EXERCISE 1: Solution

Werner Herr, CERN, AB Division

(Solution found at: /afs/ifh.de/user/w/wfherr/public/EX1)

Exercise:

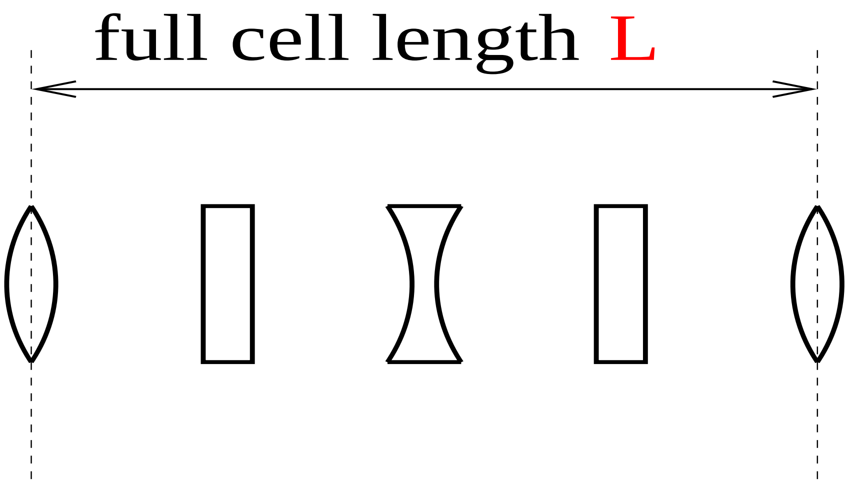
Design a machine for protons at a momentum of 20 GeV/c with the following basic parameters:

- Circumference = 1000 m,
- Quadrupole length $L_{quad} = 3.0$ m,
- 8 FODO cells.
- Dipole length is 5 m, maximum field is 3 T

Apply the knowledge from previous lectures at this school and find the optics (strength of dipoles and quadrupoles) so that betamax is around 300m. Implement it into MAD format using thin lenses for all elements and verify the calculations.

What is needed ?

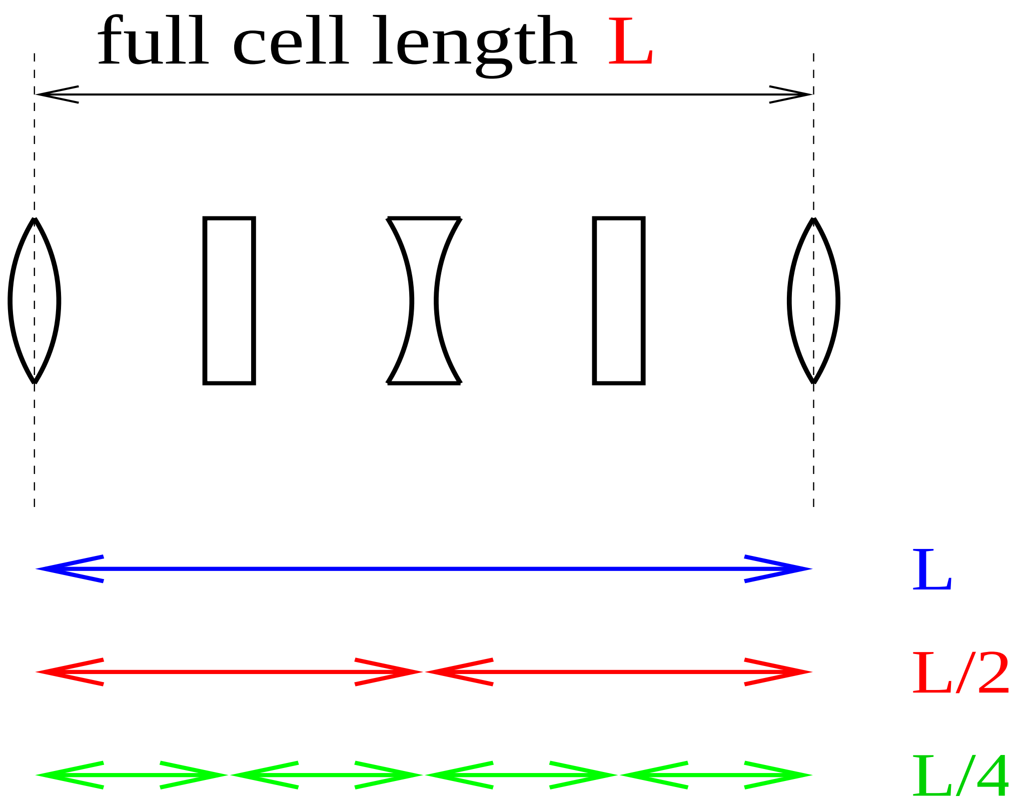
We have to define a FODO cell:



- **1 focusing quadrupole**
- **1 defocusing quadrupole**
- **2·n bending dipoles**

Fist steps

We have to define a FODO cell:



- **Make it fully symmetric**
- **Equidistant elements**

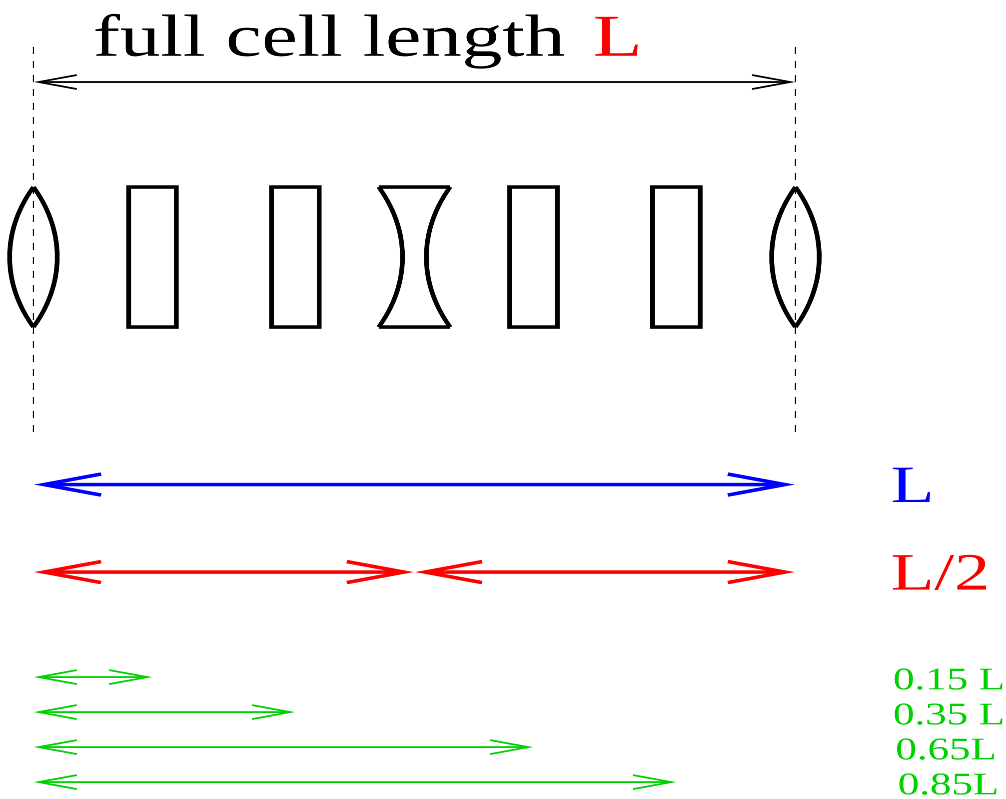
Bending magnets

Strength of the bending magnets:

- **Dipoles give 15 Tm $\rightarrow$ 0.225 rad at 20 GeV**
- **2 π/0.225 = 27.9 i.e. 28 (or more) magnets**
- **$\rightarrow$ we need 3.5, i.e. 4 magnets per cell**
- **$\rightarrow$ each with 0.1963495 rad bending angle**
- **In MAD we write them as thin lenses:**
- **mb: multipole,lrad=d,knl={2.0*pi/(4*ncell)};**

Positioning of elements

We have to define a FODO cell:



- **Not the only possibility**
- **Convenient to put them symmetric**

The cell

The cell we can write as (with lcell = 125 m):

```
qf:  qf,  at = 0.0;
mb:  mb,  at = 0.15*lcell;
mb:  mb,  at = 0.35*lcell;
qd:  qd,  at = 0.50*lcell;
mb:  mb,  at = 0.65*lcell;
mb:  mb,  at = 0.85*lcell;
```

- **We need 8 cells !**
- **We can write the positions explicitely**
- **We can loop !**

All cells

The complete machine we write as (with lcell = 125 m and ncell = 8):

```
n = 1;
while(n < ncell+1) {
  qf:  qf,  at = (n-1)*ncell +  0.0;
  mb:  mb,  at = (n-1)*ncell +  0.15*lcell;
  mb:  mb,  at = (n-1)*ncell +  0.35*lcell;
  qd:  qd,  at = (n-1)*ncell +  0.50*lcell;
  mb:  mb,  at = (n-1)*ncell +  0.65*lcell;
  mb:  mb,  at = (n-1)*ncell +  0.85*lcell;

  n = n + 1;
}
```

- **Repeats the definition 8 times**

Quadrupole strength

Strength of the quadrupole magnets →

We want to limit the maximum β-functions:

$$\hat{\beta} \;=\; f_{1/2} \cdot \frac{1 + sin(\phi/2)}{cos(\phi/2)} \;=\; L_{1/2} \cdot \frac{1 + sin(\phi/2)}{sin(\phi/2)cos(\phi/2)} \quad (1)$$

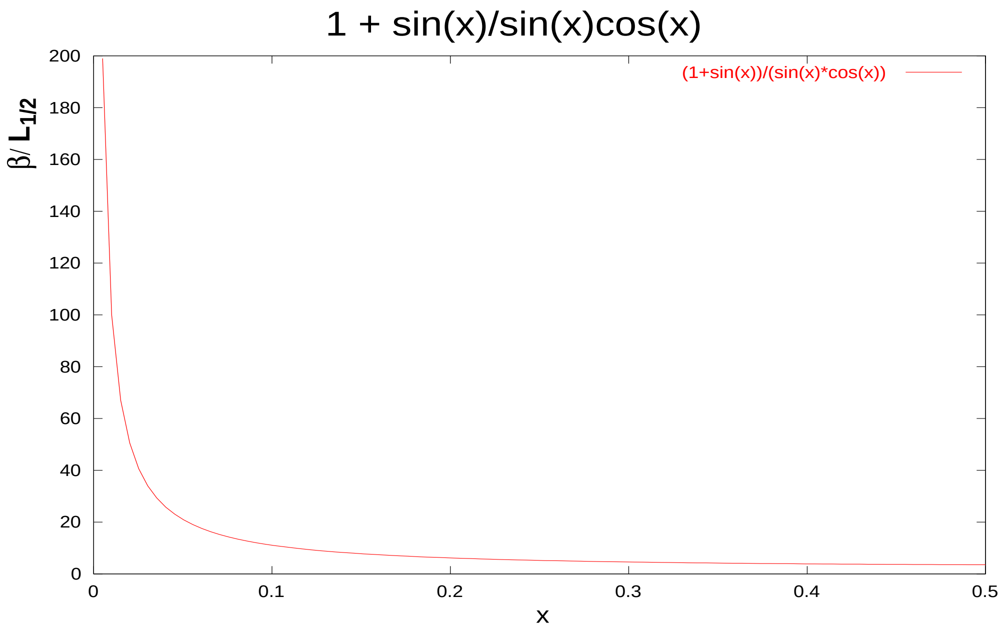
and

$$\check{\beta} \;=\; f_{1/2} \cdot \frac{1 - sin(\phi/2)}{cos(\phi/2)} \;=\; L_{1/2} \cdot \frac{1 - sin(\phi/2)}{sin(\phi/2)cos(\phi/2)} \quad (2)$$

Have to find ϕ for given $\check{\beta}$ and $\mathbf{L}_{1/2}$

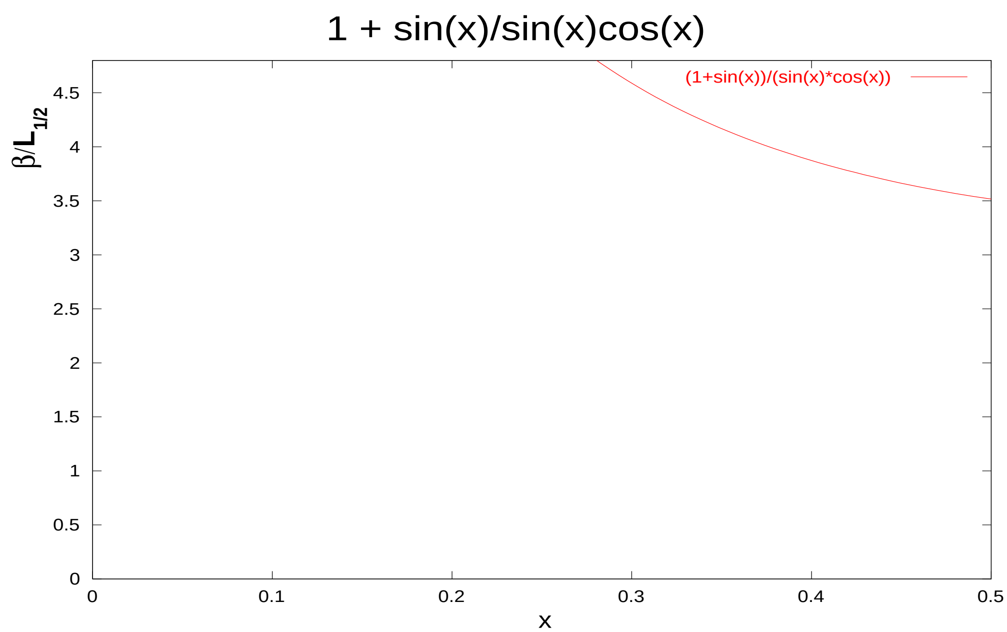
How to solve the equation?

- Analytically (rather heavy !)
- Numerically (table, Mathematica)
- Graphically



How to solve the equation?

- **We have a ratio $\hat{\beta}/\mathbf{L}_{1/2} = 4.8$**



Quadrupole strength

- **For** $\hat{\beta}$ **= 300 m,** $\mathbf{L_{1/2}}$ **= 62.5 m, lq = 3.0 m:**
- **We get** $\phi/2$ **= 16.07066**^{o}
- **With** $f_{1/2} = L_{1/2}/sin(\phi/2) \quad \rightarrow$
- $f_{1/2}$ **= 225.776 m: f = 112.888 m**
- $k_1 = \frac{1}{L_{quad} \cdot f}$ **=** $\pm$ **0.00295278**
- **qf: multipole,lrad=d,knl={0,lq*0.00295278};**
- **qd: multipole,lrad=d,knl={0,-lq*0.00295278};**

The machine

```
circum=1000.0;
ncell =  8; // Number of cells
lcell = circum/ncell;
lq = 3.00; // Length of a quadrupole

// element definitions;

// define bending magnet as multipole
// we have 4 bending magnets per cell
mb: multipole,lrad=d,knl={2.0*pi/(4*ncell)};

// define quadrupoles as multipoles
qf: multipole,lrad=d,knl={0,0.295278e-2*lq};
qd: multipole,lrad=d,knl={0,-0.295278e-2*lq};

// sequence declaration;
cascell1: sequence, refer=centre, l=circum;
start_machine: marker, at = 0;
!
   n = 1;
```

```
   while (n < ncell+1) {
   qf: qf,   at=(n-1)*lcell;
   mb: mb,   at=(n-1)*lcell+0.15*lcell;
   mb: mb,   at=(n-1)*lcell+0.35*lcell;
   qd: qd,   at=(n-1)*lcell+0.50*lcell;
   mb: mb,   at=(n-1)*lcell+0.65*lcell;
   mb: mb,   at=(n-1)*lcell+0.85*lcell;
!
   n = n + 1;
}
end_machine: marker at=circum;
endsequence;
```

The use

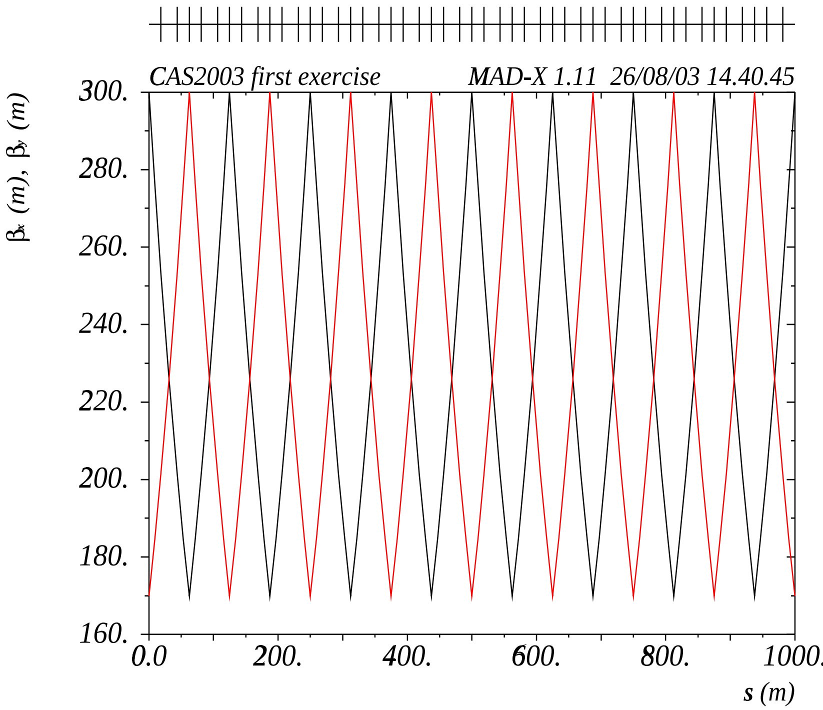
```
TITLE, 'CAS2003 first exercise';
// call the input file
call file="ex1.seq";
option,-echo;

// define the beam
Beam, particle=proton,sequence=cascell1,
      energy=20.0;

// select the sequence
use, sequence=cascell1;
// calculate Twiss parameters
select,flag=twiss,column=name,betx,bety;
twiss,save,centre,file=twiss.out;
// plot beta-functions
plot, haxis=s, vaxis=betx, bety, colour=100;

stop;
```

Graphical output



- **Plots β_x and β_y along the machine**
- **Any Twiss parameter can be plotted**

Twiss summary

```
++++++ table: summ
          length          orbit5            alfa        gammatr
  1.0000000e+03  -0.0000000e+00   1.9892714e+00   7.0901099e-01
              q1             dq1         betxmax          dxmax
  7.1425288e-01  -7.3359403e-01   2.9999965e+02   3.6461573e+02
           dxrms          xcomax          xcorms             q2
  3.2113273e+02   0.0000000e+00   0.0000000e+00   7.1425288e-01
             dq2         betymax           dymax          dyrms
 -7.3359403e-01   2.9999965e+02   0.0000000e+00   0.0000000e+00
          ycomax          ycorms          deltap
  0.0000000e+00   0.0000000e+00   0.0000000e+00
```

Twiss table

```
@ NAME              %05s "TWISS"
@ TYPE              %05s "TWISS"
@ SEQUENCE          %08s "CASCELL1"
@ PARTICLE          %06s "PROTON"
@ MASS              %le          0.938271998
@ CHARGE            %le                    1
@ ENERGY            %le                  450
@ PC                %le        449.999021827
@ GAMMA             %le        479.605062241
@ KBUNCH            %le                    1
@ BCURRENT          %le     0.00504335344393
@ SIGE              %le              0.00045
@ SIGT              %le                    0
@ NPART             %le         105000000000
@ EX                %le                    1
@ EY                %le                    1
@ ET                %le                    1
@ LENGTH            %le                 1000
@ ALFA              %le        1.98927149152
@ ORBIT5            %le                   -0
@ GAMMATR           %le       0.709010995915
@ Q1                %le         0.7142528897
@ Q2                %le         0.7142528897
@ DQ1               %le      -0.733594033286
@ DQ2               %le      -0.733594033286
@ DXMAX             %le        364.615738748
@ DYMAX             %le                    0
@ XCOMAX            %le                    0
@ YCOMAX            %le                    0
@ BETXMAX           %le        299.999654861
@ BETYMAX           %le        299.999654861
@ XCORMS            %le                    0
@ YCORMS            %le                    0
@ DXRMS             %le        321.132732943
@ DYRMS             %le                    0
@ DELTAP            %le                    0
@ TITLE             %22s "CAS2003 first exercise"
@ ORIGIN            %16s "MAD-X 1.11 Linux"
@ DATE              %08s "26/08/03"
@ TIME              %08s "15.11.09"
```

```
* NAME              S                   BETX                BETY
$ %s                %le                 %le                 %le
 "CASCELL1$START"   0                   299.9996549         169.9161056
 "START_MACHINE"    0                   299.9996549         169.9161056
 "QF"               0                   299.9996549         169.9161056
 "DRIFT_0"          9.375               275.8958303         184.8373458
 "MB"               18.75               253.4124618         201.3790421
 "DRIFT_1"          31.25               225.9553466         225.9553466
 "MB"               43.75               201.3790421         253.4124618
 "DRIFT_0"          53.125              184.8373458         275.8958303
 "QD"               62.5                169.9161056         299.9996549
 "DRIFT_0"          71.875              184.8373458         275.8958303
 "MB"               81.25               201.3790421         253.4124618
 "DRIFT_1"          93.75               225.9553466         225.9553466
 "MB"               106.25              253.4124618         201.3790421
 "DRIFT_0"          115.625             275.8958303         184.8373458
 "QF"               125                 299.9996549         169.9161056
 "DRIFT_0"          134.375             275.8958303         184.8373458
 "MB"               143.75              253.4124618         201.3790421
 "DRIFT_1"          156.25              225.9553466         225.9553466
 "MB"               168.75              201.3790421         253.4124618
 "DRIFT_0"          178.125             184.8373458         275.8958303
 "QD"               187.5               169.9161056         299.9996549
 "DRIFT_0"          196.875             184.8373458         275.8958303
 "MB"               206.25              201.3790421         253.4124618
 "DRIFT_1"          218.75              225.9553466         225.9553466
 "MB"               231.25              253.4124618         201.3790421
 "DRIFT_0"          240.625             275.8958303         184.8373458
 "QF"               250                 299.9996549         169.9161056
 "DRIFT_0"          259.375             275.8958303         184.8373458
 "MB"               268.75              253.4124618         201.3790421
 "DRIFT_1"          281.25              225.9553466         225.9553466
 "MB"               293.75              201.3790421         253.4124618
 "DRIFT_0"          303.125             184.8373458         275.8958303
 "QD"               312.5               169.9161056         299.9996549
 "DRIFT_0"          321.875             184.8373458         275.8958303
 "MB"               331.25              201.3790421         253.4124618
 "DRIFT_1"          343.75              225.9553466         225.9553466
 "MB"               356.25              253.4124618         201.3790421
 "DRIFT_0"          365.625             275.8958303         184.8373458
 "QF"               375                 299.9996549         169.9161056
 "DRIFT_0"          384.375             275.8958303         184.8373458
 "MB"               393.75              253.4124618         201.3790421
 "DRIFT_1"          406.25              225.9553466         225.9553466
```

"MB"	418.75	201.3790421	253.4124618
"DRIFT_0"	428.125	184.8373458	275.8958303
"QD"	437.5	169.9161056	299.9996549
"DRIFT_0"	446.875	184.8373458	275.8958303
"MB"	456.25	201.3790421	253.4124618
"DRIFT_1"	468.75	225.9553466	225.9553466
"MB"	481.25	253.4124618	201.3790421
"DRIFT_0"	490.625	275.8958303	184.8373458
"QF"	500	299.9996549	169.9161056
"DRIFT_0"	509.375	275.8958303	184.8373458
"MB"	518.75	253.4124618	201.3790421
"DRIFT_1"	531.25	225.9553466	225.9553466
"MB"	543.75	201.3790421	253.4124618
"DRIFT_0"	553.125	184.8373458	275.8958303
"QD"	562.5	169.9161056	299.9996549
"DRIFT_0"	571.875	184.8373458	275.8958303
"MB"	581.25	201.3790421	253.4124618
"DRIFT_1"	593.75	225.9553466	225.9553466
"MB"	606.25	253.4124618	201.3790421
"DRIFT_0"	615.625	275.8958303	184.8373458
"QF"	625	299.9996549	169.9161056
"DRIFT_0"	634.375	275.8958303	184.8373458
"MB"	643.75	253.4124618	201.3790421
"DRIFT_1"	656.25	225.9553466	225.9553466
"MB"	668.75	201.3790421	253.4124618
"DRIFT_0"	678.125	184.8373458	275.8958303
"QD"	687.5	169.9161056	299.9996549
"DRIFT_0"	696.875	184.8373458	275.8958303
"MB"	706.25	201.3790421	253.4124618
"DRIFT_1"	718.75	225.9553466	225.9553466
"MB"	731.25	253.4124618	201.3790421
"DRIFT_0"	740.625	275.8958303	184.8373458
"QF"	750	299.9996549	169.9161056
"DRIFT_0"	759.375	275.8958303	184.8373458
"MB"	768.75	253.4124618	201.3790421
"DRIFT_1"	781.25	225.9553466	225.9553466
"MB"	793.75	201.3790421	253.4124618
"DRIFT_0"	803.125	184.8373458	275.8958303
"QD"	812.5	169.9161056	299.9996549
"DRIFT_0"	821.875	184.8373458	275.8958303
"MB"	831.25	201.3790421	253.4124618
"DRIFT_1"	843.75	225.9553466	225.9553466
"MB"	856.25	253.4124618	201.3790421
"DRIFT_0"	865.625	275.8958303	184.8373458

"QF"	875	299.9996549	169.9161056
"DRIFT_0"	884.375	275.8958303	184.8373458
"MB"	893.75	253.4124618	201.3790421
"DRIFT_1"	906.25	225.9553466	225.9553466
"MB"	918.75	201.3790421	253.4124618
"DRIFT_0"	928.125	184.8373458	275.8958303
"QD"	937.5	169.9161056	299.9996549
"DRIFT_0"	946.875	184.8373458	275.8958303
"MB"	956.25	201.3790421	253.4124618
"DRIFT_1"	968.75	225.9553466	225.9553466
"MB"	981.25	253.4124618	201.3790421
"DRIFT_0"	990.625	275.8958303	184.8373458
"END_MACHINE"	1000	299.9996549	169.9161056
"CASCELL1$END"	1000	299.9996549	169.9161056

Orbit correctors

Orbit corrector arrangement in FODO cell: