

Beam Profile Monitors for Emittance measurements

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Transport lines and Linacs

Phosphor Screens
SEM Grids/Harps
OTR
Wire scanners (Linacs)

Emittance of single shots

Hadron accelerators

Wire scanners
Residual Gas Ionization
Residual Gas Scintillation
Synchrotron light (Edge effect, wigglers)
Scrapers/current (destructive)

Emittance preservation

Electron accelerators

Synchrotron light
Wire Scanners
Scrapers/current (destructive)
Laser Wire Scanner

Aspect ratio/coupling

Idea of this course: Beam Profile monitors use quite a lot of different physical effects to measure the beam size. Many effects on the beam and on the monitor have to be studied before a decision for a type of monitor can be made. In this session we will discuss emittance measurements and we will make some detailed examinations of at least two monitor types to demonstrate the wide range of physics of the profile instruments.

Synchrotron light profile monitor

In electron accelerators the effect of synchrotron radiation (SR) can be used for beam size measurements. In this course we will focus on profile determination, but SR can also be used for bunch length measurements with e.g. streak cameras with a resolution of < 1 ps. From classical electrodynamics the radiated power is given for a momentum change dp/dt and a particle with mass m_0 and charge e :

$$P_{SR} = \frac{e^2 c}{6\pi\epsilon_0 (m_0 c^2)} \left[\frac{dp}{dt} \right]^2$$

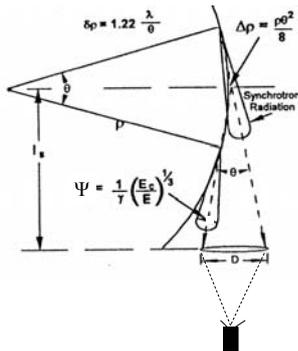
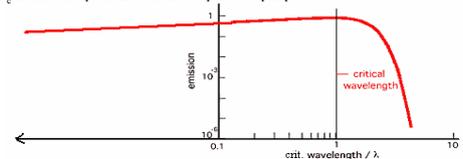
For linear accelerators $dp/dt = dW/dx$. For typical values of $dW/dx = 10 - 20$ MeV/m the SR is negligible. In circular machines an acceleration perpendicular to the velocity exists mainly in the dipole magnets (field B) with a bending radius $\rho = \beta\gamma m_0 c / (eB)$. The total power of N circulating particles with $\gamma = E/m_0 c^2$ is then

$$P_{tot} = \frac{e^2 c \gamma^4}{6\pi\epsilon_0 \rho^2} N$$

This expression is also valid for a ring having all magnets of the same strength and field-free sections in between.

The critical wavelength λ_c divides the Spectrum of SR in two parts of equal power:

$$\lambda_c = \frac{4\pi\rho}{3\gamma^3}$$

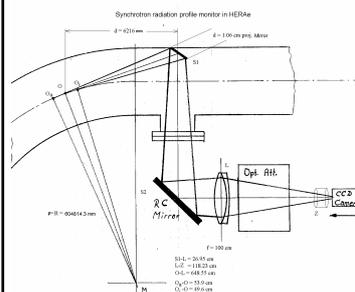


Opening angle Ψ of SR (1/2 of cone!) for $\lambda \gg \lambda_c$:
with

$$\Psi = \frac{1}{\gamma} \left(\frac{\lambda}{\lambda_c} \right)^{1/3} = \left(\frac{3\lambda}{4\pi\rho} \right)^{1/3}$$

$\gamma = E/m_0 c^2 = E [\text{MeV}] / 0.511$
 $\gamma = 23483$ at 12 GeV and
 $\gamma = 52838$ at 27 GeV
Path length s :
 $s = \rho\theta$
 ρ = Bending radius of Dipole

Example HERAe



$R = \rho = 604814.3$ mm
 $G = O-L = 6485.5$ mm
 $B = L-Z = 1182.3$ mm
 $O-S1 = 6216$ mm
 $L = O_1-O_2 = 1035$ mm
opening angle (horizontal): $\tan\theta/2 = d/26216 \Rightarrow \theta/2 = \arctan d/26216 = 0.85$ mrad
opening angle (vertical):
 $\Psi(\lambda) = 1/\gamma (\lambda/\lambda_c)^{1/3}$

$$\lambda_c = \frac{4\pi\rho}{3\gamma^3} = 0.017 \text{ nm} < \lambda_c < 0.19 \text{ nm}$$

Exercise SR1 : Which problems with the setup can be expected?:

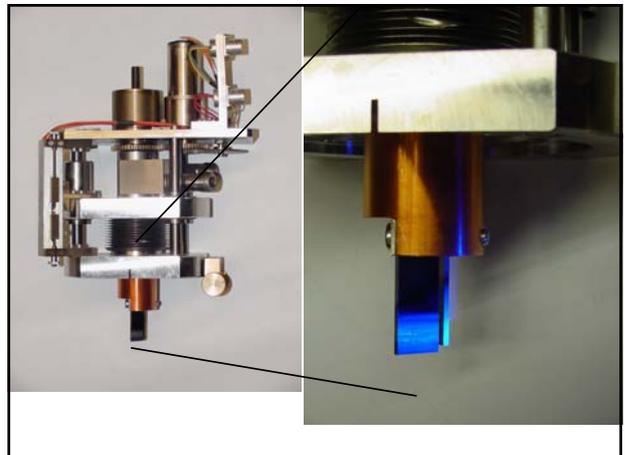
Heating of mirror:
 \Rightarrow total emitted Power per electron:

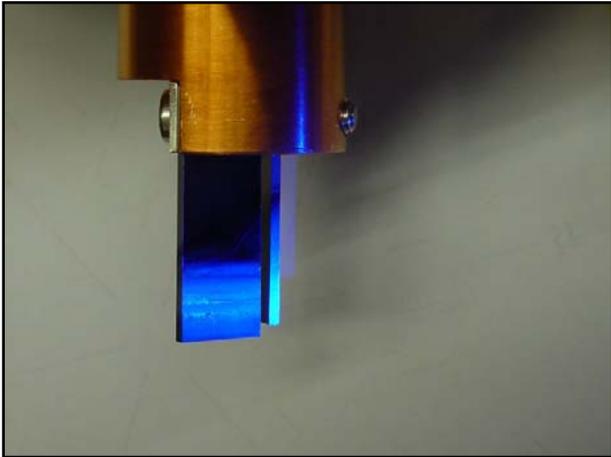
$$P = \frac{e^2 c \gamma^4}{6\pi\epsilon_0 \rho^2}$$

total Power of 46 mA circulating electrons at 27 GeV (Number of electrons $N_e = 6 \cdot 10^{12}$)

$$P_{tot} = 6 \cdot 10^6 \text{ W}$$

The mirror will get $P_{tot} \cdot \Theta / (2\pi) = 1600 \text{ W}$ (Integral over all wavelength!!!)
 \Rightarrow mirror is moveable, mirror has to be cooled
 \Rightarrow Material with low Z is nearly not visible for short wavelength \Rightarrow Beryllium
 \Rightarrow Still 100 W on mirror in HERAe



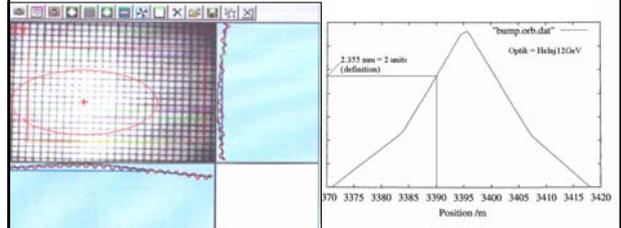


Exercise SR2: What limits the spatial resolution?

Diffraction, depth of field, arc, camera => physical
 Alignment, lenses, mirrors, vibrations => technical

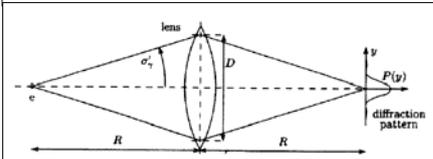
How to calibrate the optics?

Grid (yardstick) at point of emission, orbit bumps, ...



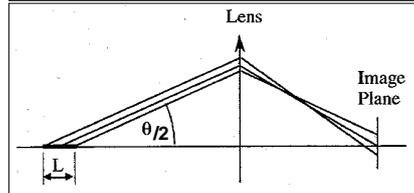
Diffraction:

EQ 1:
 Diffraction limit (for Object):
 For normal slit:
 $\sigma_{Dif} = 0.47 * \lambda / \theta / 2$ (horizontal, mirror defines opening angle θ)
 $\sigma_{Dif} \approx 0.47 * \lambda / \Psi$ (vertikal)



Depth of field:

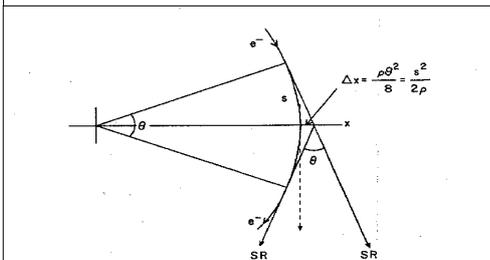
EQ 2:
 depth of field:
 Vertical: $\Delta_{depth} \approx L / 2 * \Psi = \sigma_{depth}$
 Horizontal: $\Delta_{depth} \approx L / 2 * \theta / 2 = \sigma_{depth}$ (mirror defines opening angle θ)
 $L \approx \rho \tan \theta$ or $2\rho (\theta / 2 + \Psi)$



Arc:

EQ 3:
 Arc (horizontal):
 Observation of the beam in the horizontal plane is complicated by the fact that the light is emitted by all points along the arc. The horizontal width of the apparent source is related to the observation angle as:

$\Delta x_{arc} = \rho \theta^2 / 8 = \sigma_{arc}$ (mirror defines opening angle θ)



Camera (finite pixel size)

EQ 4:
 Camera:
 image gain = G/B = 5.485
 typical resolution of camera CCD chip: $\sigma_{chip} = 6.7 \mu m$
 $\sigma_{camera} = \sigma_{chip} * G/B = 37 \mu m$

Resolution:

! not monochromatic !
 $\sigma_{Dif} = 0.47 * \lambda / \theta$ (horizontal) = ??? Depends on wavelength
 $\sigma_{Dif} = 0.47 * \lambda / \Psi$ (vertikal) = ??? Depends on wavelength
 $\sigma_{depth} = L / 2 * \theta / 2 = 440 \mu m$
 $\sigma_{arc} = \rho \theta^2 / 8 = 219 \mu m$ (horizontal)
 $\sigma_{camera} = \sigma_{chip} * G/B = 37 \mu m$

typical spectral sensitivities from CCD Sensors:

SONY

Spectral Sensitivity Characteristics
(includes lens characteristics, excludes light source characteristics)

Assume: $\lambda = 550 \text{ nm}$;
 $(\gamma = E/m_0c^2)$
 $\gamma_{12} = 2.35 * 10^4$ (E = 12 GeV)
 $\gamma_{35} = 6.85 * 10^4$ (E = 35 GeV)
 $\lambda_{c,12} = (4\pi\rho)(3\gamma^2) = 0.195 \text{ nm}$ at 12 GeV
 $\lambda_{c,35} = (4\pi\rho)(3\gamma^2) = 0.008 \text{ nm}$ at 35 GeV

opening angle (horizontal): $\tan\theta/2 = d/26216 \Rightarrow \theta/2 = \arctan d/2/6216 = 0.85 \text{ mrad}$

opening angle (vertical):
 $\Psi(\lambda) = 1/\gamma (\lambda/\lambda_c)^{1/3} = [(3\lambda)(4\pi\rho)]^{1/3} = 0.6 \text{ mrad}$

(mirror has to be larger than spot size on mirror)
 \Rightarrow

| | | |
|-------------------------------------|--------------------------------|---------------------|
| σ_{diff} (horizontal) | $= 0.47 * \lambda/\theta/2$ | $= 304 \mu\text{m}$ |
| σ_{diff} (vertical) | $= 0.47 * \lambda/\Psi$ | $= 431 \mu\text{m}$ |
| σ_{depth} | $= L/2 * \theta/2$ | $= 440 \mu\text{m}$ |
| σ_{arc} (horizontal) | $= \rho \theta^2/8$ | $= 219 \mu\text{m}$ |
| σ_{camera} | $= \sigma_{\text{chip}} * G/B$ | $= 37 \mu\text{m}$ |

$$\sigma_{\text{cor}} = (\sigma_{\text{diff}}^2 + \sigma_{\text{depth}}^2 + \sigma_{\text{arc}}^2 + \sigma_{\text{camera}}^2)^{1/2} = 579 \mu\text{m} ; (\text{horizontal})$$

$$\sigma_{\text{cor}} = (\sigma_{\text{diff}}^2 + \sigma_{\text{depth}}^2 + \sigma_{\text{camera}}^2)^{1/2} = 617 \mu\text{m} ; (\text{vertical})$$

Horizontal:
 $\sigma_{\text{cor}} = [(\rho \theta^2/8)^2 + (L/2 * \theta/2)^2 + (0.47 * \lambda/\theta/2)^2]^{1/2}$ with $L \approx \rho \tan \theta \approx \rho \theta$

Vertical:
 $\sigma_{\text{cor}} = [(L/2 * \Psi)^2 + (0.47 * \lambda/\Psi)^2]^{1/2}$ with $L \approx \rho \tan \theta \approx \rho \theta$

Not the whole truth:

1) Diffraction:
 a) Ψ_{exact} is larger than the Gauss approximation (e.g. $0.79 \rightarrow 1.08 \text{ mrad}$ at Tristan)
 b) For a gaussian beam the diffraction width is $\sigma_{\text{diff}} \approx 1/\pi * \lambda/\Psi$

(Ref: ON OPTICAL RESOLUTION OF BEAM SIZE MEASUREMENTS BY MEANS OF SYNCHROTRON RADIATION. By A. Ogata (KEK, Tsukuba). 1991. Published in Nucl Instrum.Meth.A301:596-598,1991)

$\Rightarrow \sigma_{\text{diff}} \approx 1/\pi * \lambda/\Psi_{\text{exact}} = 218 \mu\text{m}$ ($\Psi_{\text{exact}} = 0.8 \text{ mrad}$, $\lambda = 550 \text{ nm}$) vertical

Fig. 1 Angular distribution of the 500 nm component of the synchrotron radiation from the TRISTAN MR bending magnet (246 m bending radius) operated at 30 GeV, and its Gaussian approximation.

8: Fraunhofer-diffraction for synchrotron radiation from long magnets

2) Depth of field:
 The formula $R_{\text{depth}} = L/2 * \theta/2$ describes the radius of the distribution due to the depth of field effect. It is not gaussian and has long tails. The resolution of an image is probably much better than the formula above. A gaussian approximation with the same integral is shown in the figure below resulting in a width of $\sigma_{\text{depth}} = 61 \mu\text{m}$.

| | | | |
|--------------------------|--------------------------------|----------------------------------|---|
| σ_{diff} | $= 0.47 * \lambda/\theta/2$ | $= 304 \mu\text{m}$ (horizontal) | before: (431 μm) (440 μm) |
| σ_{diff} | $= 1/\pi * \lambda/\Psi$ | $= 218 \mu\text{m}$ (vertical) | |
| σ_{depth} | $= L/2 * \theta/2$ | $= 61 \mu\text{m}$ | |
| σ_{arc} | $= \rho \theta^2/8$ | $= 219 \mu\text{m}$ (horizontal) | |
| σ_{camera} | $= \sigma_{\text{chip}} * G/B$ | $= 37 \mu\text{m}$ | |

| | | | |
|-----------------------|---|------------------------------------|--|
| σ_{cor} | $= (\sigma_{\text{diff}}^2 + \sigma_{\text{depth}}^2 + \sigma_{\text{arc}}^2 + \sigma_{\text{camera}}^2)^{1/2}$ | $= 381 \mu\text{m}$; (horizontal) | (579 μm) (617 μm) |
| σ_{cor} | $= (\sigma_{\text{diff}}^2 + \sigma_{\text{depth}}^2 + \sigma_{\text{camera}}^2)^{1/2}$ | $= 229 \mu\text{m}$; (vertical) | |

Beam width $\sigma_{\text{beam}} = (\sigma_{\text{fil,measured}}^2 + \sigma_{\text{cor}}^2)^{1/2}$

Injection Optics
 SIZEA0=1.09
 EMEAN= .36
 PWHM2 =2.95
 PWTIME =1.02
 EMEZ =51.08*10^-9 m rad
 EMEZ =24.78*10^-9 m rad

Exercise SR3: Discuss possible improvements of an SR-monitor:

- Monochromator at shorter wavelength (x-rays, need special optic) \rightarrow
- Use optimum readout angle
- Polarization - filter
- Use x-ray ($\lambda < 0.1 \text{ nm}$) (LEP)

More:
 Interferometer
 The principle of measurement of the profile of an object by means of spatial coherency was first proposed by H.Fizeau and is now known as the Van Cittert-Zernike theorem. It is well known that A.A. Michelson measured the angular dimension (extent) of a star with this method.

References
 SPATIAL COHERENCY OF THE SYNCHROTRON RADIATION AT THE VISIBLE LIGHT REGION AND ITS APPLICATION FOR THE ELECTRON BEAM PROFILE MEASUREMENT.
 By T. Mitsuhashi (KEK, Tsukuba). KEK-PREPRINT-97-56, May 1997. 4pp. Talk given at 17th IEEE Particle Accelerator Conference (PAC 97), Accelerator Science, Technology and Applications, Vancouver, Canada, 12-16 May 1997.

Intensity Interferometer and its application to Beam Diagnostics, Elfm Gluskin, ANL, publ. PAC 1991 San Francisco

MEASUREMENT OF SMALL TRANSVERSE BEAM SIZE USING INTERFEROMETRY
 T. Mitsuhashi
 High Energy Accelerator Research Organization, Oho, Tsukuba, Baraki, 305-0801 Japan
 DIPAC 2001 Proceedings - ESRF, Grenoble

Imaging: Compound Refractive Lens

lens-maker formula: $1/f = 2(n-1) / R$
 X-ray refraction index: $n = 1 - \delta + i\beta$, $\delta \approx 10^{-6}$

- concave lens shape
- strong surface bending R
- small Z (Be, Al, ...)
- small d
- many lenses

Number of lenses: $N = 10 \dots 300$

PETRA III @ 15 keV:

- $R = 201.8 \text{ } \mu\text{m}$, $R_0 = 447 \text{ } \mu\text{m}$, $d = 10 \text{ } \mu\text{m}$
- $N = 20$
- material: beryllium

$f = 3.33 \text{ m}$
 $\sigma_{res} \approx 1 \text{ } \mu\text{m}$

Gero Kube, DESY / MDI | FLS 2006 (Hamburg), 15.-19. May 2006

Interference: ATF (KEK)

ATF-Damping Ring Vertical Beam Size Measurement

Visibility vs. SlitDistance (mm) graph showing $\sigma = 6.2 \mu\text{m}$

smallest result: $4.7 \mu\text{m}$ with 400 nm @ ATF, KEK
 accuracy $\sim 1 \mu\text{m}$

Gero Kube, DESY / MDI | FLS 2006 (Hamburg), 15.-19. May 2006

Back to an imaging SR-Monitor: Still not the whole truth:

(Diffraction Limited) Resolution

Liinaard-Weichert potentials: $\varphi(x) = \frac{-e}{R(1-\delta-\beta)}$, $A(x) = \frac{-e\beta}{R(1-\delta-\beta)}$

Liinaard-Weichert fields: $E(x) = -e \left(\frac{1-\beta^2}{R^2(1-\delta-\beta)^3} + \frac{\delta \times (\delta-\beta \times \beta)}{eR(1-\delta-\beta)^3} \right)$, $B(x) = \delta \times E(x)$

Fourier transform: $\varphi_{\omega} = -e \int \frac{\varphi(x)}{R(x)} e^{i\omega(x+R(\tau)/c)} dx$, $A_{\omega} = -e \int \frac{A(x)}{R(x)} e^{i\omega(x+R(\tau)/c)} dx$

Electric field: $E_{\omega} = -i\omega \int \frac{E(x)}{R(x)} - \frac{\delta \times \delta}{\omega} \frac{\delta}{R^2(\tau)} e^{i\omega(x+R(\tau)/c)} dx$

numerical near field calculation depending on field geometry
 free codes available (SRW, SPECTRA, ...)

propagation in frame of scalar diffraction theory
 $H_{\omega,1,2} = -i \frac{\omega}{c} \int \int \frac{H_{\omega,1,2}}{r} \exp(i\omega(r/\rho)) d\Omega$

Numerical way
 Includes real electron path
 (depth of field and curve)

Classical way: approximation
 spectral flux density: $\frac{d^2 N_{\omega}}{d\lambda d\omega} = \frac{-\omega^2}{4\pi^2} |H_{\omega,1,2}|^2$

Fourier diffraction pattern by spherical waves
 accounts for the fact that SR is actually a spherical wave
 depth of field, orbit curvature: additional contributions
 depth of field and orbit curvature included

G. Kube and F. Fischer, Appl. Phys. Lett. 80, 1185-1187 (2002)
 G. Kube and F. Fischer, Proceedings of the EPAC2004, Stockholm, Sweden (2004) 1177

Resolution Broadening Effects for the HERAe Emittance Monitor

determination:

- calculation of spatial SR intensity distribution including beam emittance
- quadratical subtraction of beam size

For ∞ mirror size

horizontal profile: $\sigma_{res} = 203 \mu\text{m}$ (numerical), $\sigma_{res} = 171 \mu\text{m}$ (analytical)

vertical profile: $\sigma_{diff+fract} = 188 \mu\text{m}$, $\sigma_{dof+curv} = 275 \mu\text{m}$, $\sigma_{dof} = 134 \mu\text{m}$, $\sigma_{res} = 138 \mu\text{m}$

579 \rightarrow 381 \rightarrow 203 μm | 617 \rightarrow 229 \rightarrow 138 μm

G. Kube, F. Fischer, K. Wittenburg, in Proceedings of BW2004, AIP Conf. Proc. 732 (2004), p.350-357
 G. Kube, F. Fischer, Ch. Wiebers, K. Wittenburg, in Proceedings of DIPAC2005 (in press)

Vertical Resolution for Off-Axis Observation at HERAe

motivation:

- high heat load of extraction mirror
- opening angle Ψ_c for optical SR larger than for X-rays \rightarrow x-ray miss the mirror

solution:

- observation in off-axis geometry ($\theta_m \geq 0$)
- modification of lower bound in Kirchhoff integral (SR propagation through optical system)

consequence: additional resolution broadening

point spread function: σ -polarization, π -polarization

comparison: σ -polarization, π -polarization

G. Kube, F. Fischer, K. Wittenburg, in Proceedings of BW2004, AIP Conf. Proc. 732 (2004), p.350-357
 G. Kube, F. Fischer, Ch. Wiebers, K. Wittenburg, in Proceedings of DIPAC2005 (in press)

Comparison SR-monitor vs Wire scanner

Data - Frame

Resolution: $\sigma_v = 542 \mu\text{m}$

Correction: $617 \rightarrow 229 \rightarrow 138 \mu\text{m}$

Emittance Cycle: $\sigma_{MEAN} = 87$, $\sigma_{RMS} = 15$, $\sigma_{WHICH} = 20$
 $\sigma_{EMITX} = 37.62 \times 10^{-9} \text{ m rad}$, $\sigma_{EMITY} = 11.22 \times 10^{-9} \text{ m rad}$

Proton Synchrotron Radiation Diagnostics

"Frequency Boost" of Synchrotron Radiation

task: production of sufficient synchrotron radiation intensity at high frequencies $\omega \gg \omega_c$

\rightarrow intensity: $\frac{d^2W}{dt d\omega} \propto |\dot{E}_\omega|^2$ with $\dot{E}_\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt \dot{E}(t) e^{i\omega t}$

intuitive approach: synchrotron radiation in time domain

1.) field shape for on-axis observation ($\theta = E_\omega = 0$)

2.) pulse length estimation at fixed observation point

truncation of wave train:

$\Delta t = \frac{1}{\omega_c}$

- R. Chazan, Phys. Rev. A **20** (1979) 524
 - F. Med, Particle Accelerator **2** (1979) 215
 - A. Heilmann, CAS proceedings 88-04

Generation of Frequency Boost

sharp cut-off of wavetrain in time domain

Central Field

Fringe Field

Sheet Magnet

$\frac{d^2N}{d\Omega dt d\omega} |\dot{E}_\omega|^2$
 with $\dot{E}_\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt \dot{E}(t) e^{i\omega t}$

still requires high beam energies (CERN, Tevatron, HERA)

Gero Kube, DESY / MDI DIPAC 2007 (Venice), May 21, 2007

Wire Scanners

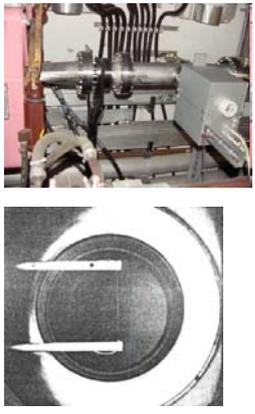
Introduction

Conventional wire scanners with thin solid wires (conventional compared with new techniques using, for example, Lasers) are widely used for beam size measurements in particle accelerators.

Their advantages:

- 1) Resolution of down to 1 μm
- 2) Trusty, reliable
- 3) Direct

0.1 micron position resolution is possible



CERN/DESY 1990-2003

Speed: 1 m/s
 Scanning area: approx. 10 cm
 Wire material: Carbon/Quartz
 Wire diameter: 7 microns
 Signal: shower

1 micron resolution

Where one should locate the Scintillator?

Projected angular distribution could be approximated by Gaussian with a width given by

$$\Theta_{\text{mean}} = \frac{0.014 \text{ GeV}}{pc} \cdot \sqrt{\frac{d'}{L_{\text{rad}}}} \cdot \left(1 + 1/9 \cdot \log_{10} \frac{d'}{L_{\text{rad}}}\right)$$

$d' = 1.5 \times 10^{-3} \text{ cm}$ – the thickness of the target, $X_0 = 12.3 \text{ cm}$ – quartz-wire radiation length, $x/X_0 = 1.22 \times 10^{-4}$

It is corresponding to:

$$\Theta_{\text{mean}} \approx 3.0 \times 10^{-6} \text{ rad}$$

for electron momentum of 30 GeV/c.

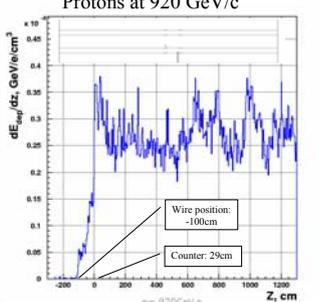
Scattered particles will arrive vacuum chamber of radius $R = 2 \text{ cm}$ at:

$$z \approx \frac{R}{\Theta_{\text{mean}} \sqrt{2}} \approx 4.9 \text{ km!!!!}$$

What to do?

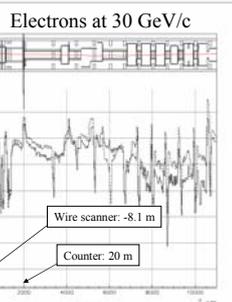
Monte Carlo simulations of best location for scintillators

Protons at 920 GeV/c



Wire position: -100cm
Counter: 29cm

Electrons at 30 GeV/c



Wire scanner: -8.1 m
Counter: 20 m

Simulation includes all magnetic fields
as well as all elastic and inelastic scattering cross sections

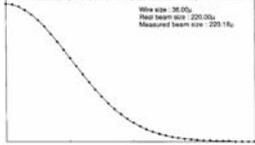
Wire scanner's known limitations are:

1. The smallest measurable beam size is limited by the finite wire diameter of a few microns,
2. Higher Order Modes may couple to conductive wires and can destroy them,
3. High beam intensities combined with small beam sizes will destroy the wire due to the high heat load.
4. Emittance blow up

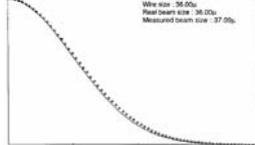
Limitations:

1. Wire size
The smallest achievable wires have a diameter of about 5-6 μm .
An example of the error in the beam width determination is shown for a 36 μm wire.

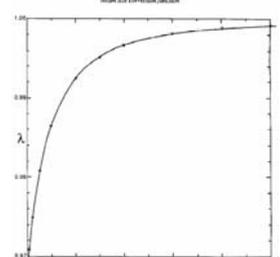
Wire size: 36.00 μm
Real beam size: 220.00 μm
Measured beam size: 220.18 μm



Wire size: 36.00 μm
Real beam size: 36.00 μm
Measured beam size: 37.05 μm



Beam size correction function



Influence of the wire diameter on the measured beam width.
(All figures from: Q. King; Analysis of the Influence of Fibre Diameter on Wire-scanner Beam Profile Measurements, SPS-ABM-TM/Note 8802 (1988))

Limitations:

2. Higher Order modes
An early observation (1972 DORIS) with wire scanners in electron accelerators was, that the wire was always broken, even without moving the scanner into the beam. An explanation was that Higher Order Modes (HOM) were coupled into the cavity of the vacuum chamber extension housing the wire scanner fork. The wire absorbs part of the RF which led to strong RF heating.

Exercise WIRE1: Discuss methods of proving this behavior. What are possible solutions against the RF coupling?

Methods:

1. Measurement of wire resistivity
2. Measurement of thermo-ionic emission
3. Optical observation of glowing wire
4. Measurement of RF coupling in Laboratory with spectrum analyzer

1. Measurement of wire resistivity

The wire resistivity will change depending on the temperature of the wire, even without scanning.

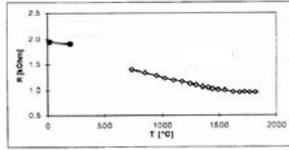
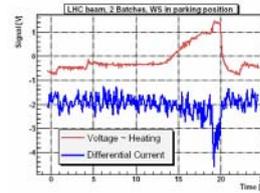


Fig 4: Measured wire resistance variations with temperature.

Here: 8 μm Carbon wire
(from OBSERVATION OF THERMAL EFFECTS ON THE LEP WIRE SCANNERS. By J. Camas, C. Fischer, J.J. Graa, R. Jung, J. Koopman (CERN). CERN-SL-95-20-BI, May 1995. 4pp. Presented at the 16th Particle Accelerator Conference - PAC-95, Dallas, TX, USA, 1 - 5 May 1995. Published in IEEE PAC 1995:2649-2651)

2. Measurement of thermo-ionic emission

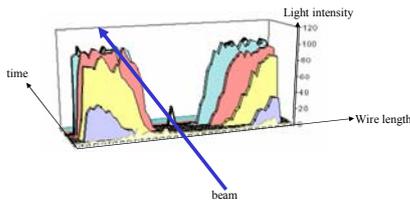


Wire heating due to the LHC beam injection in the SPS (No scan, wire in parking position). The beam energy ramp/bunch length decreasing begin t=11 s.

A constant current was supplied to the wire and the voltage drop across it was fed to a digital scope together with the difference between the input and output currents. The differential current ($I_{out}-I_{in}$) grow up is due to the wire heating and consequent emission of electrons for thermionic effect. Fig. WIRE5 shows such voltage and differential current evolutions during the SPS cycle with LHC type beam. No scans were performed along this cycle. It is thus evident that the wire heating does not depend on the direct wire-beam interaction only.

(From CAVITY MODE RELATED WIRE BREAKING OF THE SPS WIRE SCANNERS AND LOSS MEASUREMENTS OF WIRE MATERIALS F. Caspers, B. Dehning, E. Jensen, J. Koopman, J.F. Mala, CERN, Geneva, Switzerland F. Ronzavolo, CERN/University of Lausanne, Switzerland, DIPAC03)

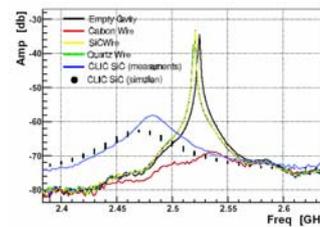
3. Optical observation of glowing wire



Digitized video recording of an 8 μm carbon wire scanning a 0.8 mA beam. The wire is parallel to the horizontal axis, and the light intensity is plotted along the vertical axis (arbitrary units). Successive profiles are separated by 20 ms. The central spot corresponds to the passage of the wire through the beam. Thus, RF heating led to (huge) thermal glowing before the beam interacts with the wire.

(from: QUARTZ WIRES VERSUS CARBON FIBERS FOR IMPROVED BEAM HANDLING CAPACITY OF THE LEP WIRE SCANNERS. By C. Fischer, R. Jung, J. Koopman (CERN). CERN-SL-96-09-BI, May 1996. 8pp. Talk given at 7th Beam Instrumentation Workshop (BIW 96), Argonne, IL, 6-9 May 1996.

4. Measurement of RF coupling with spectrum analyzer



Resonant cavity signal in presence of Carbon (36 μm), Silicon Carbide and Quartz wires

The plot qualitatively proves the RF power absorption of Carbon, and the non-absorption of Silicon Carbide and Quartz. Absorbed energy is mainly converted into heat.

What are solutions for the problems 1-4?

Damping of Higher Order Modes with Ferrites etc.
 Non conducting wires

Limitations:

3. Wire heat load

According to Bethe-Bloch formula, a fraction of energy dE/dx of high energy particles crossing the wire is deposited in the wire. Each beam particle which crosses the wire deposits energy inside the wire. The energy loss is defined by dE/dx (minimum ionization loss) and is taken to be that of a minimum ionizing particle. In this case the temperature increase of the wire can be calculated by:

$$T = C \cdot dE / dx_m \cdot d' \cdot N \cdot \frac{1}{c_p \cdot G} \quad [^{\circ}C]$$

unknown

where N is the number of particles hitting the wire during one scan, d' is the thickness of a quadratic wire with the same area as a round one and G [g] is the mass of the part of the wire interacting with the beam. The mass G is defined by the beam dimension in the direction of the wire (perpendicular to the measuring direction):

Exercise WIRE2: Which kind of wire Material you will prefer for a wire scanner in this accelerator?

Estimate the wire temperature after one scan with a speed v (assume no cooling mechanisms).

Solving G: G [g] is the mass of the part of the wire interacting with the beam. The mass G is defined by the beam dimension in the direction of the wire (perpendicular to the measuring direction) and by the wire diameter d' :

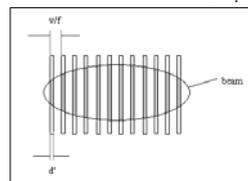
$$G = \text{wire volume} \cdot \rho = 2 \cdot \sigma_v \cdot d'^2 \cdot \rho \quad [g]$$

Solving N:

The number of particles N hitting the wire during one scan depends on the speed of the scan ($\sim 1/v$), the revolution frequency ($\sim f_{rev}$), the wire diameter ($\sim d'$) and the beam current ($\sim NB \cdot n_{bunch}$):

$$N = \frac{d' \cdot f_{rev}}{v} \cdot (NB \cdot n_{bunch})$$

The figure shows a graphical representation of the parameters. The quotient d'/v is the ratio of the scanned beam area or, in other words, like a grid seen by one bunch, assuming that all bunches are equal. However, the ratio can exceed the value 1 (a foil) if the scanning distance between two bunches is smaller than the wire diameter. Note that N does not depend on the beam widths σ .



Geometrical meaning of the parameters v/f and d'

Therefore, the temperature increase of the wire after one scan becomes:

$$T = C \cdot dE / dx_m \cdot d' \cdot N \cdot \frac{1}{c_p \cdot G} \quad [^{\circ}\text{C}] \quad \text{Mass } G = \text{wire volume} \cdot \rho = 2 \cdot \sigma_v \cdot d^2 \cdot \rho \quad [\text{g}]$$

$$N = \frac{d' \cdot f_{rev}}{v} \cdot (NB \cdot n_{bunch})$$

$$T_h = C \cdot dE / dx_m \cdot d' \cdot \frac{f_{rev}}{v} \cdot (NB \cdot n_{bunch}) \cdot \frac{1}{c_p \cdot 2 \cdot \sigma_v \cdot d^2 \cdot \rho} \cdot \alpha \quad [^{\circ}\text{C}]$$

with $\frac{dE}{dx_m} = dE / dx \quad \left[\frac{\text{MeV} \cdot \text{cm}^2}{\text{g}} \right]$ and $f_{rev} \cdot NB = f_{bunch}$

$$T_h = C \cdot dE / dx \cdot n_{bunch} \cdot \frac{f_{bunch}}{v} \cdot \frac{1}{c_p \cdot 2 \cdot \sigma_v} \cdot \alpha \quad [^{\circ}\text{C}]$$

Parameter table

Where h, denotes the horizontal (h) scanning direction. The cooling factor 'α' is described in the next section. Note that the temperature does not depend on the wire diameter and that it depends on the beam dimension perpendicular to the measuring direction. The temperature increase is inverse proportional to the scanning speed, therefore a faster scanner has a correspondingly smaller temperature increase.

The wire parameters $dE/dx / cp$ and the Quotient T_h/T_m should be minimal for a choice of the material ($\alpha = 1$):

| Material | $dE/dx / c_p$ | $T_m [^{\circ}\text{C}]$ | T_h/T_m |
|----------|---------------|--------------------------|-----------|
| AL | 7.7 | $1.1 \cdot 10^4$ | 16.9 |
| W | 50.6 | $7.1 \cdot 10^4$ | 20.9 |
| C | 5.4 | $0.77 \cdot 10^4$ | 2.2 |
| Be | 4.1 | $0.58 \cdot 10^4$ | 4.8 |
| SiO2 | 12.9 | $1.8 \cdot 10^4$ | 10.6 |

TableWire3: calculated Temperatures

From Table WIRE3 follows, that even the best material (Carbon) will be a Factor 2.2 above its melting temperature.

Small pit marks seen near the end of the wire are further evidence for arcing.

Burned by the e-beam at SLC

Exercise WIRE2a: Discuss cooling mechanisms which will cool the wire.

- Secondary particles emitted from the wire
- Heat transport along the wire
- Black body radiation
- Change of c_p with temperature

1) Secondaries:
Some energy is lost from the wire by secondary particles. In the work in J. Bossert et al., The micron wire scanner at the SPS, CERN SPS/86-26 (MS) (1986) about 70% is assumed. In DESY III (example above) no carbon wire was broken during more than 10 years of operation. At HERA, the theoretical temperature of the carbon wire (without secondaries) exceeds the melting temperature after a scan by far ($T = 12800 \text{ }^{\circ}\text{C}$). Considering the loss by secondaries of 70%, the temperature reaches nearly the melting point. In practice, the wire breaks about once in 2 months. The observation is that the wire becomes thinner at the beam center. This may indicate, that during a scan some material of the wire is emitted because of nuclear interactions or is vaporized because it is very close to the melting temperature. This supports the estimate of the 70% loss and one has to multiply the factor $\alpha = 0.3$ in the equation above

| Wire type | Approximation Q (μJ) | Monte Carlo Q (μJ) |
|----------------|----------------------|--------------------|
| 10 μm graphite | 0.140 | 0.169 ± 0.002 |
| 10 μm tungsten | 1.01 | 2.19 ± 0.04 |
| 15 μm tungsten | 2.27 | 4.90 ± 0.05 |
| 50 μm tungsten | 25.3 | 32.0 ± 0.1 |

Heat deposited in various wire types by a passing bunch of 1 nC. The approximated value follows (1), the "Monte Carlo" result is obtained by the simulation of an electromagnetic shower with Fluka.

Thermal Load on Wirecanners
Lars Fröhlich
37th ICFE Advanced Beam Dynamics Workshop on Future Light Sources; May 15-19, 2006 in Hamburg, Germany

2) Heat transport: The transport of heat along the wire does not contribute to short time cooling of the wire (P. Lefevre; CERN PS/DL/Note 78-8). However, frequent use of the scanner heats up also the ends of the wire and its connection to the wire holders (fork).

For low repetition rates (LINACs) this is the major cooling mechanism.

10 μm graphite wire bombarded with 800 bunches at 1 MHz; simulated with various combinations of cooling mechanisms, 5 Hz rep. rate.

Thermal Load on Wirecanners; Lars Fröhlich; 37th ICFE Advanced Beam Dynamics Workshop on Future Light Sources; May 15-19, 2006 in Hamburg, Germany

3) Black body radiation: The temperature T_{bb} at which the radiated power is equal to the deposited power in the wire during one scan $P_{dep} [\text{MeV/s}]$ can be calculated from the Stefan-Boltzmann-law:

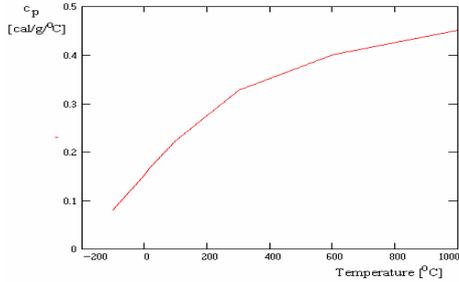
$$T_{bb} = \sqrt[4]{\frac{P_{dep}}{s \cdot A}}$$

where $s = 35.4 \text{ MeV} / (\text{s}^1 \text{ cm}^2 \text{ }^{\circ}\text{K}^4)$ is the Stefan-Boltzmann-constant and A is the area of radiating surface. The surface of the heated wire portion A is $2 \cdot \sigma_v \cdot d \cdot \pi [\text{cm}^2]$. The power can be calculated by:

$$P_{dep h,v} = \alpha \cdot dE / dx \cdot d' \cdot n_{bunch} \cdot \frac{f_{bunch} \cdot d'}{v} \cdot \frac{1}{t_{scan}} \quad [\text{MeV} / \text{s}]$$

where $t_{scan} = 2 \cdot \sigma_{h,v} / v$ is the time for a scan (in the assumption of 2σ it is neglected that only about 70% of the power is concentrated within 2σ). α is the expected loss from secondaries.
For the example above $T_{bb} = 3900 \text{ }^{\circ}\text{C}$. Therefore the black body radiation is only a fraction of cooling in case of fast scans.

4) $c_p(T)$: The heat capacitance is a function of the temperature. Fig. 2 shows the increase of c_p for Carbon with T. The expected temperature after a scan is inversely proportional to c_p . Therefore one can expect a slightly smaller resulting temperature because of this dependence.



Temperature of the wire (v=1m/s)

| | Num. of part. | Typ. Beam diam. | Temp. after scan [C] | Eq. - Temp [Celsius] |
|----------|----------------------|-----------------|----------------------|----------------------|
| HERAp | 1.1*10 ¹³ | 0.7 mm | 3930 | 5160 |
| HERAe | 6.5*10 ¹² | 0.2 mm | 4800 | 4500 |
| PETRAp | 4.8*10 ¹² | 2 mm | 980 | 3500 |
| PETRAe | 1.5*10 ¹² | 0.1 mm | 4700 | 6800 |
| DESY III | 1.2*10 ¹² | 1 mm | 3400 | 5300 |
| TTF fast | 2.8*10 ¹³ | 0.05 mm | 4000 | 7400 |
| TTF slow | 2.8*10 ¹³ | 0.05 mm | 286 000 | 2900 |

Melting temperature = 3500 °C for Carbon
= 1700 °C for Quartz

The wire in DESY III still exists with 200 mA = 1.25·10¹² p
In HERA we exchange the wires every 2 month after "normal" use. Unusual frequent use will destroy the wires much earlier.

Limitations

4: Emittance blow up

Exercise WIRE3: Calculate the emittance blowup of the proton beam after one scan at a position with $\beta = 11.8$ m for $p = 0.3$ and 7 GeV/c (Carbon wire). Assume a measurement position close to a Quadrupole ($\alpha = 0$)

For small deflection angles a good approximation for average root mean square scattering angle is given by:

$$\delta\Theta = \frac{0.014 \text{ GeV}}{pc} \cdot \sqrt{\frac{d'}{L_{rad}} \cdot \left(1 + \frac{1}{9} \cdot \log_{10} \frac{d'}{L_{rad}}\right)}$$

Remember: $\gamma(s)^2 y + 2\alpha(s)yy' + \beta(s)y'^2 = \epsilon$

A fraction Ψ of the circulating beam particles will hit the wire:

$$\Psi = \frac{d' \cdot f_{rev}}{v}$$

(see exercise WIRE2)

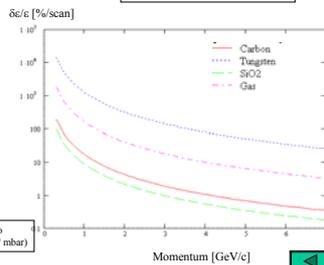
The resulting emittance blowup is than:

$$\delta\epsilon_{rms} = \sqrt{2\pi} \cdot \delta\Theta^2 \cdot \Psi^2 \cdot \beta$$

$$= 5.1 \cdot 10^{-2} \pi \text{ mm mrad}$$

$\sqrt{2\pi}$ from literature

Gas: emittance growth due to residual gas per hour ($P=10^{-7}$ mbar)



D. Möhl, Sources of emittance growth (also P. Bryant; CAS, Beam transfer lines):

$$\delta\epsilon = \pi \frac{1}{2} \cdot \Theta_{rms}^2 \cdot \beta$$

Averaging over all Betatron-phases
Unit of phase space emittance

M. Giovannozzi (CAS 2005)

$$\delta\epsilon = \pi \frac{1}{4} \cdot \Theta_{rms}^2 \cdot \beta$$

D. Möhl, Sources of emittance growth, 2007:

$$\delta\epsilon = \frac{1}{2} \cdot \Theta_{rms}^2 \cdot \beta$$

Wire Parameters

| Parameter | Symbol | Unit | wire material | | | | |
|----------------------|-------------------------|-------------------|---------------|---------|--|-----------|----------------------------|
| | | | AL | Wf | Carbon | Beryllium | Quartz (SiO ₂) |
| wire diameter | d | cm | | | | | |
| min. diameter | d = d ₀ | cm | | | | | |
| density | ρ | g/cm ³ | 2.7 | 19.3 | 2.3 | 1.85 | 2.29 |
| Conversions | c | cal/oxide | | | | | |
| Conversion factor | 0.18 · 10 ¹⁴ | MeV/cad | | | | | |
| Speed of wire | v = 100 | cm/s | | | | | |
| specific capacity* | heat | cal/g/°C | 0.21 | 0.036 | 0.42 (+400/°C) 0.17 (-400/°C) | 0.43 | 0.18 |
| Energy loss of wire | dE/dz | MeV/cm/g | 1.62 | 1.02 | 2.3 | 1.78 | 2.33 |
| loss (dEP) | dE/dz _w | MeV/cm | 4.37 | 35.13 | 5.3 | 3.3 | 5.3 |
| melting temp. | T _m | °C | 650 | 3400 | ca. 3500 | 1200 | 1700 |
| heat conductivity | λ | W/(m·K) | 230 | 100-160 | 30-3000 | 200 | 1.2-1.4 |
| radiation length | X ₀ | cm | 8.9 | 0.36 | 18.8 | 34.7 | 35.3 |
| Nuclear cross length | λ _{nuc} | cm | 20 | 9.6 | 34 | 30 | 25.4 |

Table WIRE1: Parameters of wire materials. * >= 500 °C

The beam parameters used in this exercise are shown in the following table:

| Parameter | Symbol | Unit | Value |
|-------------------------------------|---------------------------------|-----------|------------------------|
| circumference of accel. | circ. | m | 300 |
| particle | p | Proton | |
| Beam particle momentum | p | GeV/c | 0.3-7 |
| Beta function | β _x = β _y | m | 11.8 |
| Emittance | ε _x = ε _y | π mm mrad | 15 |
| revolution Frequency | f _{rev} | MHz | 0.93 |
| Bunch spacing | t _{sech} | ns | 98 |
| | f _{sech} | MHz | 10.2 |
| Number of bunches in accel. | NB | | 11 |
| Bunch charge | Q _{bunch} | 1/e | 1.1 · 10 ¹¹ |
| Beam width measurement 1 | σ _x | mm | 1.5 |
| Beam width perpendicular to meas. 1 | σ _y | mm | 1 |

Table WIRE2: Parameters of Beam

LINACS/Transport Lines Emittance Measurement

In a transfer line, the beam passes once and the shape of the ellipse at the line determines its shape at the exit. Exactly the same transfer line/Linac injected first with one emittance ellipse and then different ellipses has to be accredited with different α and β , γ functions to describe the cases. Thus α and β , γ depend on the input beam and their propagation depends on the structure. Any change in the structure will only change the α and β , γ values downstream of that point. ... The input ellipse must be chosen by the designer and should describe the configuration of all the particles in the beam.

In the following let's assume a transport line or the part of the Linac where no acceleration takes place. What about the emittance?

If no energy is transferred to the beam (Hamiltonian systems), the emittance is conserved.

Explain ways of measuring the emittance of a charged particle beam in a Linear Accelerator or a transport line without knowing the beam optic parameters α , β , γ .

Exercise L1: Which one is the preferable method for a high energy proton transport line ($p > 5$ GeV)?

Solution: 3 (thin) screens or SEM grids or varying quadrupole which measure the different beam widths σ . For pepper pot or slits one needs a full absorbing aperture.



Substrate: Aluminum Foil, 0.001" thick

<http://accelconf.web.cern.ch/AccelConf/p95/ARTICLE/TPB/TPB26.PDF> <http://www.rhicome.bnl.gov/RHIC/Instrumentation/Systems/InProfile/flags.html>



Secondary Emission Detectors

Introduction
When electrically charged particles with sufficient kinetic energy hit the surface of a solid, the latter emits electrons. These electrons are called *secondary electrons*, and the bombarding electrons are called *primary* electrons.

SE Grids
When a beam passes through a foil or a wire, a few percent of low energy electrons, with respect to the incoming particles, are emitted from the superficial layers. This charge depletion is proportional to the local density of the beam and can be used to measure a beam density profile.

The main problems encountered with these monitors are the small useful signal (pC) generated under a very high source impedance, and the collection of unwanted parasitic charges.

The monitors are, in general, built with the thinnest possible foils to minimize disturbance to the beam.

The main limitations of these monitors are their resolution, owing to the finite number and dimension of the strips, and the overall gain spread from channel to channel.

The resolution can be increased in single-shot operations by inclining the grid with respect to the beam direction. In a multi pulse measurement, the resolution can be increased by displacing the grid between measurements.



SEM grid with 15 foils to measure the beam profile. The large foils at either side of the grid are connected electrically together to form a 16-th channel.



Scheme of the pepperpot plate in a cross sectional view and relevant dimensions

Light spots observed on the viewing screen of a Pepperpot device. Top left: Spots generated for calibration using a laser beam. Top right: Spots from an oxygen beam. Bottom: Intensity distribution along one line.

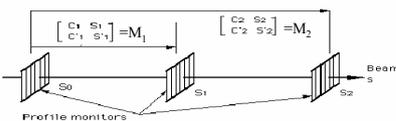


Three screen method:

If β is known unambiguously as in a circular machine, then a single profile measurement determines ϵ by

$$\sigma_y^2 = \epsilon \beta_y$$

But it is not easy to be sure in a transfer line which β to use, or rather, whether the beam that has been measured is matched to the β -values used for the line. This problem can be resolved by using three monitors (see Fig. 1), i.e. the three width measurement determines the three unknowns α , β and ϵ of the incoming beam.



Exercise L2: Assuming that the geometry between the measurement stations and the transport matrices $M_{1,2}$ of the transport line are well defined (including magnetic elements), describe a way to get the emittance using the 3 screens and the σ -matrix.

Unfair, not introduced in the lessons, sorry

Introduction of σ -Matrix

(see for example: K. Wille; Physik der Teilchenbeschleuniger, Teubner)

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_y'^2 \end{pmatrix} = \epsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma \text{ matrix}$$

$$\epsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \quad (\beta\gamma - \alpha^2 = 1)$$

$$\text{Beam width}_{rms} \text{ of measured profile} = \sigma_y = \sqrt{\beta(s) \cdot \epsilon}$$

Transformation of σ -Matrix through the elements of an accelerator:

$$\sigma_s = M \cdot \sigma_0 \cdot M^t \quad M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}; M^t = \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix}$$

L_1, L_2 = distances between screens or from Quadrupole to screen and Quadrupole field strength are given, therefore the transport matrix M is known.
Applying the transport matrix gives (now time for exercise):

Exercise L2: Assuming that the geometry between the measurement stations and the transport matrices $M_{1,2}$ of the transport line are well defined (including magnetic elements), describe a way to get the emittance using the 3 screens and the σ -matrix.

$$\sigma_x = M \cdot \sigma_0 \cdot M^T$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \cdot \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix} = \sigma^{\text{measured}} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy} \\ \sigma_{yy} & \sigma_y^2 \end{pmatrix}^{\text{measured}} = \epsilon_{\text{rms}} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11} M_{11} + \sigma_{12} M_{12} & \sigma_{11} M_{21} + \sigma_{12} M_{22} \\ \sigma_{21} M_{11} + \sigma_{22} M_{12} & \sigma_{21} M_{21} + \sigma_{22} M_{22} \end{pmatrix}$$

$$= \begin{pmatrix} M_{11}(\sigma_{11} M_{11} + \sigma_{12} M_{12}) + M_{12}(\sigma_{21} M_{11} + \sigma_{22} M_{12}) & \dots \\ \dots & \dots \end{pmatrix}$$

$$\sigma_{11}^{\text{measured}} = \sigma_y^2 = M_{11}^2 \sigma_{11} + 2M_{11} M_{12} \sigma_{12} + M_{12}^2 \sigma_{22} \quad (\sigma_{12} = \sigma_{21}) \quad (1)$$

Solving σ_{11} σ_{12} and σ_{22} while Matrix elements are known: Needs minimum of three different measurements, either three screens or three different Quadrupole settings with different field strength.

$$\epsilon_{\text{rms}} = \sqrt{\det \sigma} = \sqrt{\sigma_{11} \sigma_{22} - \sigma_{12}^2} \quad (2)$$

Exercise L3: In a transport line for $p = 7.5$ GeV/c protons are two measurement stations. The first is located exactly in the waist of the beam and shows a beam width of $\sigma_y = 3$ mm, the second at a distance of $s = 10$ m shows a width of $\sigma_y = 9$ mm. Assuming no optical elements in this part, calculate the emittance and the normalized emittance of the beam.

No optical elements $\Rightarrow M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$ (3)

Waist $\Rightarrow \alpha = \sigma_{12} = \sigma_{21} = 0$ $\left(= -\frac{1}{2} \frac{d\beta}{ds} \right)$ $(\epsilon_{\text{rms}} = \sqrt{\sigma_{11} \sigma_{22} - \sigma_{12}^2} = \sqrt{\sigma_{11} \sigma_{22}})$ (4)

Momentum $p = 7.5$ GeV/c \Rightarrow relativistic $\gamma \beta \approx 7.5$

Measured width at $s = 0 \Rightarrow (3 \text{ mm})^2 = \sigma_y^2(0) = \sigma_{11}$ (5)

Calculate σ_{22} with width measured at $s = 10$ m and with (1) and $\alpha=0 \Rightarrow$

$$(9 \text{ mm})^2 = \sigma_y^2(10) = M_{11}^2 \sigma_{11} + M_{12}^2 \sigma_{22} = \sigma_{11} + s^2 \sigma_{22} \quad (\sigma_{11} \text{ and } \sigma_{22} \text{ at } s=0)$$
 (6)

with (5) $\Rightarrow \sigma_{22} = \frac{\sigma_y^2(10) - \sigma_y^2(0)}{s^2}$ (7)

With (4) and (7) $\Rightarrow \epsilon_{\text{rms}} = \sqrt{\sigma_{11} \sigma_{22}} = \sqrt{\sigma_y^2(0) \cdot \frac{\sigma_y^2(10) - \sigma_y^2(0)}{s^2}} = \frac{\sigma_y(0)}{s} \sqrt{\sigma_y^2(10) - \sigma_y^2(0)}$

$\epsilon_{\text{rms}} = 2.5 \cdot 10^{-6} \pi \text{ m rad}$ or $\epsilon_{\text{normalized}} = \epsilon_{\text{rms}} \gamma \beta = 19 \cdot 10^{-6} \pi \text{ m rad} = 19 \pi \text{ mm mrad}$

$$\epsilon_{\text{rms}} = 2.5 \cdot 10^{-6} \pi \text{ m rad} \quad \text{or}$$

$$\epsilon_{\text{normalized}} = \epsilon_{\text{rms}} \gamma \beta = 19 \cdot 10^{-6} \pi \text{ m rad} = 19 \pi \text{ mm mrad}$$

Additional exercise: Calculate $\beta(s=0$ and $s=10\text{m})$

Beam width $\sigma_{\text{rms}} = \sqrt{\beta(s) \cdot \epsilon}$

At $s=10$ m: $\sigma^2 = \beta \epsilon \Rightarrow \beta = 32.4$ m

At $s = 0$ m : $\beta = 3.6$ m

Still more:
What is the influence on the emittance ϵ assuming at $s = 10$ m this β , a dispersion of $D = 1$ m and a momentum spread of $\Delta p/p = 10^{-2}$?

$$\epsilon = \frac{\sigma^2 - \left(D \cdot \frac{\Delta p}{p} \right)^2}{\beta} = \frac{81 \cdot 10^{-6} - 1 \cdot 10^{-6}}{32.4} = 2.469 \pi \text{ mm mrad}$$

or $\approx 1\%$ which is less than the typical accuracy of a profile measurement

End of emittance calculations, let's go to some instruments



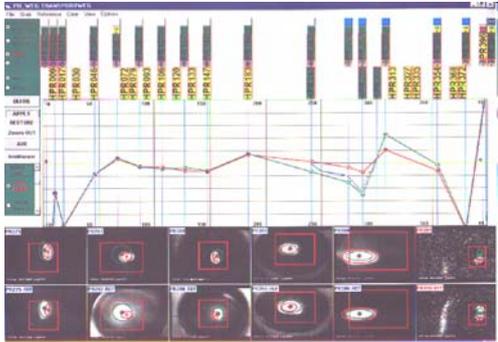
Resolution

The overall resolution is limited by: Depth of Field, Camera (H and V) and lens resolution. Phosphor screen grain size, air waves and mechanical vibration, a factor in measuring beams of tens of micron size, are not significant for beam sizes in the Millimeter range.

Depth of field is a factor because the screen is tilted at 45° with the top at a different distance from the lens than the bottom. This matters if the beam is well off center or large. In the latter case, however, finer resolution isn't required.

Camera resolution is limited by the number of pixels in the array and the readout electronics bandwidth. The overall resolution can be calculated using the manufacturer's data for the cameras and lenses and the parameters of the beam and optical path for each location.

Ref:
 Design of the Beam Profile Monitor System for the RHIC Injection Line *
 R. L. Witkover
 PAC95
<http://accelconf.web.cern.ch/accelconf95/ARTICLES/TPB/TPB26.PDF>




That's the end of the screen session

