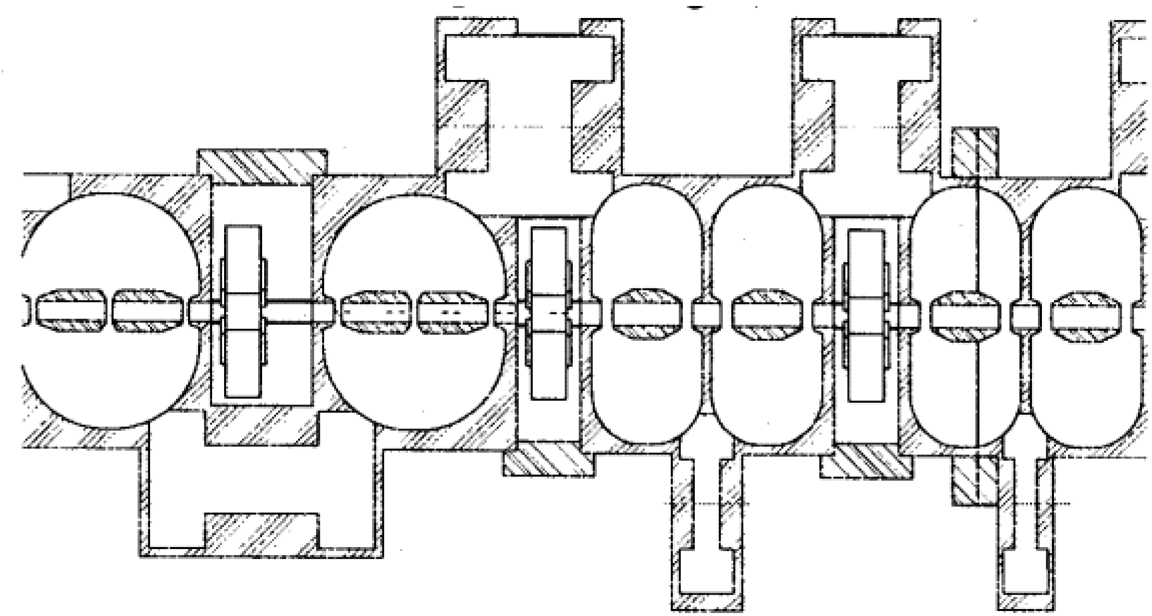
Drawbacks and advantage of DTL

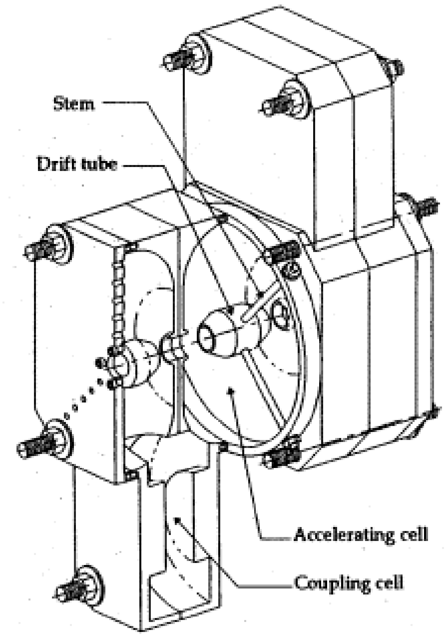
The main **drawbacks** of classical DTL is that the drift tube should be able to contain a quadrupole. This gives several difficulties :
D1) Drift tubes are mechanically more difficult to install as the alignment of quadrupoles should be the better as possible. The DTL needs a large number of joins leading to a decrease of the shunt impedance (by ~20%).
D2) Drift tubes are bigger, increasing the power losses on it, and decreasing the shunt impedance.
D3) Stems are bigger as the quadrupole cooling and power feeding should pass through it. This induces an other reduction of shunt impedance.
D4) Large tanks induces large sensitivity to RF instabilities (stabilised by post couplers).

The main **advantages** are :
A1) its short lattice period (2 $\beta\lambda$) allows a small beam size and a large acceleration rate.
A2) Large tanks allows to feed one tank with a high power supply.
A3) High packing factor.

CC-DTLs

The main difference with the S-DTL is that some tank (or cavities) are coupled. This allow to feed many cavities with only one power supply. The transverse focalisation period is adiabatically changing.

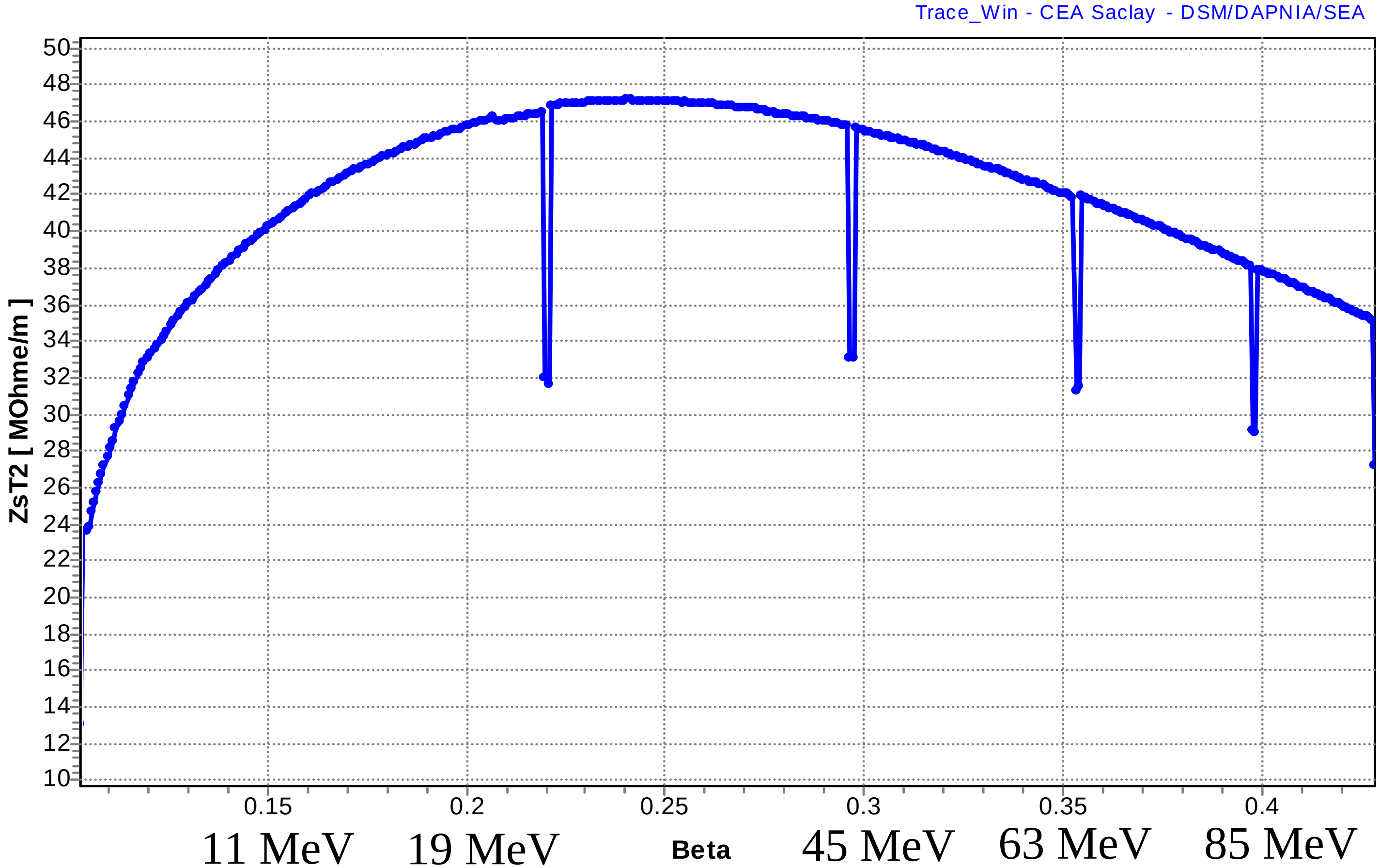




LANL-APT CCDTL (from 6.7 MeV)

This structure appears to be mechanically very expensive and very difficult to tune. To be followed …

Effective impedance



DTL effective shunt impedance (*80%) for ASH project (*CEA-Saclay*). Note the differences between figures shown in previous graphs.

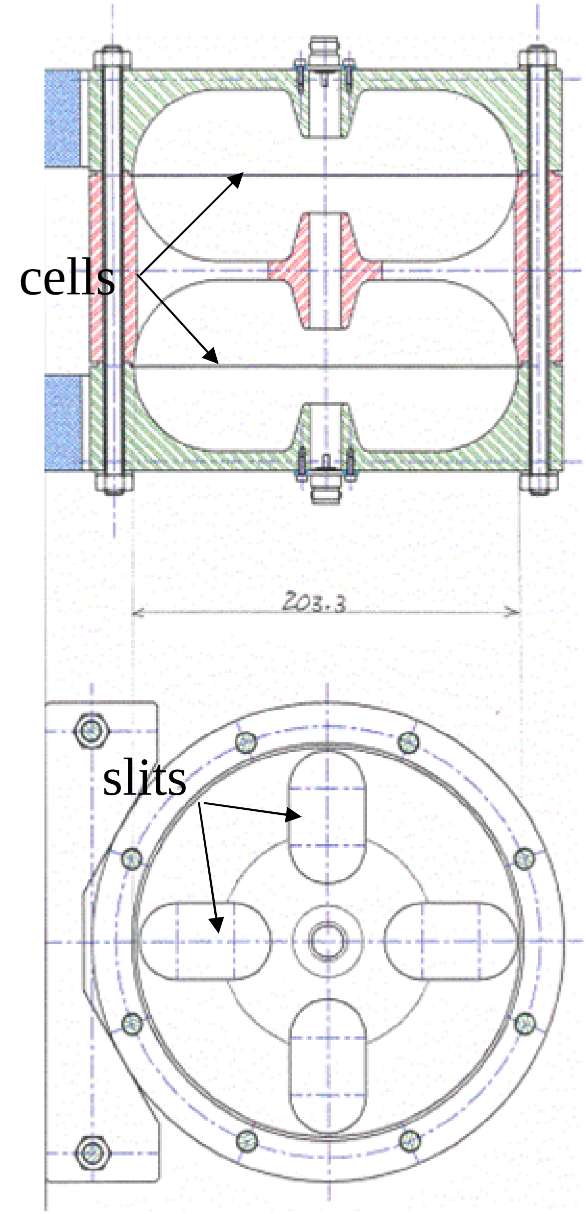
Main Challenge in DTL Design

• chose mean electric field minimising the DTL cost (high field : low linac length but high RF losses).
• keep low peak field,
• maximise the effective shunt impedance,
• good field stability,
• define tolerances which give no particle losses,
• low emittance growth,
• Low sensitivity to errors (misalignment …) : tolerances calculations,
• Good vacuum properties,
• Easy cooling,
• Easy fabrication and alignment,
• Easy tuning.

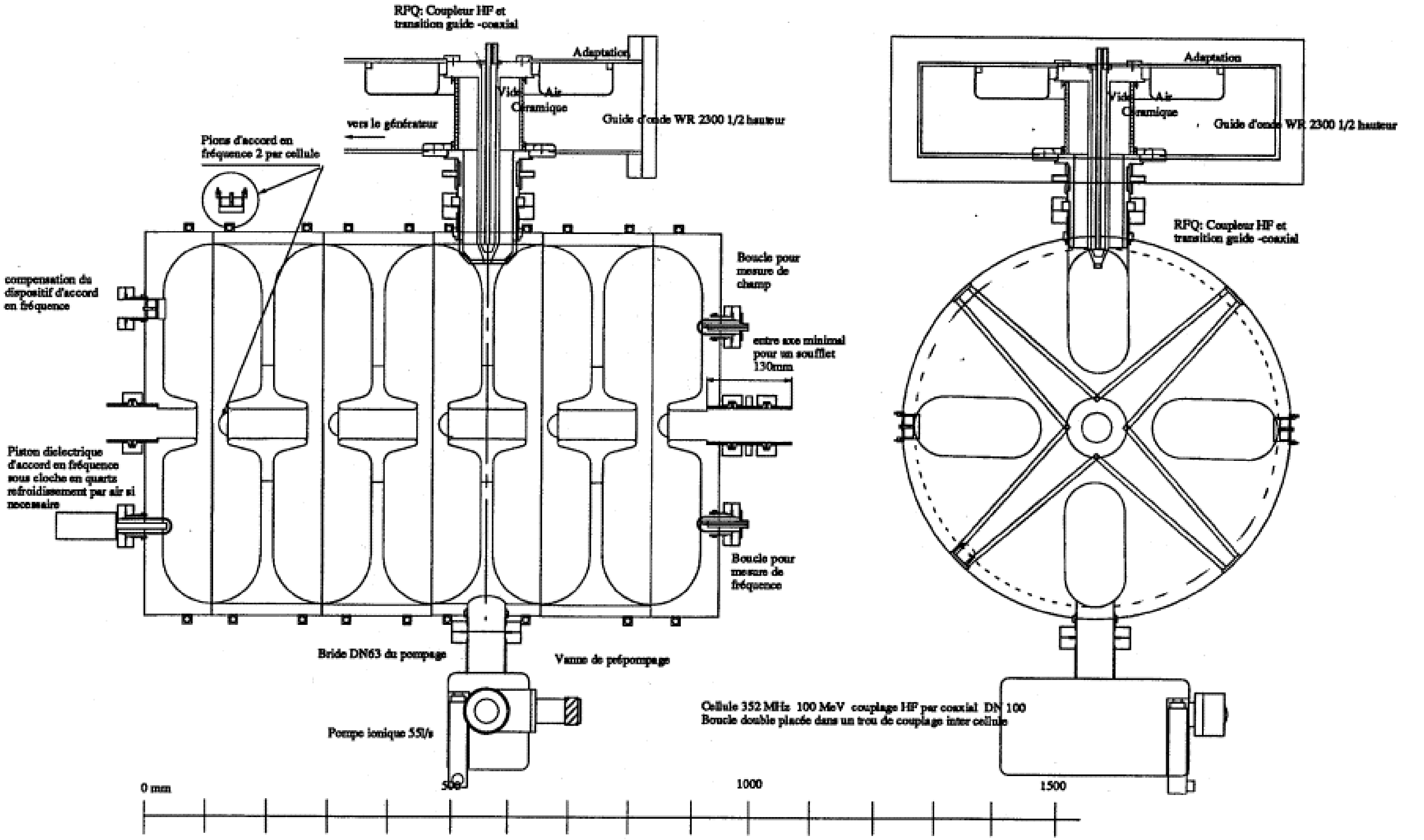
The CCL (Coupled Cavity Linac)

Principles

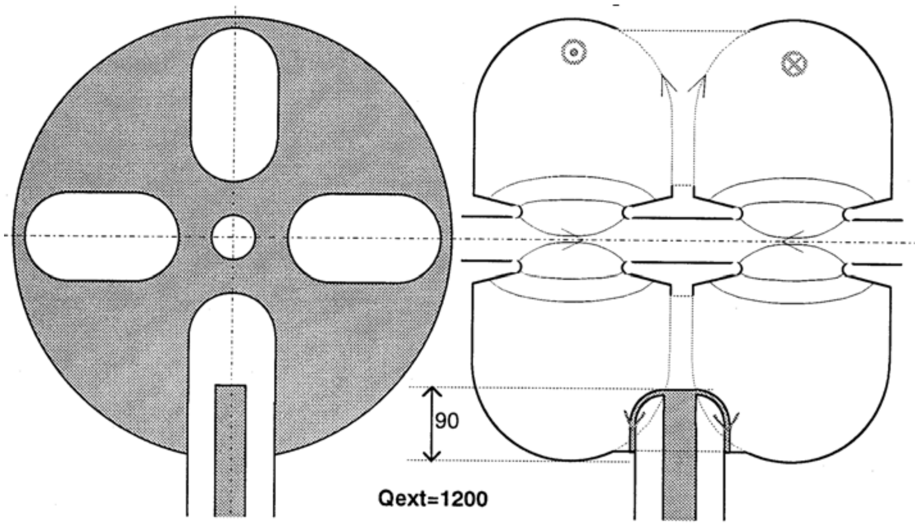
- The CCL cavity is a resonant cavity.
- The cavity is a set of identical cells coupled by slits,
- The phase difference between cells is π.
- The excited mode in cells is the TM_{010}.
- Between cells, a drift tube without field.
- Quadrupoles are outside cavities.



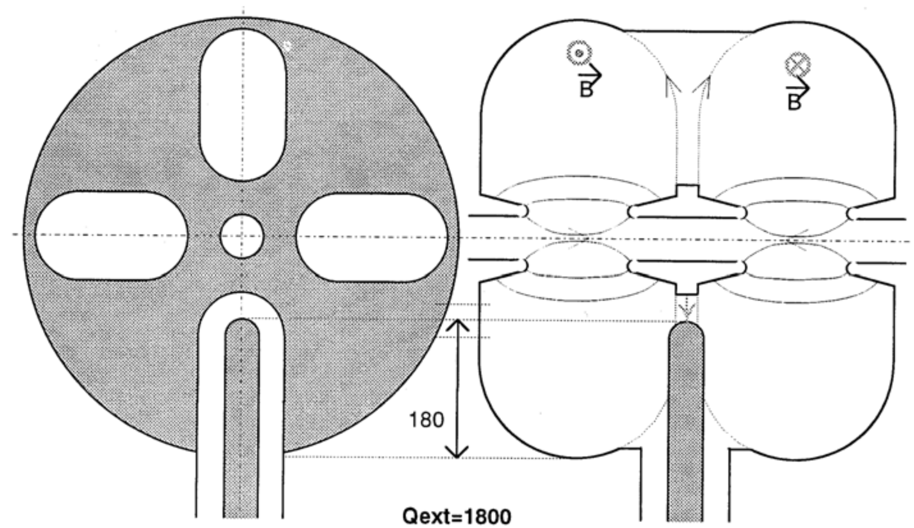
The cavity feeding (1)



The cavity feeding (2)



Magnetic coupling



Electric coupling

The power transits to neighbour cells through the coupling slits.
These slits should be optimised for a good transfer.

The families of cavities

The β of one cavity is defined as the reduced velocity of one particle going from the centre of one cell to the centre of the following one in half a RF period.

A family of cavity is a set of cavities having the same number of cells and the same β_n. A finite number a families is used to reduced the fabrication cost. Unfortunately, the transit time factor of one cavity decreases if the particle β_s is not the optimum one (see tutorial).

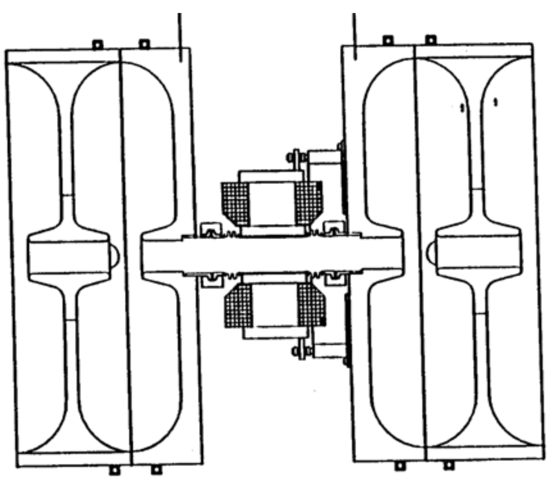
The optimum number of cavity families (with their β_n) has to be found :
High number of families $\Leftrightarrow$ High transit time factor <u>but</u> high fabrication cost.

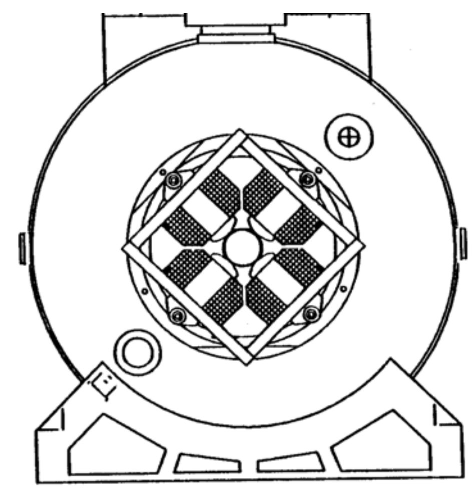
The focusing scheme

Quadrupoles are placed outside of cavities.
Different schemes are possible :
FODO, FDO, FDFODFDO, … every n cavities.

• Short lattice period is better to minimise the beam size limited by the phase advance per lattice which should be smaller than 90°.
• For H^- beam, low magnetic field is better to avoid lorentz stripping.
• A low number of quadrupoles increases the packing factor.

Generally, FOFO or FDO schemes are preferred.





The SCL (Superconducting Cavity Linac)

Why superconducting cavities

• Power losses in copper has a very high cost.

• Moreover, the cooling limited capabilities reduces the electric field for high duty-cycle beams.

• In superconducting cavities, the power losses are a negligible part of the total RF power. But these losses are not negligible with respect to the helium cooling system which has a very low (10^{-3}) efficiency.

Principles of elliptical cavity

• The cavity is made of superconducting material (generally Niobium).

• Its shape is optimised for a easy fabrication and surface treatments.

• It is made in several cells, in π mode, coupled through the beam path (large aperture).

• RF power in injected at one side through a coupler.

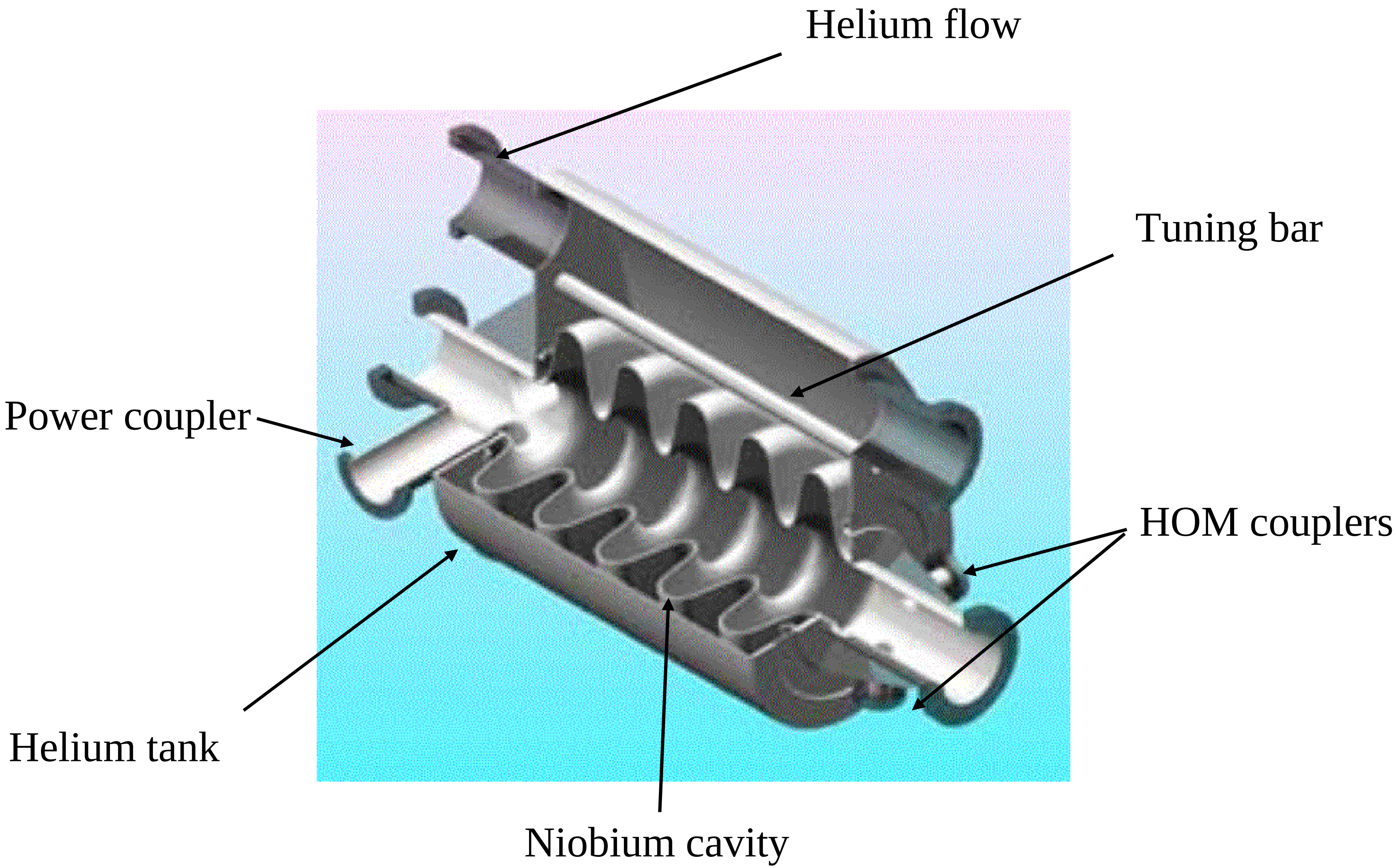
• The coupler has to be optimised in respect of the beam current.

• Each cavity is placed in a helium tank.

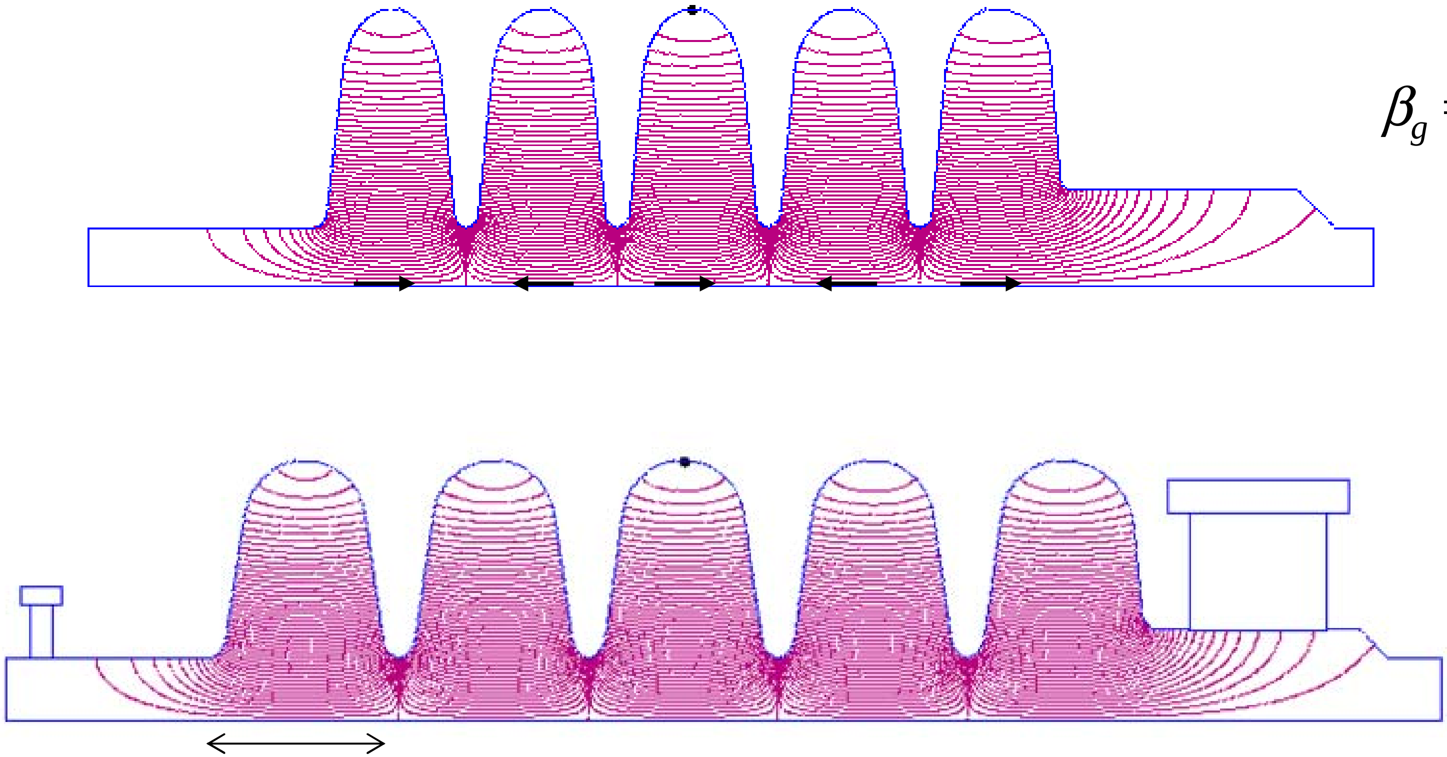
• Several cavities are placed in the same cryomodule.

• Each cavity is individually tuned.

The Elliptical cavity



Electric field lines



$\beta_g = 0.5$

$\beta_g = 0.65$

$\frac{\beta_g \lambda}{2}$

$f_{RF} = 704.4$ MHz

Cavity field limitation

The energy gain of a particle with $\beta_g c$ velocity in one cavity is defined as :

$$G = q \cdot E_{acc} \cdot n \frac{\beta_g \lambda}{2} \cdot \cos(\varphi_p)$$

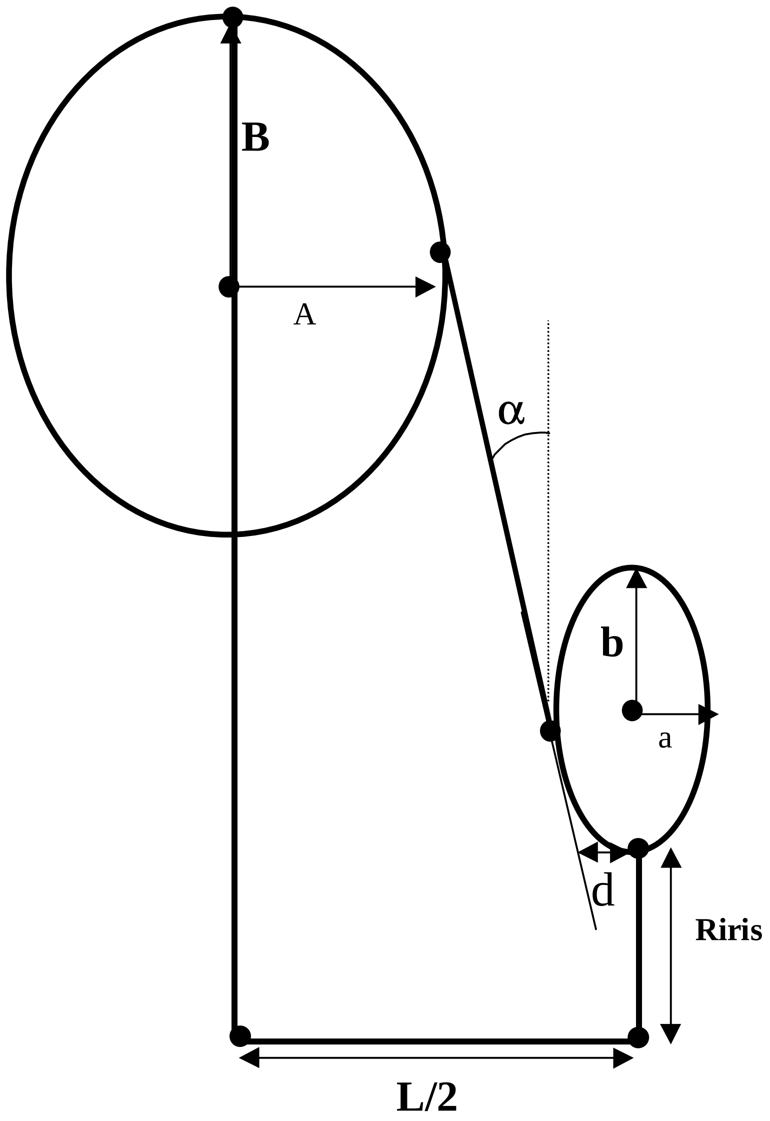
$\boldsymbol{E_{acc}}$ is the accelerating field in the cavity, n is the cavity number of cells.

The accelerating field is limited either by the maximum peak electric field $\boldsymbol{E_{peak}}$ and the maximum peak magnetic field $\boldsymbol{B_{peak}}$. They both can be responsible of quenches (depending on the surface quality), multipactor (depending on the cavity geometry).

Conservative values are E_{peak}=27.5 MV/m and B_{peak}=50 mT.

This corresponds to (order of magnitude, @ 700 MHz) :
~ E_{acc} = 8.5 MV/m for β_g = 0.5,
~ E_{acc} = 10.5 MV/m for β_g = 0.65,
~ E_{acc} = 12.5 MV/m for β_g = 0.8.

Optimisation of the cavity shape

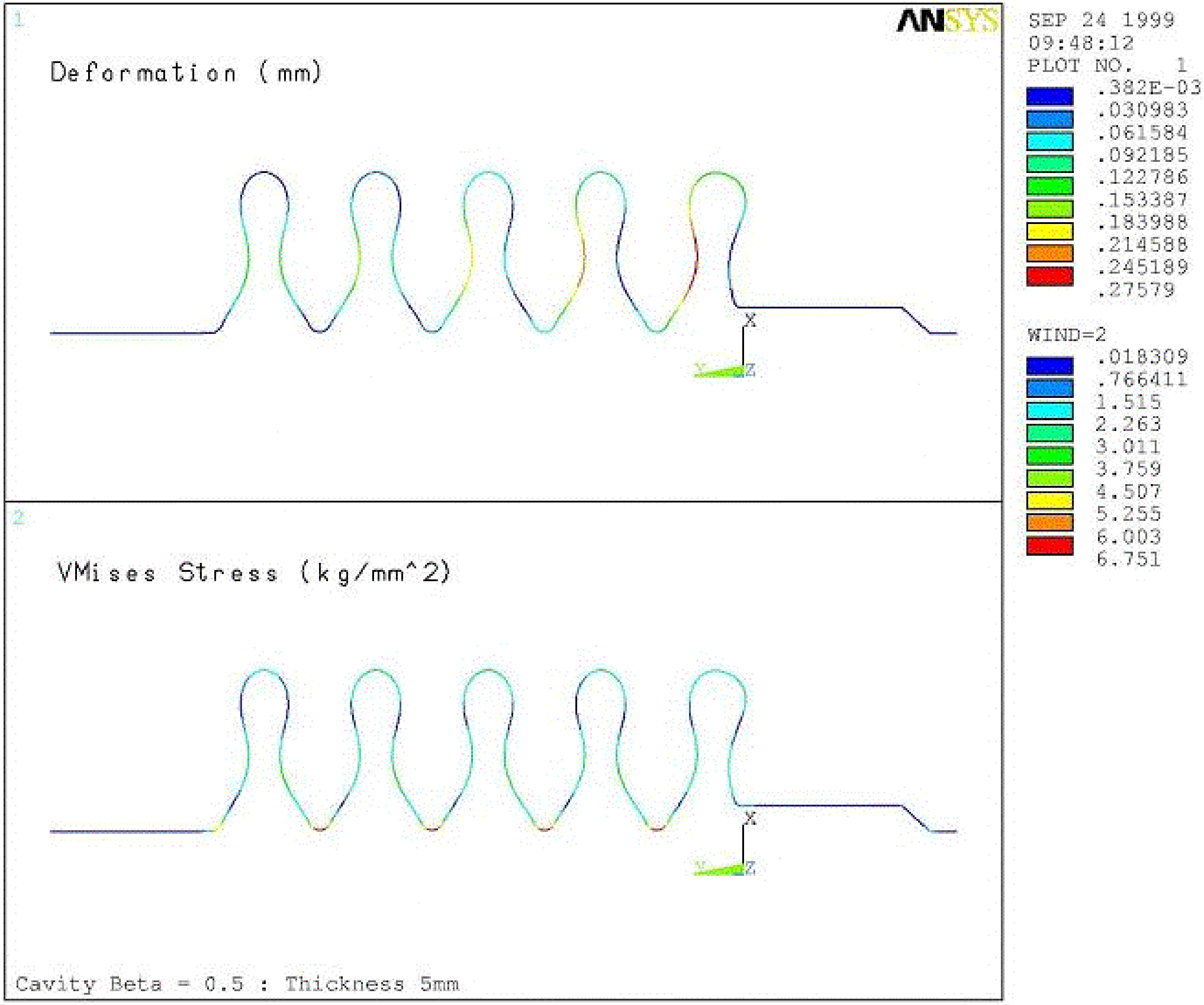


Can be chosen : A, B, a, b, α, Riris, L (These parameters are not all independent).

Should be optimised :
- Cavity geometrical β_g (Fixed),
- Cavity frequency (Fixed , will be adjusted with tuners),
- E_{peak}/E_{acc} and B_{peak}/E_{acc} should be minimised,
- Inter-cell coupling (~1%).

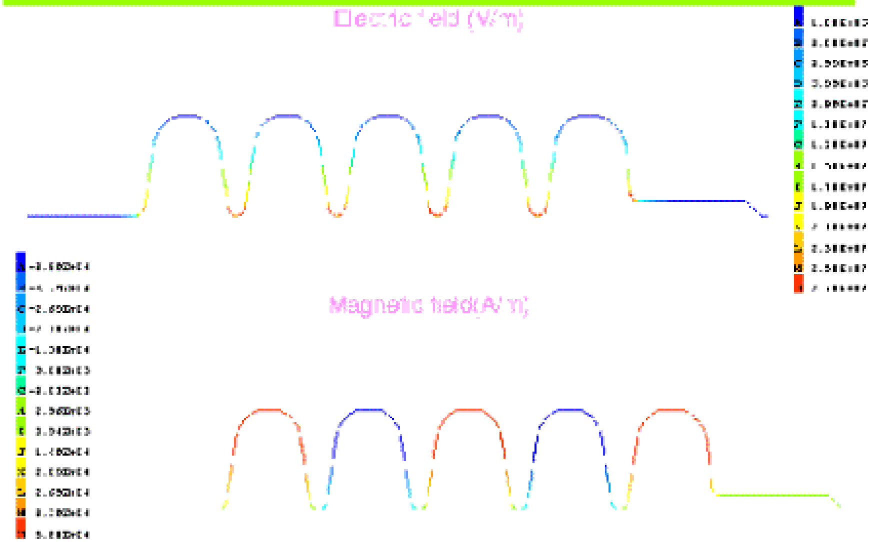
The Niobium width is the minimum allowing good mechanical properties :
- Lorentz detuning inside bandwidth (stiffening can be added),
- Elasticity limit should not be exceeded.

Deformation study

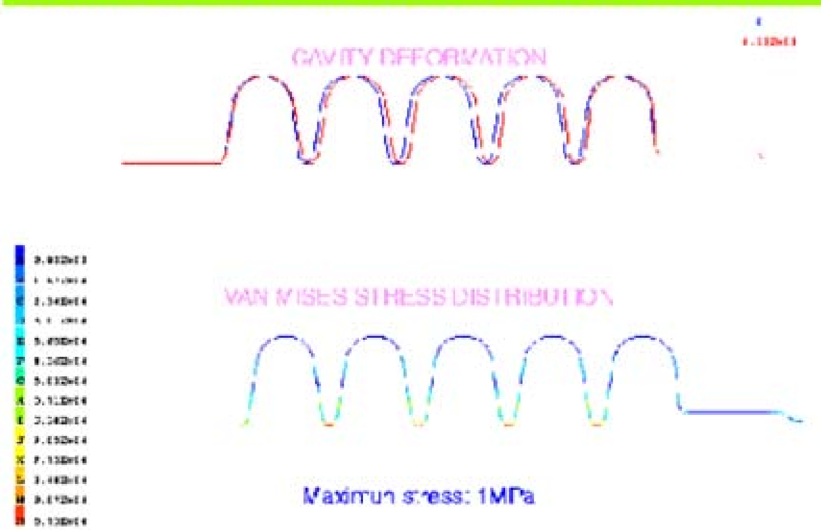


Lorentz forces studies

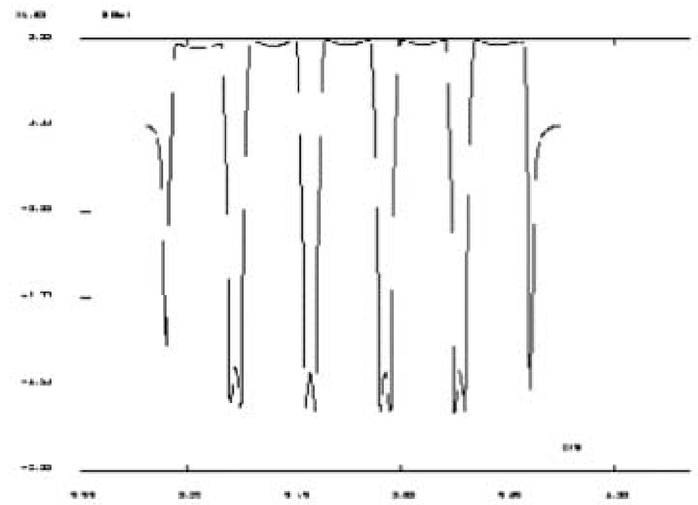
ELECTROMAGNETIC FIELDS ON β=0.65 MULTICELL CAVITIES



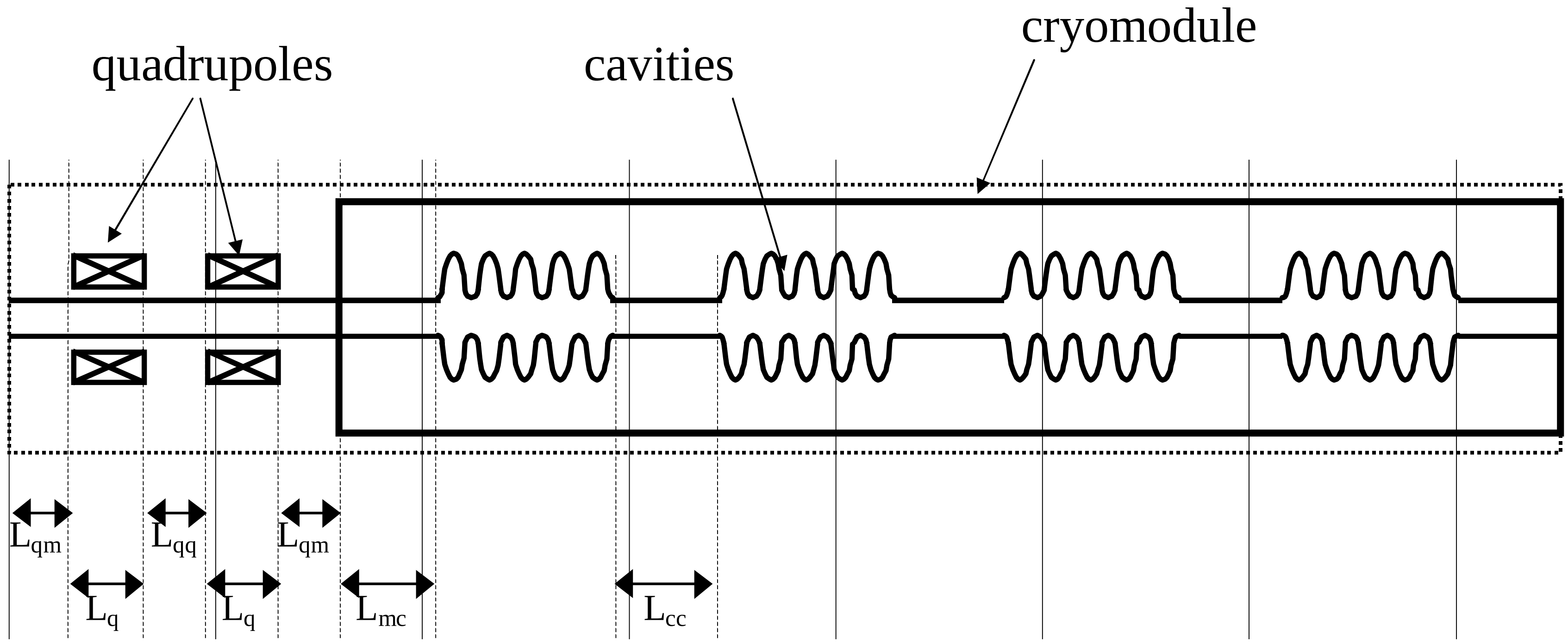
LORENTZ FORCES EFFECTS ON β=0.65 MULTICELL CAVITIES



LORENTZ FORCES ON β=0.65 MULTICELL CAVITIES



A typical SCL Lattice



- Doublet focusing,
- Given distances are generally fixed (minimum needed).
- The number of cavities per cryomodule and number of cell par cavity should be optimised.

SC Linac design

• The transverse focusing scheme has to be chosen (generally, doublet focusing seems to be the best),
• Given distances are generally fixed (minimum needed).
• The number of cavity families and the transition energy have to be chosen.
• For each section (or family), the number of cavities per cryomodule and number of cell par cavity have to be chosen.
• A synchronous phase law, keeping sufficiently large bucket has to be chosen.

The linac is then generated around a synchronous particle, lattice after lattice, cavity after cavity. 3 phenomena limits the accelerating gradient :
• The peak fields in the cavity,
• The maximum power that can be injected through the coupler,
• The longitudinal phase advance per lattice that should stay below 90°

Choice of the section

At a given beam energy, different type of sections are compared.

The cheapest (studies, construction and operation) **is chosen** !

Studies :

prototyping : the lowest number of section types as possible,
manpower : well-known technology.

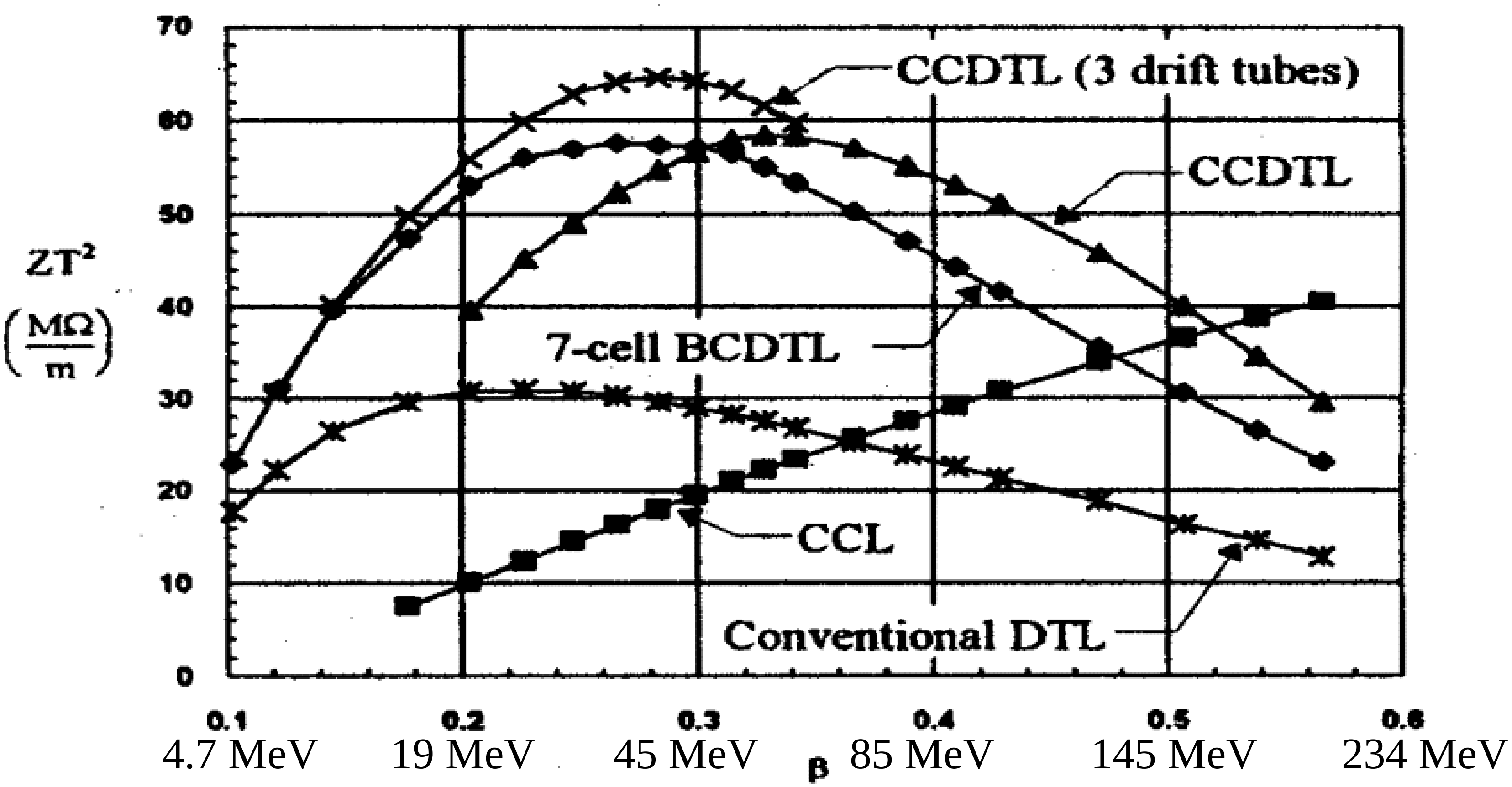
Construction :

Building, Structures : highest field as possible,
Power, cooling : lowest field as possible.
Large number of manufacturers.

Operation :

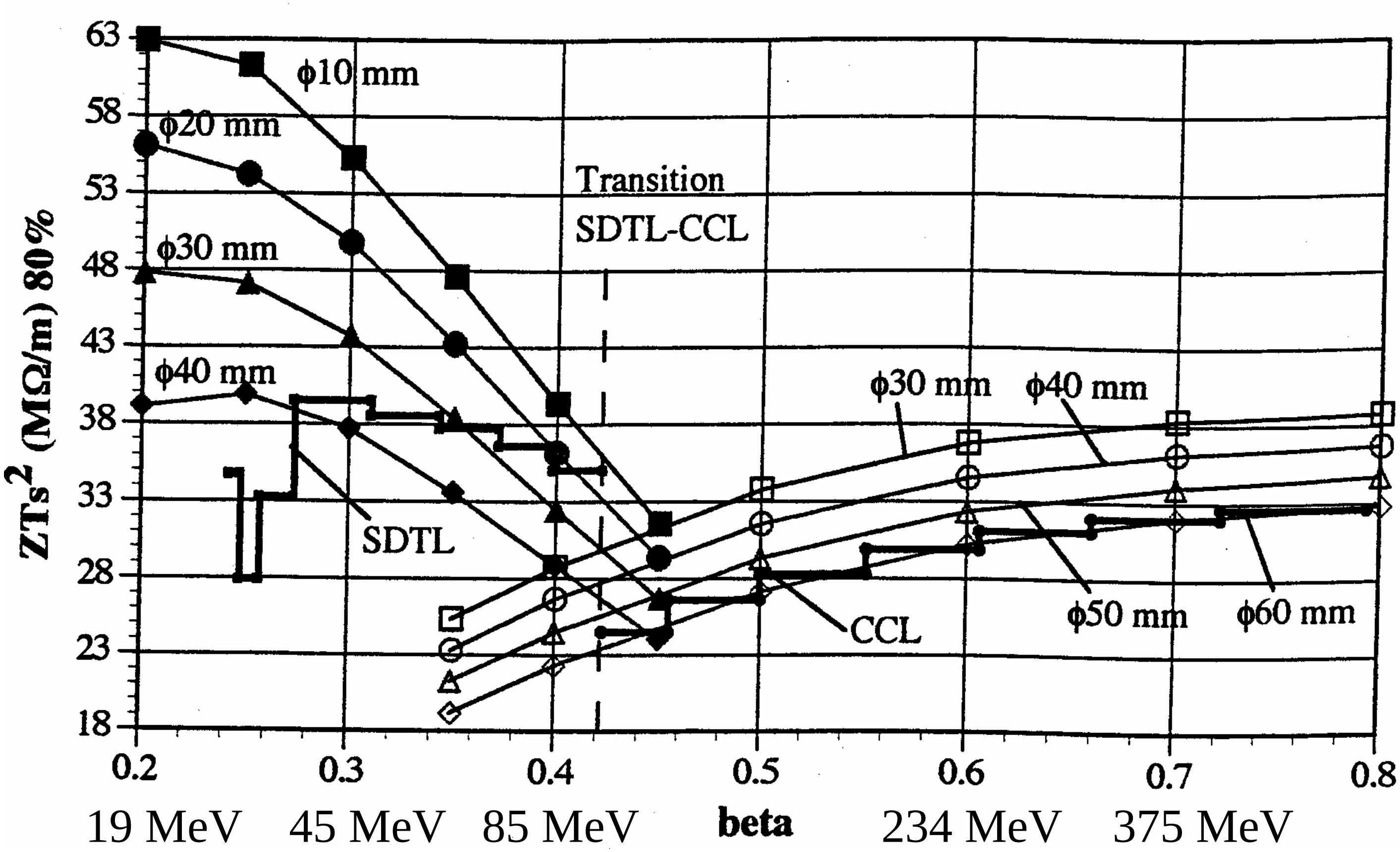
Electricity : lower shunt impedance, lower field,
manpower : easy technology,
availlability : robust technology.

Effective impedance (1)



Here are the effective shunt impedances of different structures (*J. Bilhen, LANL-APT*). These curves, revealing the power consumption in the structures, can be used to chose the structure.

Effective impedance (2)



Here are the effective shunt impedances for TRISPAL project (*C. Bourra, Thomson*). They have been chosen to set the transition energy between sections.

Normal conducting or super conducting ?

Power : Cooling a cavity has a cost (at 2K, ~500-1000 W per W deposited in the cavity) which has to be compensated by the gain obtained from chosing SC rather than NC.
NC are limited in field by cooling capability.
SC field filling time is long (a few 100 μs, depends on beam current)

Technology : SC is newer, needs higher qualified staff, needs better RF control, needs more RF power, is more difficult to align BUT has a lot of manufacturers, is very stable (cooled), is shorter, is more acceptant in term of beam dynamics.

⇒ **SC is more interesting with a low current, long pulse (or cw) beam**

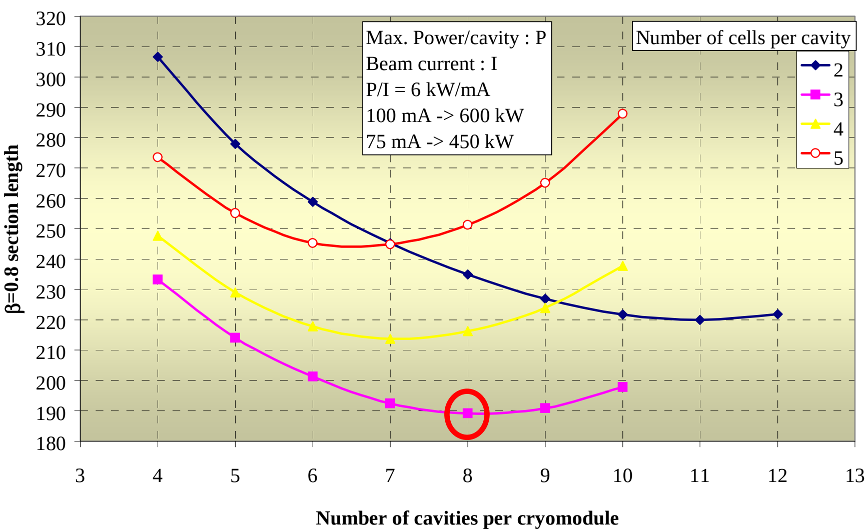
SCL optimisation

Some parameters can be optimised to reduced the linac length or cost (which are not necessarily compatible). These parameters are :

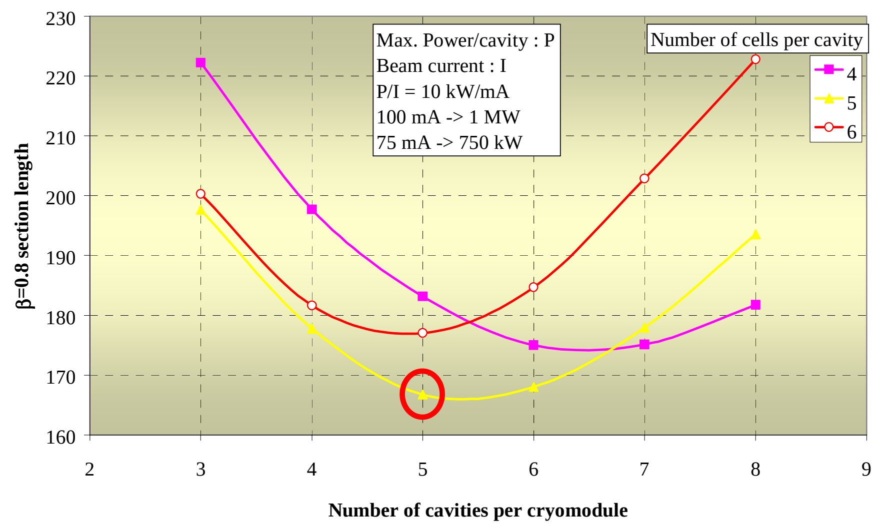
- the number of famillies, their geometrical beta and the transition energies,
- the number of cavities per cryomodule and their number of cells.

The optimum strongly depends on the maximum field and maximum power *P* (correlated to the beam current *I*) limitations.

<u>Example</u> of beam length optimisation of a 420 - 1330 MeV β=0.8 section :



$P/I = 6$ kW/mA



$P/I = 10$ kW/mA

Beam matching

At each time the focusing scheme (transverse and longitudinal) is changed, the beam has to be matched between both structures.

There are basically two ways of matching the beam :

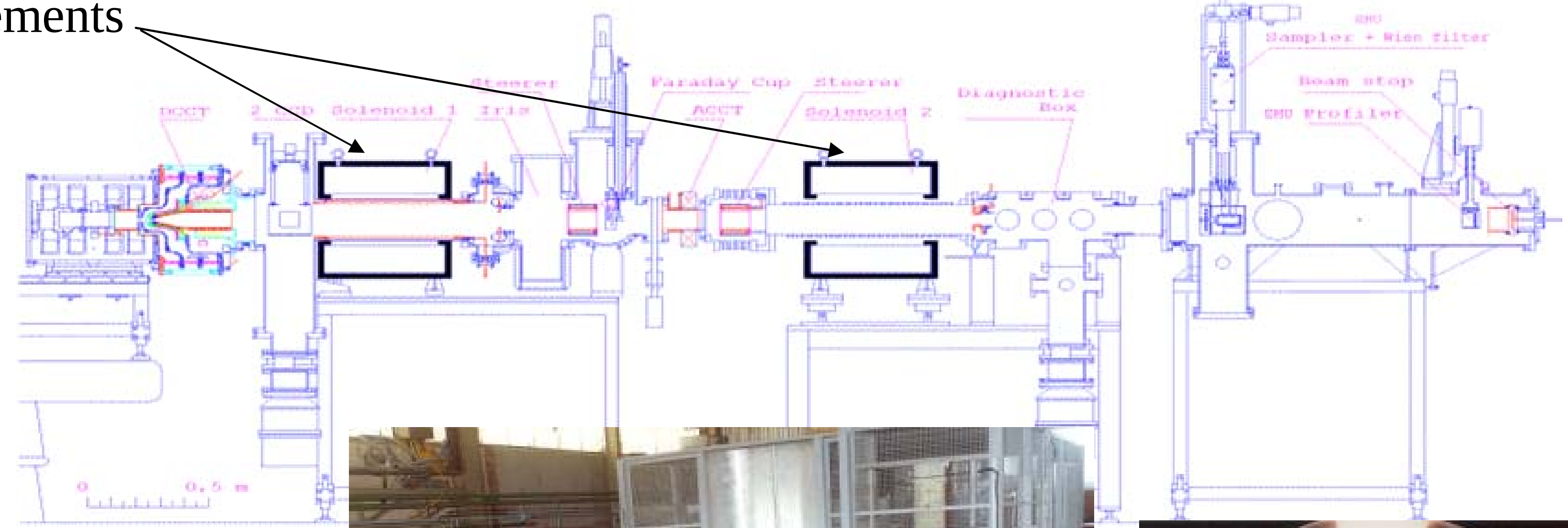
- Using a dedicated line (Examples : LEBT, matching line between RFQ and DTL)

- Changing some elements characteristics at the transition (Example : Between two SCL sections)

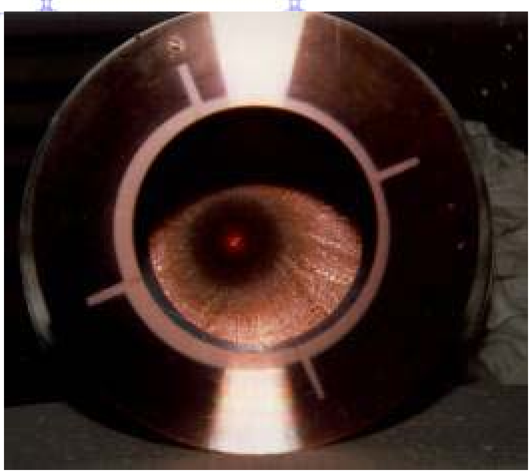
The LEBT (Low Energy Beam Transport)

A cylindrical continuous beam is matched from the source to the RFQ.

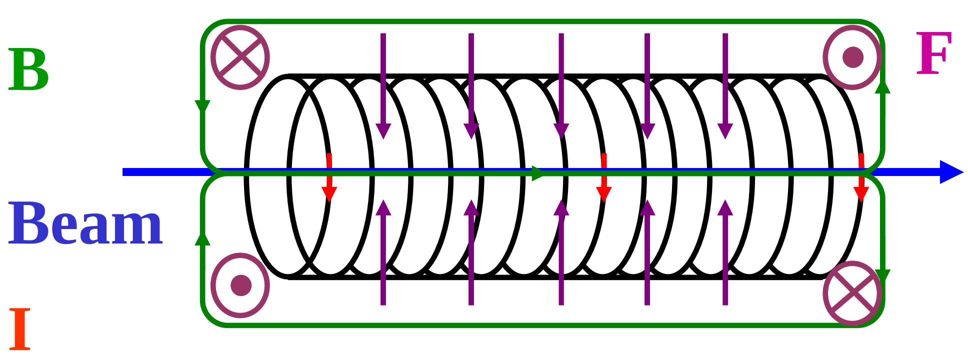
There are two transverse parameters to match, one needs at least two focusing elements

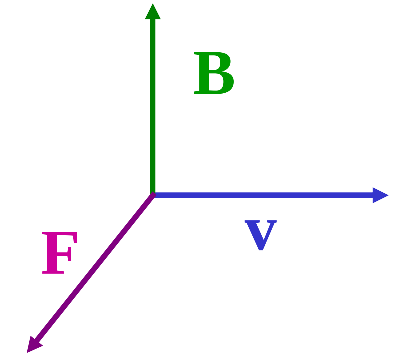


IPHI LEBT



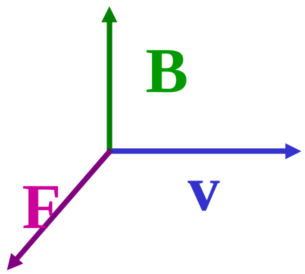
Focalisation with a solenoid





Input : B = $B_\perp$

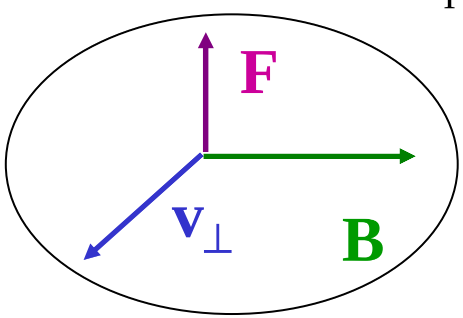
$F \propto v \cdot B$



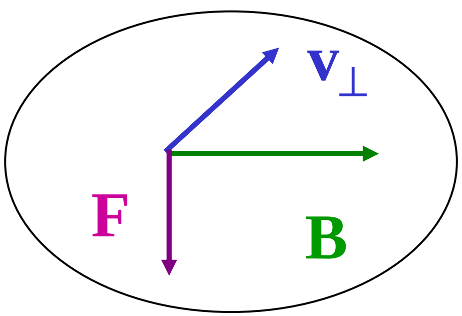
Beam transverse rotation :

$v_\perp \propto v \cdot B \cdot r$

Middle : B = B_l



x < 0



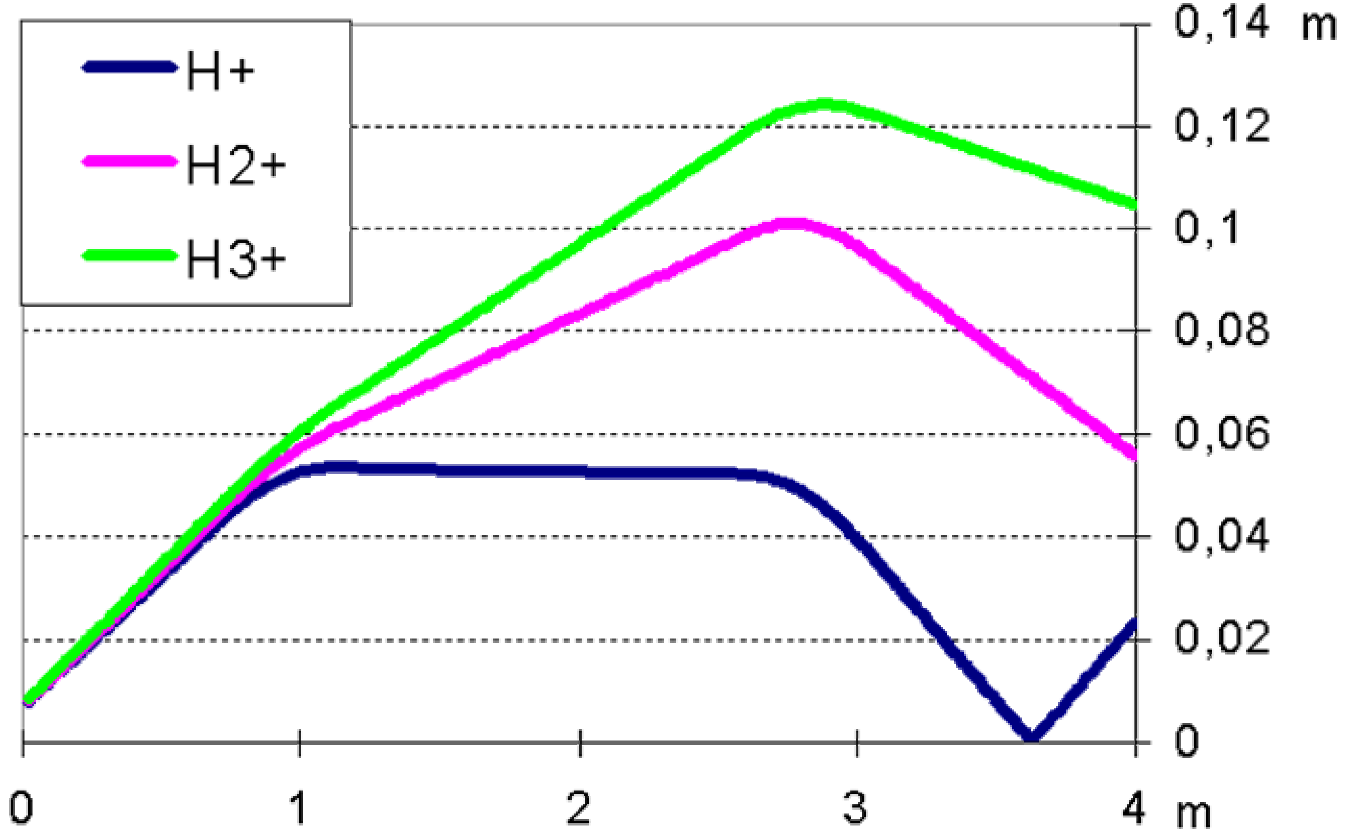
x > 0

$F \propto v_\perp \cdot B \propto v \cdot B^2 \cdot r$

Beam linear focusing

Beam matching to the RFQ

Two solenoids are required to match the beam (2 Twiss parameters should be achieved at the input of the RFQ).

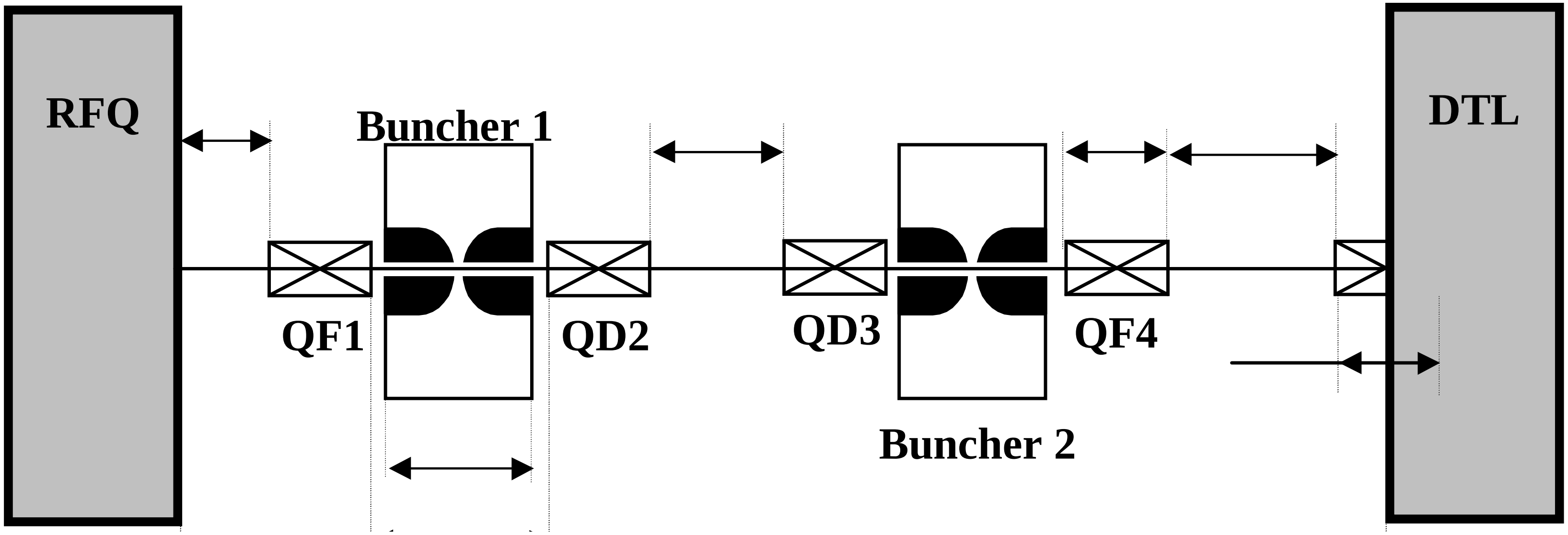


Matching the beam from RFQ to the DTL

The beam coming from the RFQ should be matched to the DTL.
A matching section is used. It should contain at least 4 quads and 2 bunchers.
Their values are automatically calculated by an envelope code.

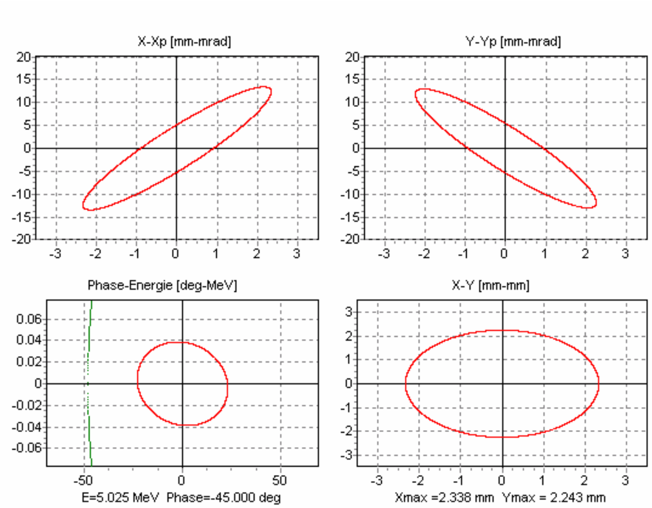
Why ?

6 Twiss parameters have to be matched (β and α en x, y and z directions)

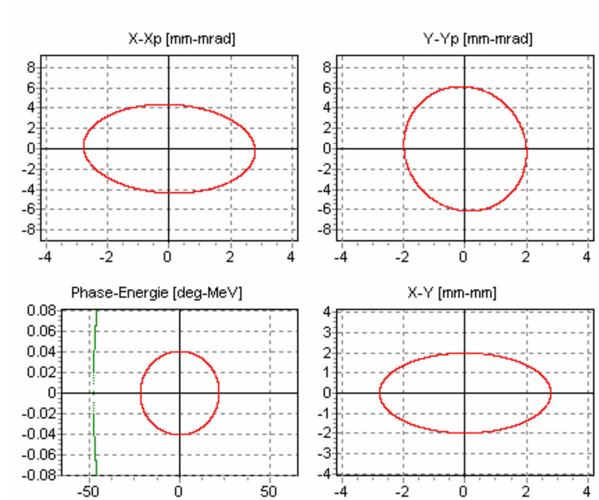


A matching is easier and lead to less emittance growth if the transverse and longitudinal phase advances per unit length are the same in both structures.

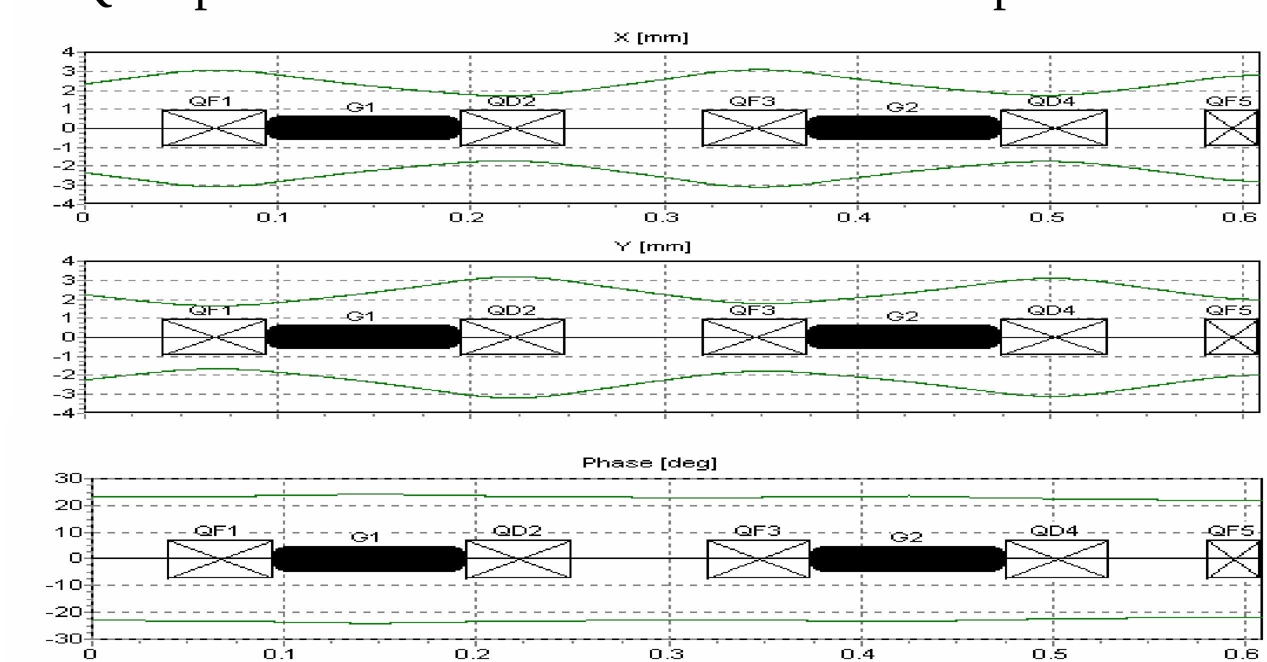
Matching Section : Envelopes



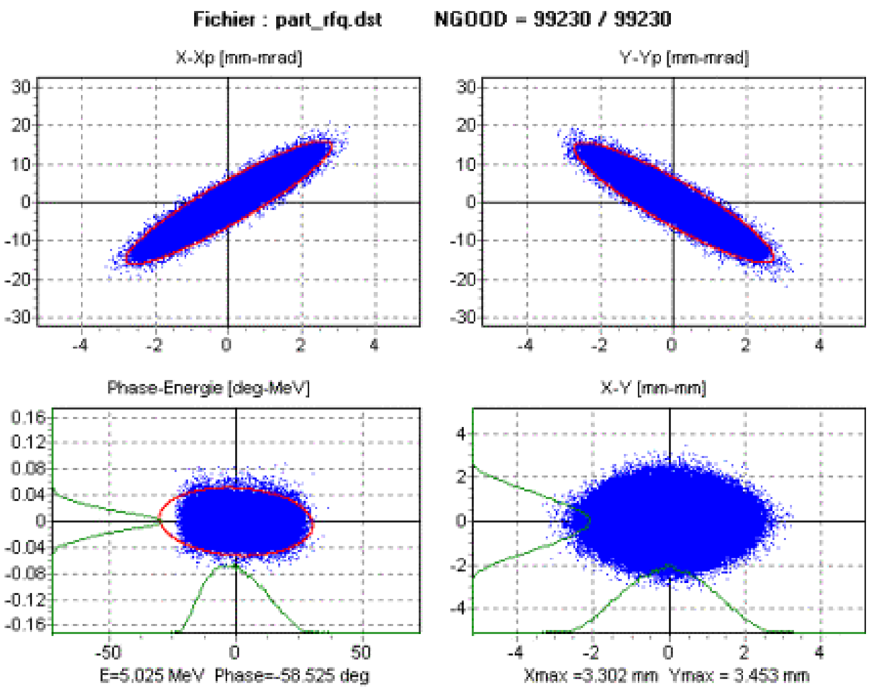
RFQ output



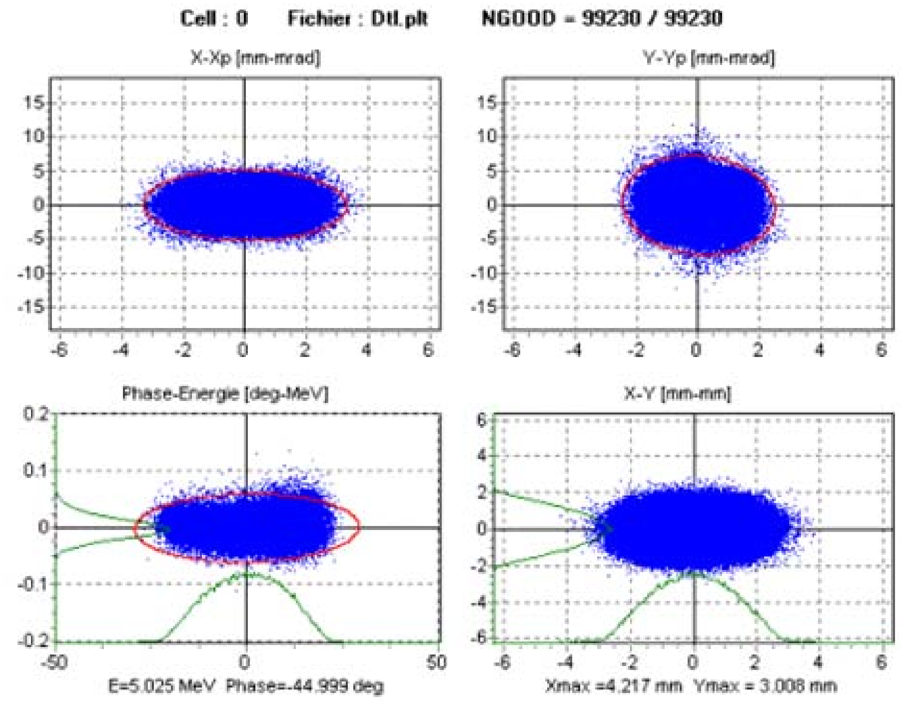
DTL input matched beam



Matching Section : Multiparticles



RFQ output



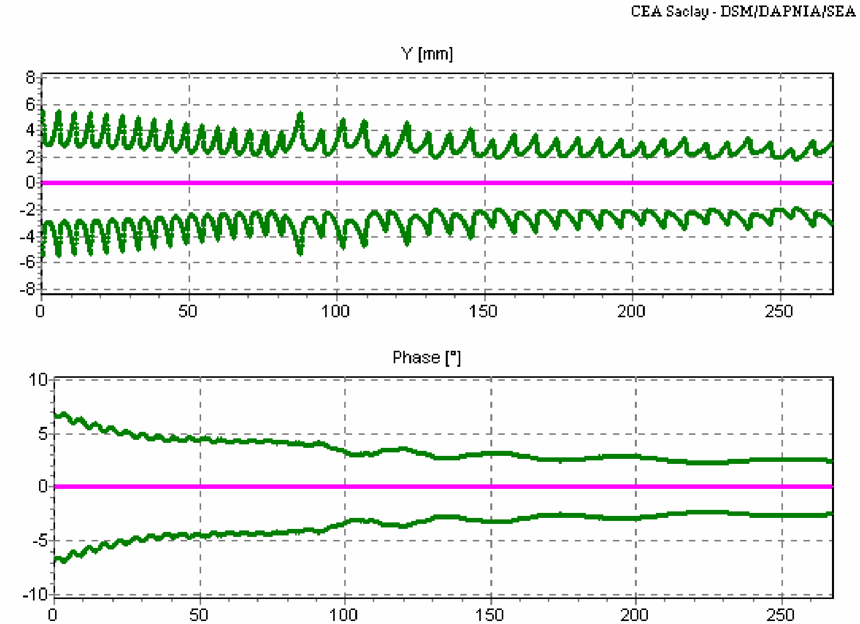
DTL input

Matching the beam between SCL sections

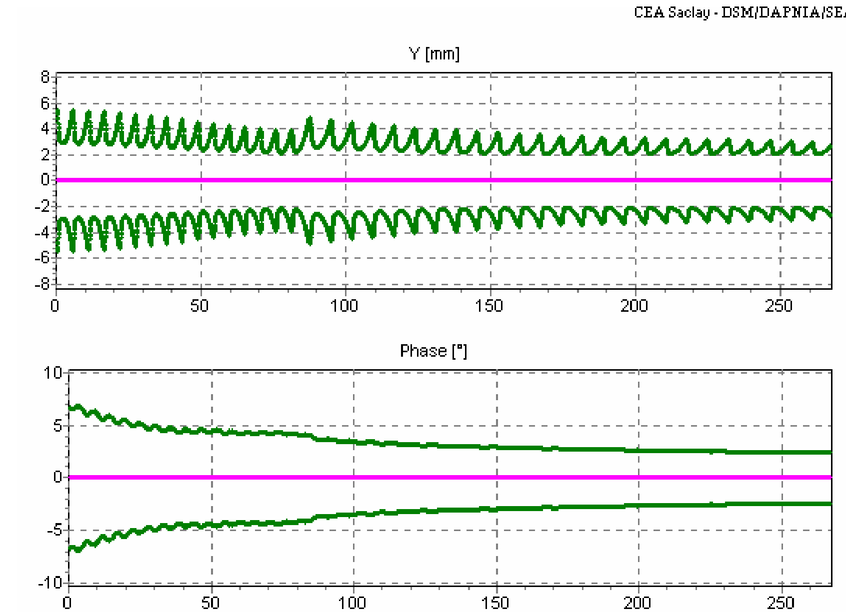
The matching between two SCL sections (with different cavity lengths and number of cavities per tank) is realised by adjusting quadrupole gradient and cavity phase or field at the interface.

After the matching, as acceleration rate has changed in some cavities, the phases of all the cavities should be readjusted.

This procedures are realised with envelope code and verified with PIC code



Non matched beam



Matched beam

<u>Demonstration</u> : ESS/Concert accelerator with TraceWIN.

Space-charge force

Force between beam particles

In beam frame, it is induced by electrostatic field

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_0} \cdot \frac{\vec{r} - \vec{r}_i}{\left\|\vec{r} - \vec{r}_i\right\|^3} = \iiint \frac{\rho(\vec{r}\,')}{4\pi\varepsilon_0} \cdot \frac{\vec{r} - \vec{r}\,'}{\left\|\vec{r} - \vec{r}\,'\right\|^3} \cdot d^3\vec{r}$$

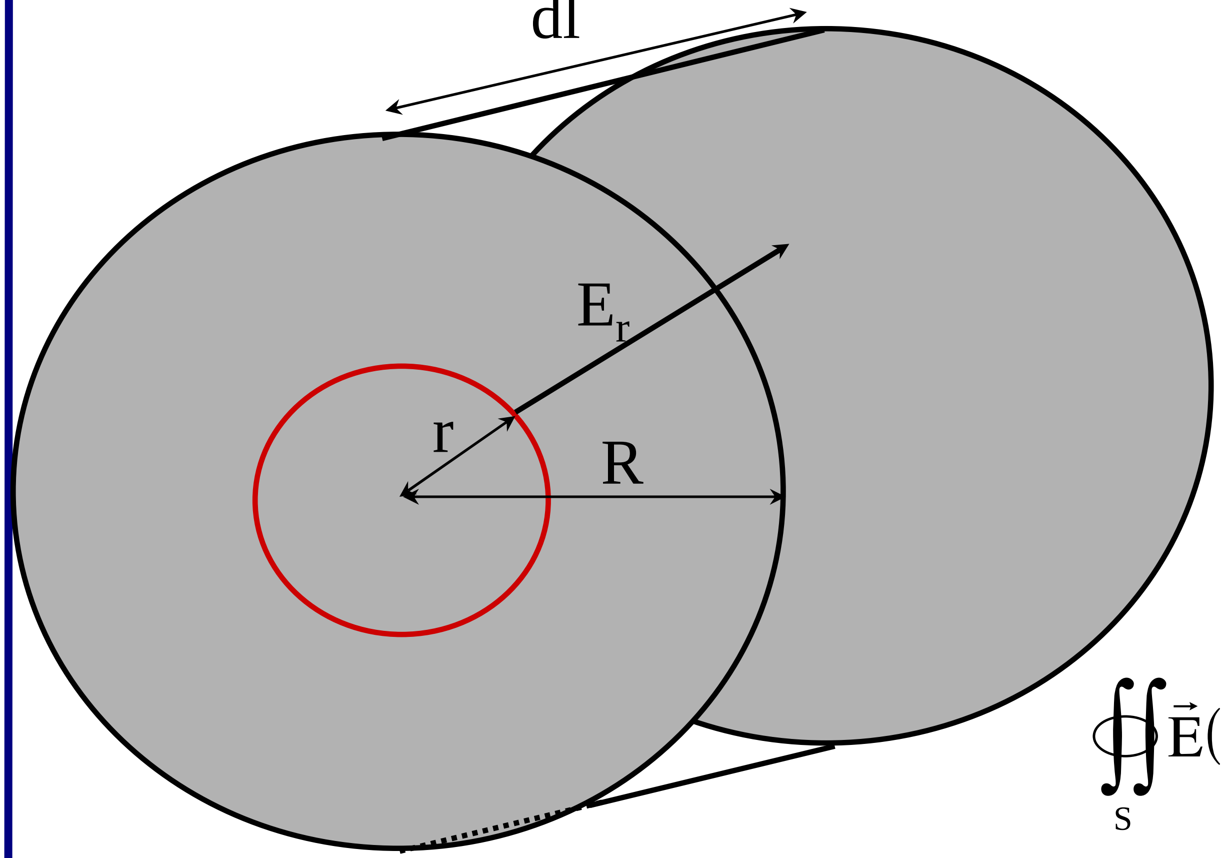
or $\Delta V = -\dfrac{\rho(\vec{r})}{\varepsilon_0}$ and $\vec{E}(\vec{r}) = -\vec{\nabla} V$

Back to the lab frame, it is reduced by a factor :

$1-\beta^2 = 1/\gamma^2$ (due to magnetic field)

- Defocusing effect
- Phase advance reduction

Electric field in a homogenous cylinder



$$\rho = \frac{I}{\pi R^2 \cdot \beta c}$$

Gauss theorem :

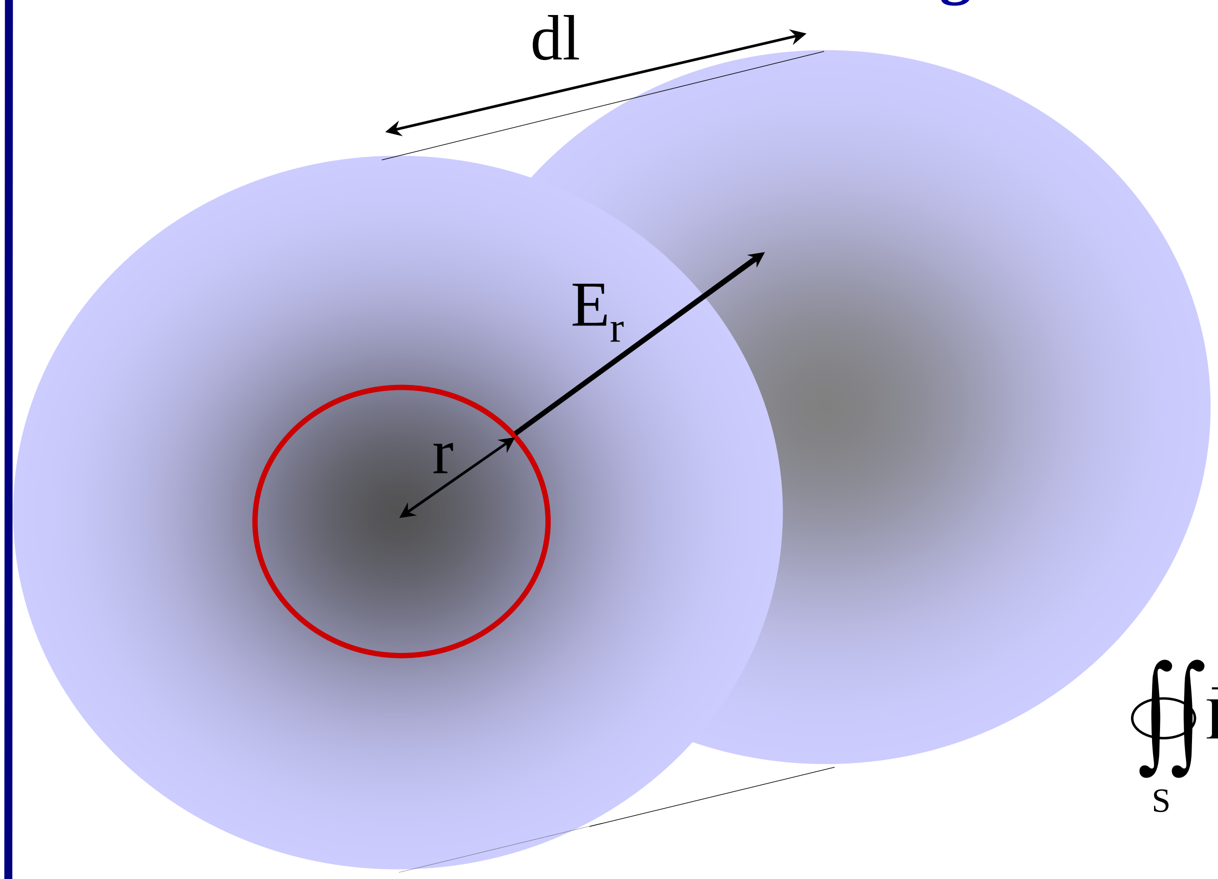
$$\oiint_S \vec{E}(\vec{r}) \cdot d\vec{s} = \frac{1}{\varepsilon_0} \iiint_V \rho(\vec{r}) \cdot dv$$

$$2\pi r \cdot E_r \cdot dl = \frac{1}{\varepsilon_0} \cdot \pi r^2 \cdot \rho \cdot dl$$

$$E_r(r) = \frac{I}{2\pi\varepsilon_0 R^2 \cdot \beta c} \cdot r$$

Linear force

Electric field in a gaussian cylinder



$$\rho(r) = \frac{I}{2\pi \cdot \sigma_r^2 \cdot \beta c} \cdot e^{\frac{-r^2}{2 \cdot \sigma_r^2}}$$

Gauss theorem :

$$\oiint_S \vec{E}(\vec{r}) \cdot d\vec{s} = \frac{1}{\varepsilon_0} \iiint_V \rho(\vec{r}) \cdot dv$$

$$E_r(r) = \frac{I}{2\pi\varepsilon_0 \cdot \beta c} \cdot \frac{\left(1 - \exp\left(-\frac{r^2}{2 \cdot \sigma_r^2}\right)\right)}{r}$$

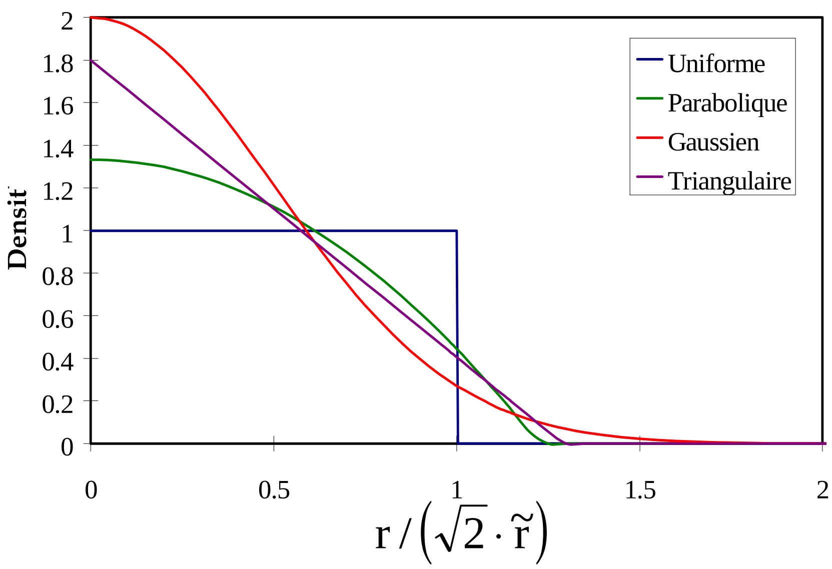
Non-linear force

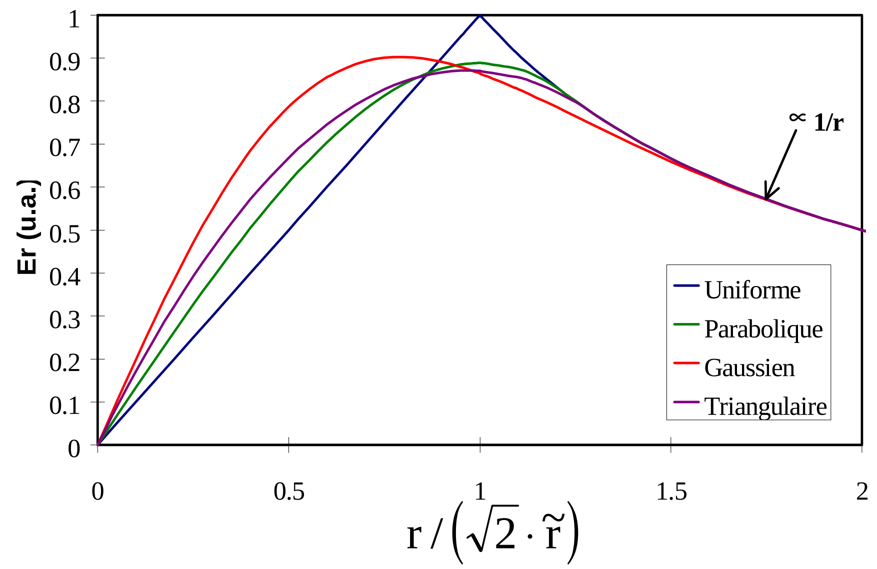
Equivalent beams

- Two beam are equivalent if they have the same current and the same RMS parameters (emittance, Twiss).

- Their dynamics (including space-charge) can be linearised and modeled by the same equation : the envelop equation used to match the beam

Example of equivalent continuous axisymétric beams.





Emittance growth : filamentation

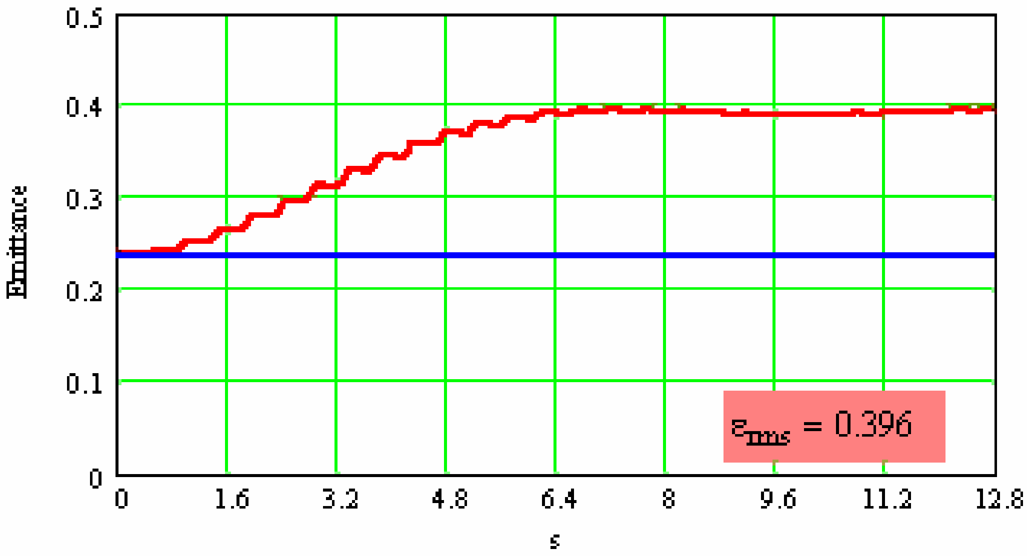
When the confinement force is non linear (multipole, longitudinal, space-charge) , the particle phase advance depends on the oscillation amplitude A :

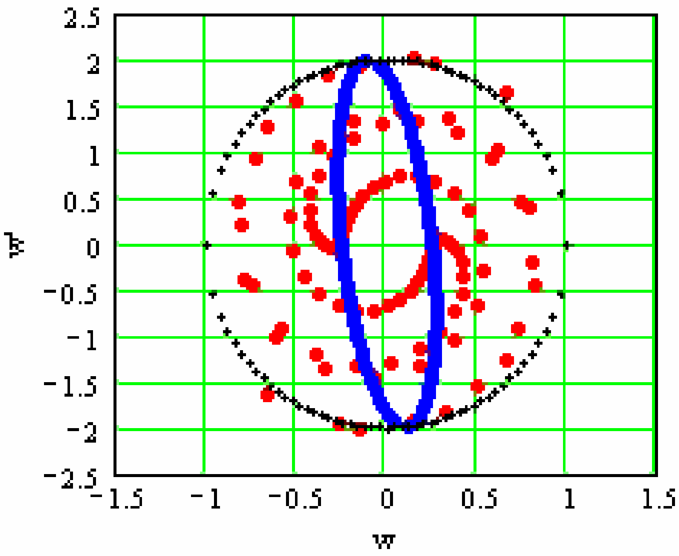
$$\sigma = \sigma(A)$$

$$\frac{d^2 w}{ds^2} + k_w(s,w) \cdot w = 0$$

This phenomenon is known as the tune spread.

Particle do not rotate at the same speed in the phase-space : possible filamentation





Linear force — Non linear force